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Optimum design criterion and multilevel coding for radio systems over AWGN and Rayleigh fading channels

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Summary

For the narrow band Wireless Code Division Multiple Access (WCDMA) system, there are some channel coding schemes proposed and applied like Turbo code and convolutional codes. But for the 4G Code Division Multiple Access (CDMA) wideband systems, we have to use new channel coding schemes with high bandwidth efficiency. In this case, multilevel coding (MLC) scheme is easy to map to Multiple Quardrature Amplitude Modulation (MQAM) modulation strategy to be used for 4G, and MLC+MQAM will be a potential channel coding scheme for the error correcting of next generation of mobile systems.

A novel criterion, that is 'capacity rule' plus 'mapping rule', for the design of the optimum MLC scheme for radio systems over Rayleigh fading channels is proposed in this paper. Based on this theory, a few of key issues related to design an optimum MLC system are investigated. These include a novel optimum design criterion proposed, different mapping strategies, different decoding methods of MLC/MSD and MLC/Parallel Decoding on Levels (PDL) and their performance comparison over Additive White Gaussian Noise (AWGN) and Rayleigh fading channels respectively. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: multilevel coded modulation; multistage decoding; optimum criterion; mapping strategies; capacity rule; Rayleigh fading channel

1. Introduction

For the narrow band WCDMA system, there are some channel coding schemes proposed and applied, like Turbo code and convolutional codes. But for the 4G CDMA wideband systems, we have to use new channel coding schemes with high bandwidth efficiency. In this case, MLC scheme is one of the best solutions that will be easy to map to MQAM modulation strategy to be used in 4G wireless radio systems.

Set-partitioning strategy, called design criterion, is the key technique in designing an optimum multilevel codes (MLC) system. The traditional set-partitioning strategy is Ungerboeck partitioning (UP) which is known from trellis-coded modulation (TCM) proposed by Ungerboeck [1,2]. It is well known that UP strategy, which enlarges the intra subset's minimum Euclidean distance with each step of set partitioning, constitutes the best criterion for AWGN channels [3]. But for mobile fading channels, this criterion for code optimisation is completely different [4]. Thus, it is of great significance to study another suitable design criterion with some special considerations on different set-partitioning strategies

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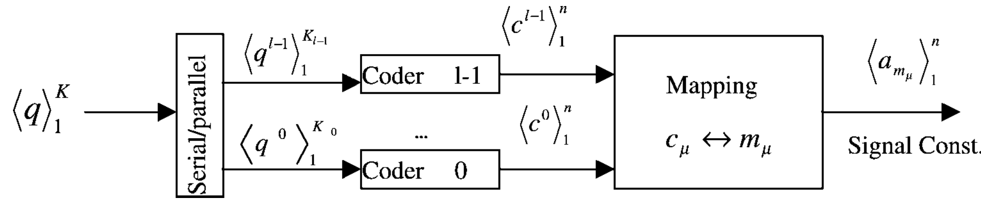


Fig. 1. Multilevel coding scheme.

applied to mobile fading channels and find out the optimum design criterion for mobile fading channels.

A sub-optimal decoding technique called multi-stage decoding (MSD) was introduced in Reference [5] for the decoding of MLC. This decoding procedure is carried out stage by stage and is accomplished by decoding the component codes one at a time. The reliability of MLC system can be improved greatly by using MSD method, which is to decode each component code individually, starting from the lowest level and using decisions of previous decoding stages. Because of the advantages of MSD many publications have concentrated on it [6–12].

Another decoding method for MLC proposed by P. Schramm in 1997 is parallel decoding on levels (PDL) or independent decoding on levels (IDL) [13]. The complexity and time delay of this decoding method is lower than MSD, and it has robustness to different channels [14].

Based on the capacity calculation results we proposed and showed before in references [15–17] (Table II), we are able to design an optimum MLC scheme for AWGN and Rayleigh fading channels respectively. In this paper, we first propose a new 'Non-Euclidean distance' for designing an optimum MLC coded modulation scheme. It is called 'capacity rule +BP (block partitioning)'. And then we compare the different performances of two decoding methods for MLC—MLC/MSD and MLC/PDL in AWGN and Rayleigh fading channels respectively.

The significance of using 8ASK instead of 8PSK is that we are able to extend our results for MLC with 8ASK to MLC with MQAM, for example, 64QAM because 64QAM can be constructed by two components of I and Q branches theoretically. And in the next generation of mobile radio systems with high transmission rate (let us say 4G), we really need some challenge channel coding schemes, with not only power efficiency but also bandwidth efficiency like joint MLC (6 levels) with 64QAM, in which we can get higher bandwidth efficiency up to 6 bits/second!

2. Multilevel Coding Scheme and Capacity Rule

The structure of an MLC system is shown in Figure 1. Through a binary set-partitioning of the signal set $\mathbf{A} = \{a_m \mid m \in \{0, 1, 2, \ldots, 2^{l-1}\}\}$, a mapping $\mathbf{m} \leftrightarrow \mathbf{C}$ of binary labels $\mathbf{C} = (c^0, c^1, \ldots, c^{l-1})$, $c^i \in \{0, 1\}$, $i \in \{0, 1, \ldots, 1-1\}$, to signal points a_m is defined. The subsets of signal points at level i are denoted by the path in the set-partitioning tree from the root to the subsets, i.e.

$$\boldsymbol{A}_{c^0 \cdots c^i} = \{\boldsymbol{a}_m | \boldsymbol{m} \longleftrightarrow (c^0, c^1, \ldots c^i, x^{i+1}, \ldots x^{l-1}), \\ x^j \in \{0,1\}, j \in \{i+1, \ldots, l-1\}\} \tag{1}$$

For conciseness, we restrict our considerations to MLC schemes with binary component codes as shown in Figure 1. A sequence $\langle q \rangle_1^k$ of source data symbols is demultiplexed in l sequences $\langle q^i \rangle_l^{K_i}$ $(i \in \{0, 1, \ldots, l-1\}, \sum_i K_i = K)$, which are fed into individual binary encoders. Then, the encoders produce code sequences $\langle c_1^{in} \rangle$ of uniform length n. The resulting binary labels $c_\mu = (c_\mu{}^0, \ldots, c_\mu^{l-1})$, $\mu = 1, \ldots, n$, are mapped to signal constellation a_{m_μ}. Therefore, the code rate R of the MLC scheme is:

$$R = \frac{k}{n} = \sum_0^{l-1} R_i = \frac{\sum_{i=0}^{l-1} k_i}{n}. \tag{2}$$

For component codes c^i at level i, block codes, concatenated codes or even no code can be chosen. In this paper, we select BCH codes as component codes and take three levels for our computer simulations, i.e. $l = 3$.

2.1. Capacity Rule

Since the mapping M is bijective and hence without loss in the sense of information theory, the mutual information $I(Y; A)$ between the transmitted signal point $a \in A$ and the received signal $y \in Y$ equals the mutual information $I(Y; X^0, X^1, \ldots, X^{l-1})$ between the address vector $X \in \{0,1\}^l$ and the received signal point:

$$I(Y; A) = I(Y; X^0, X^1, \ldots, X^{l-1}), \tag{3}$$

where, we denoted the random variables corresponding to the transmitted and received signal point, the binary address vector and its components by capital letters.

Then, applying the chain rule to the mutual information it yields [11]:

$$I(Y; X^0, X^1, \ldots, X^{l-1}) = I(Y; X^0) + I(Y; X^1 \mid X^0) + \cdots + I(Y; X^{l-1} \mid X^0 X^1, \ldots, X^{l-2}). \tag{4}$$

This equation can be interpreted in the following way: the transmission of vectors with binary digits X^i, $i = 0, 1, \ldots, l-1$, over the physical channel can be virtually separated into the parallel transmission of the digits x^i over ℓ equivalent channels. The equivalent channel i consists of the equivalent mapper i, provided that the digits $X^0, \ldots, X^{i-1}$ and the noise channel are known. The binary symbol x^i is multiply represented in the signal set of the equivalent mapper i for $i < \ell - 2$.

From the chain rule, the mutual information of the equivalent channel i can be calculated by Equation (5):

$$\begin{aligned} &I(Y; X^i, \ldots, X^{l-1}/X^0, \ldots, X^{i-l}) \\ &= I(Y; X^i/X^0, \ldots, X^{i-l}) \\ &\quad + I(Y; X^{i+1}, \ldots, X^{l-1}/X^0, \ldots, X^i). \end{aligned} \tag{5}$$

However, the capacity C^i for given *a priori* probabilities of signal points yields:

$$\begin{aligned} C^i &= I(Y; X^i/X^0, \ldots, X^{i-1}) \\ &= I(Y; X^i, \ldots, X^{l-1}/X^0, \ldots, X^{i-1}) \\ &\quad - I(Y; X^{i+1}, \ldots, X^{l-1}/X^0, \ldots, X^i), \end{aligned} \tag{6}$$

where the mutual information $I(Y; X^i, \ldots, X^{l-1}/X^0, \ldots, X^{i-1})$ is calculated by averaging with respect to all possible combinations of $x^0, \ldots, x^{i-1}$:

$$\begin{aligned} &I(Y; X^i, \ldots, X^{l-1}/X^0, \ldots, X^{i-1}) \\ &= E_{x^0, \ldots, x^{i-1} \in \{0,1\}^i}\{I(Y; X^i, \ldots, X^{l-1}/X^0, \ldots, X^{i-1})\}. \end{aligned} \tag{7}$$

Thus the capacities of the equivalent channel i can be obtained [19,20]:

$$\begin{cases} C^i = E_{x^0, \ldots, x^{i-1}}\{C(\mathrm{A}(x^0, \ldots, x^{i-1}))\} \\ -E_{x^0, \ldots, x^i}\{C(\mathrm{A}(x^0, \ldots, x^i))\}, \quad i = 1, \ldots, l-1 \\ C^0 = C(\mathrm{A}) - E_{x^0}\{C(\mathrm{A}(x^0))\}, \quad i = 0. \end{cases} \tag{8}$$

Based on the concept of the equivalent channels and its capacities, we can easily draw our 'capacity rule' or 'rate rule'. Given a binary digital modulation scheme, choose the rate R^i at the individual coding level i of an MLC scheme to equal the capacity C^i of the corresponding equivalent channel i:

$$R^i = C^i \qquad i = 0, 1, \ldots, l-1. \tag{9}$$

The basis of the capacity rule is to characterize the transmission properties of the equivalent channels by its capacities. Operating at the capacity limit of MLC scheme, the capacity rule provides the maximum individual rates to be transmitted with arbitrarily low error probability. Thus, the design of MLC scheme with an optimum trade-off between power and bandwidth efficiency has to be based on the capacity rule.

3. Different Mapping Strategies

There are three kinds of mapping (set-partitioning) strategies for the given signal constellation. Traditional UP is aimed at maximizing the intra-subset minimum Euclidean distance. As an inverse strategy, we call block partitioning (BP). This scheme minimizes the intra-subset minimum Euclidean distance. Last strategy is a kind of combination of UP and BP strategy called mixed partitioning (MP).

Because BP [19] (Figure 2) goes absolutely in inverse direction compared with UP (and UP has been proved not to be a good and efficient set-partitioning strategy to Rayleigh fading channels), it will be worthily considered as an efficient set-partitioning strategy and a better criterion in designing an optimal MLC system for Rayleigh fading channels. This assumption has been proved by our calculations of capacities for different set-partitioning strategies in both Rayleigh and AWGN channels [15–17].

MP results from a combination. In this paper, it is defined in this way: BP-UP-UP which means the first

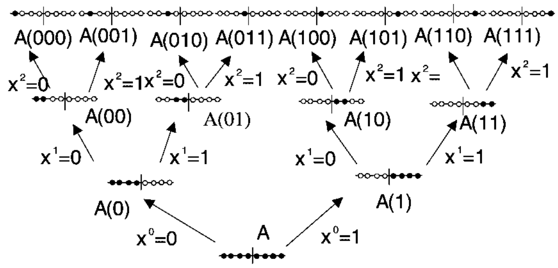


Fig. 2. BP of 8ASK signal set.

Table I. Different rates of MLC/MSD schemes with 8ASK modulation based on capacity rule (R = 2.5 bits/symbol).

	UP	BP	MP
AWGN channel	$C_0 = R_0 = 0.5$ $C_1 = R_1 = 1$ $C_2 = R_2 = 1$	$C_0 = R_0 = 0.95$ $C_1 = R_1 = 0.85$ $C_2 = R_2 = 0.7$	$C_0 = R_0 = 0.875$ $C_1 = R_1 = 0.625$ $C_2 = R_2 = 1$

Table II. Different rates of MLC/MSD schemes with 8ASK modulation based on capacity rule (R = 2 bits/symbol).

	UP	BP	MP
Rayleigh channel	$C_0 = R_0 = 0.3125$ $C_1 = R_1 = 0.75$ $C_2 = R_2 = 0.9375$	$C_0 = R_0 = 0.8125$ $C_1 = R_1 = 0.6875$ $C_2 = R_2 = 0.5$	$C_0 = R_0 = 0.8475$ $C_1 = R_1 = 0.35$ $C_2 = R_2 = 0.8025$

partitioning step is done by the rule of BP and followed by UP and UP again.

From the results of capacities for MLC/MSD scheme with three mapping strategies and 8ASK modulation, the rate design values of MLC/MSD over different channels are obtained. Table I lists the rate design values over AWGN channels when R is 2.5 bits/symbol, while Table II lists the results over Rayleigh channels when R is 2 bits/symbol. Based on our discussions above and the capacity rule described in Equation (9), the two tables are very important and useful in the designing of optimum MLC systems.

4. Different Decoding Methods for Multilevel Coding

For both AWGN and Rayleigh fading channels, optimal decoding of multilevel codes can be performed by a maximum-likelihood (ML) decoder that finds the best input sequence that maximizes the probability of receiving the observed sequence. But this decoder has to work at a very huge complexity when the number of states becomes quite large. In this case, the ratio between performance and decoding complexity is poor even for very simple codes in each level. Thus, good sub-optimal decoding techniques are needed to obtain the good trade-off between performance and complexity.

4.1. Multistage Decoding for MLC (MLC/MSD)

The right side of the chain rule (4) suggests the rule for a low-complex-staged decoding procedure that is well known as multistage decoding (MSD) proposed by Imai in his original work [5]. The component codes C^i are successively decoded by the corresponding decoders D_i, see Figure 3. At stage i, decoder D_i processes depend on not only the block $y = \big(y[1], \ldots, y[N]\big),\ y[\mu] \in \mathrm{Y}$, of received signal points, but also on decisions $x^j, j = 0, \ldots, i-1$, of previous decoding stages j. The use of previous decoding decisions accomplishes the selection of the current subsets of the equivalent mapper i for the different time instants $\mu = 1, \ldots, N$.

Actually, the staged decoding according to the chain rule in Equation (4) would require the transmitted symbol x^j instead of the estimate $\hat{x}^j$. But if we assume error-free decisions $\hat{x}^j = X^j$ of decoder D_j, MSD can be interpreted as an implementation of the chain rule. Clearly, in practice, erroneous decisions occur and errors propagate from lower levels to higher ones. But it is shown later that error propagation in MSD does not significantly influence the performance of the system.

Obviously, multistage decoding is not identical to ML decoding, although each level can and should be ML decoded. Therefore, the performance will be degraded compared to the super-decoder. However, the decoding complexity is significantly reduced because now the complexity is the sum of decoding complexity of each level instead of the product. Of course, additional delay is imposed on the decoding process because the single decoders cannot work in parallel.

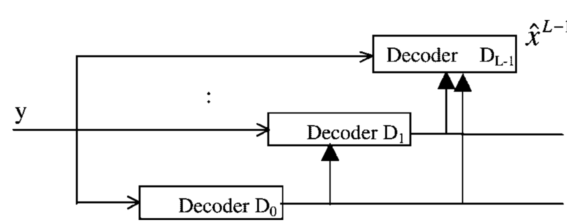


Fig. 3. Multistage decoder (MSD).

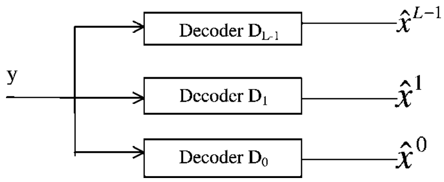


Fig. 4. Parallel decoding of levels (PDL).

4.2. Parallel Decoding for MLC (MLC/PDL)

The use of estimates on lower levels may be unsuitable in practice, e.g. due to memory requirements. In this case, the codes on each level could be decoded independently, i.e. without feedback of estimates. Therefore, an alternative decoding strategy for MLC transmission is parallel decoding of the individual levels (PDL) [13]. Thereby, in contrast to the MSD approach, decoder D_i makes no use of decisions of other levels $i \neq j$. All decoders D_i, $i = 0, 1, \ldots, l-1$, are working in parallel. The PDL approach is sketched in Figure 4. For MLC/PDL, the transmission of each address symbol x^i, $i = 0, 1, \ldots, l-1$, over the equivalent channel i is based on the entire signal constellation, since there is no preselection of signal points at higher levels due to decoding decisions of other levels. Of course, information is lost by not using estimates from lower levels. Thus, the sum of the capacities C_{sum} of all levels is less than (or equal to) the total capacity of the signal set, i.e.

$$C_{\text{sum}} = \sum_i C_i \leq C_{\text{set.}} \tag{10}$$

To be more accurate, the concept of the equivalent channel and its characterising pdf have to be adopted appropriately for an MLC/PDL scheme. While in the case of MLC/MSD, the signal set of the equivalent mapper i is time variant for $i > 0$ depending on the binary digits x^j of lower levels j, $j = 0, \ldots, i-1$, the equivalent mapper i for the MLC/PDL scheme is time invariant for all $i = 0, \ldots, l-1$. Since the decoding at one level is done independently of other levels, the equivalent mappers for MLC/PDL comprise the entire signal set $\mathbf{A}$ in every case. In the signal set of equivalent mapper i, the binary symbol b^i is multiply represented by all signal points with address digit $x^i = b^i$, $b^i \in \{0, 1\}$.

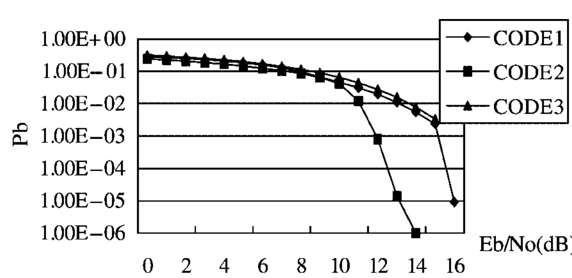
An advantage of the PDL decoding approach is certainly that error propagation from lower to higher levels can be avoided since the levels are decoded independently. Additionally, PDL is favorable in terms of decoding delay since the individual decoders are working in parallel instead of serial in the staged decoding approach MSD. If transmission over time-variant channels is considered, where both the static AWGN and the Rayleigh fading channel are present, MLC/PDL turned out to provide the best robustness to both channel situations among other competing coded modulation schemes.

5. Numerical Results and Discussions

5.1. Non-Euclidean Metric of MLC Scheme for AWGN Channels

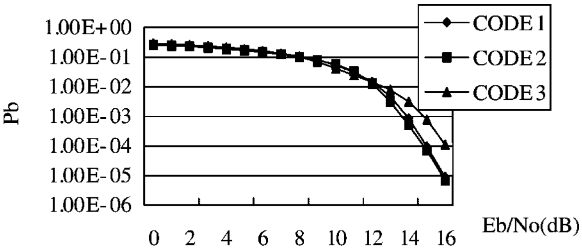
According to the results in Table I, the performance of MLC/MSD scheme in 8ASK signal constellation over AWGN channels is investigated in which BCH codes with code length of 127 are used as component codes. Simulations results are shown in Figures 5–7. There are three MLC/MSD schemes in Figures 5–7, i.e. CODE1, CODE2 and CODE3. For CODE2, the rates of binary component codes on three levels are designed according to 'capacity rule' shown in Table I, while 'capacity rule' is not obeyed for CODE1 and CODE3. For comparison on performance, the total rates of three schemes are all chosen as $R = 2.5$ bits/symbol. From the simulation results, we can see:

(a) The performance of MLC/MSD scheme according to 'capacity rule' is optimal at same bandwidth efficiency. As Figures 5–7 show, CODE2 scheme is superior to CODE1 and CODE3 schemes.
(b) The schemes that do not obey 'capacity rule' have different affection to performance at the same total rates. As CODE1 scheme in Figure 5 shows, the rate R_0 of the first level is greater than the



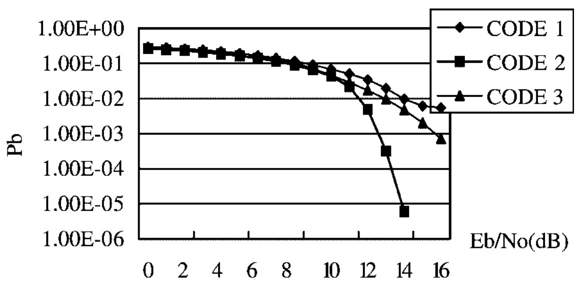
CODE 1-C0: (127,127); C1: (127,57,11); C2: (127,127)
CODE 2-C0: (127,57,11); C1: (127,127); C2: (127,127)
CODE 3-C0: (127,127); C1: (127,127); C2: (127,57,11)

Fig. 5. Performance comparison of different rates (AWGN channel, UP, MLC/MSD£©).



CODE 1-C0: (127,120,1); C1: (127,85,6); C2: (127,106,3)
CODE 2-C0: (127,120,1); C1: (127,106,3); C2: (127,85,6)
CODE 3-C0: (127,106,3); C1: (127,85,6); C2: (127,120,1)

Fig. 6. Performance comparison of different rates (AWGN channel, BP, MLC/MSD£©).



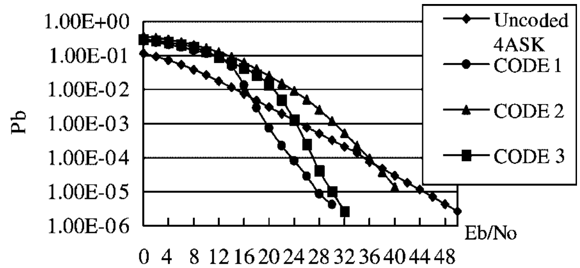
CODE1-C0:(127,78,7);C1:(127,106,3);C2:(127,127)
CODE2--C0:(127,106,3);C1:(127,78,7);C2:(127,127)
CODE3--C0:(127,106,3);C1:(127,127);C2:(127,78,7)

Fig. 7. Performance comparison of different rates (AWGN channel, MP, MLC/MSD£©).

capacity, R_1 of the second level is lower than the capacity, therefore, the performance of CODE1 is inferior to CODE2 scheme. In CODE1 scheme of Figure 6, the rate R_0 equals the capacity, R_1 is lower and R_2 is greater than capacity, therefore, the performance of CODE1 scheme is nearly the same as that of CODE2 scheme. Thus, the rate of the first level R_0 must be designed according to the capacity of equivalent channel comparing with other levels. If R_0 is greater than the capacity, the performance will degrade more greatly.

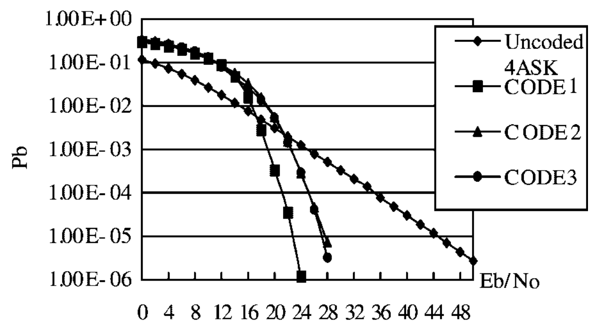
5.2. Non-Euclidean Metric of MLC Scheme for Mobile Fading Channels

In Rayleigh fading channels, the performance of three-level MLC/MSD scheme with BCH codes ($n = 127$) as component codes is investigated. Results are shown in Figures 8–10 in which there are three MLC/MSD schemes. CODE1 scheme is based on 'capacity rule' results shown in Table II, CODE2 and CODE3 schemes are not. The total rates of three



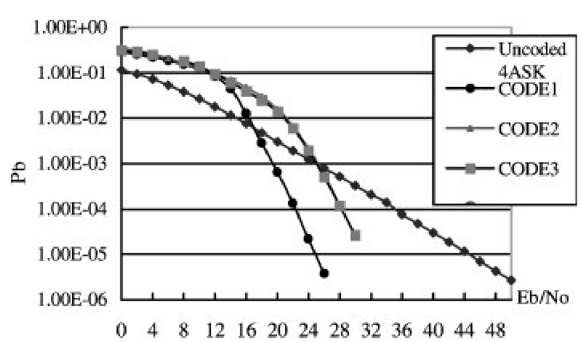
CODE 1:C0(127,36,15);C1(127,99,4);C2(127,120,1)
CODE 2:C0(127,120,1);C1(127,99,4);C2(127,36,15)
CODE 3:C0(127,99,4);C1(127,36,15);C2(127,120,1)

Fig. 8. Performance comparison of different rates (Rayleigh channel, UP, MLC/MSD).



CODE1:C0(127,99,4); C1(127,92,5); C2(127,64,10)
CODE2:C0(127,64,10); C1(127,92,5); C2(127,99,4)
CODE3:C0(127,92,5); C1(127,64,10);C2(127,99,4)

Fig. 9. Performance comparison of different rates (Rayleigh channel, BP, MLC/MSD).



CODE1: C0 (127,106,3); C1 (127,43,14); C2 (127,106,3)
CODE2: C0 (127,43,14); C1 (127,106,3); C2 (127,106,3)
CODE3: C0 (127,106,3); C1 (127,106,3); C2 (127,43,14)

Fig. 10. Performance comparison of different rates (Rayleigh channel, MP, MLC/MSD).

schemes are $R = 2$ bit/symbol. For comparison, the performance of uncoded 4ASK modulation scheme is also given. From the results, we can see:

(a) 'Capacity rule' can be proved with any set-partitioning strategy as well as the results in AWGN channels. When code rates of component codes in MLC scheme are designed according to 'capacity rule', the system performance is optimal. As Figures 8–10 show, CODE1 schemes are always superior to CODE2 and CODE3 schemes.
(b) With any mapping strategy, if code rates of component codes are not designed based on 'capacity rule', rates of different levels can be chosen arbitrarily at given bandwidth efficiency and there is no obvious rule to obtain optimal system performance.

5.3. System Performance Designed According to Capacity Rule

According to 'capacity rule' and the rate design values required according to calculated results in Table I and Table II, the performance of three-level MLC/MSD schemes with three different set-partitioning strategies and two different decoding methods is investigated. In each scheme, BCH codes with code length 127 or 255 are chosen as component codes and 8ASK modulation is used.

5.3.1. *Performance comparison of UP, BP and MP schemes in AWGN channels*

Figure 11 shows the performance comparison of these three set-partitioning strategies in AWGN channels. The code rates of component codes in MLC scheme are designed based on 'capacity rule' calculated and

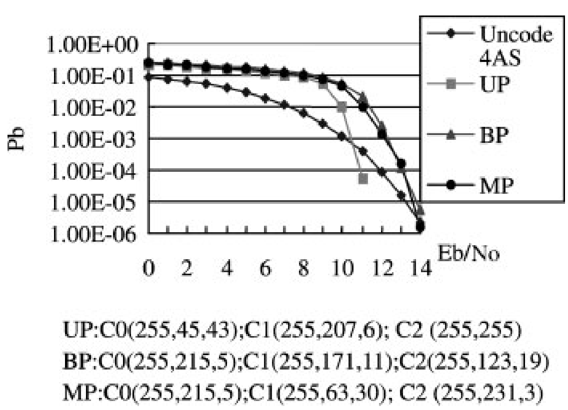


Fig. 11. Performance comparison of MLC/MSD with UP, BP and MP (AWGN channels).

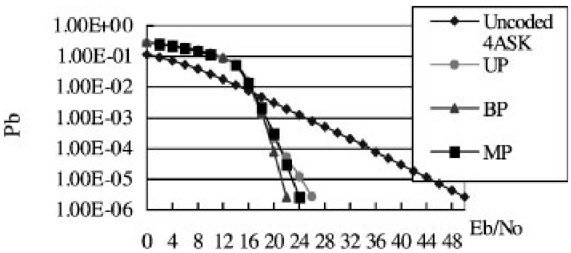
shown in Table I. And, the total rate of three schemes is chosen as $R = 2$ bits/symbol in order to compare power efficiency at same bandwidth efficiency. The performance of uncoded 4ASK scheme is also given for comparison. From the simulation results, we observe the following:

(a) In AWGN channels, the performance of MLC system with UP is optimal as compared to BP and MP according to individual 'capacity rule' at the same bandwidth efficiency. The performance of MP is a little superior to BP.
(b) The performance of three set-partitioning strategies is nearly the same at lower signal-to-noise ratio (SNR) when E_b/N_0 is lower than 9 dB; with the increase of SNR ($E_b/N_0 > 9$ dB), the performance difference of these three schemes will be larger. As Figure 11 shows, the performance of UP scheme is better than BP and MP schemes by 2–3 dB coding gain when $P_b = 10^{-4}$. When SNR is between 9 dB and 13 dB, the performance of BP scheme is superior to MP scheme and MP scheme is superior to BP scheme at higher SNR ($E_b/N_0 > 13$ dB).
(c) Because ASK modulation is amplitude modulation, the performance of uncoded 4ASK scheme is not bad in AWGN channels because of the less interference. As Figure 11 shows, only UP scheme with 8ASK signal constellation has coding gain as compared to uncoded 4ASK scheme. BP and MP schemes would not have coding gain until very high SNR. Therefore, it has little significance to employ BP and MP schemes in MLC system with 8ASK signal constellation over AWGN channels. Thus, the importance of the selection of set-partitioning strategies in MLC system can be proved.

5.3.2. *Performance comparison of UP, BP and MP schemes in Rayleigh fading channels*

Figure 12 shows the performance comparison of MLC/MSD schemes with three set-partitioning strategies in Rayleigh fading channels, in which 8ASK signal constellation is used and BCH codes are chosen as component codes. Table II is used to design the optimal MLC systems with three different mapping strategies.

(a) From the analysis in Section 3, we know that UP is the best set-partitioning strategy for AWGN channels but not for Rayleigh fading channels.



UP:C0(255,79,27);C1(255,191,8); C2(255,239,2)
BP:C0(255,207,6);C1(255,171,11);C2(255,131,18)
MP:C0(255,215,5);C1(255,87,26); C2(255,207,6)

Fig. 12. Performance comparison of MLC/MSD with UP, BP and MP (Rayleigh channels).

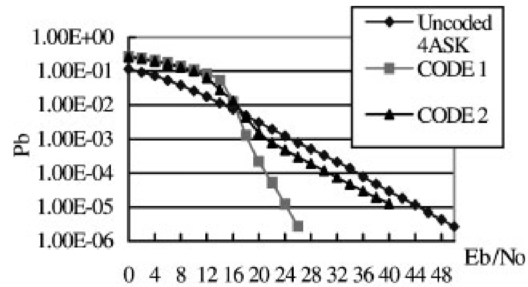
Because BP goes in absolute inverse direction as compared to UP, it might be a good method for Rayleigh fading channels. Here, this assumption is proved. As Figure 12 shows, MLC system with BP is optimal in Rayleigh channels, while UP is inferior to BP and MP schemes.

(b) The system performance designed according to 'capacity rule' is improved greatly with high coding gains. When the code length is 255, the MLC scheme with BP strategy and 8ASK modulation has coding gain of 23.5 dB compared with uncoded 4ASK system.

5.3.3. The Robustness of BP strategy in AWGN and Rayleigh fading channels

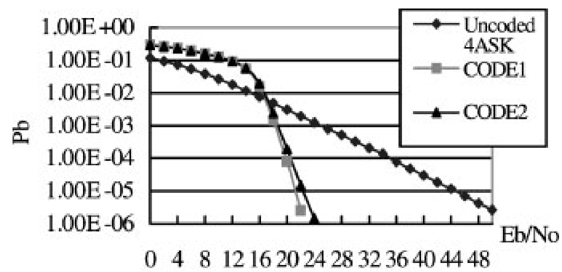
Comparing the results we got in Table I in Reference [15] with three different set-partitioning strategies, it is easy to find that BP is a kind of robust set-partitioning strategy from the point of view of 'capacity rule' in all of the three. The reason is that you can choose almost the same design rates for three different levels when the channel characteristics are completely changeable from AWGN to Rayleigh fading or *vice versa*. This discovery should be very important for the design of an optimum MLC scheme that is suitable either for AWGN channel or for Rayleigh fading channel.

Figures 13–15 show the performance comparison of MLC/MSD schemes with three different set-partitioning strategies in Rayleigh fading channels. In each scheme, BCH codes with code length of 255 are chosen as component codes and different design rates are used. There are two kind of rates in Figures 13–15. The code rates in CODE1 scheme are designed according to 'capacity rule' for Rayleigh fading channels, while the code rates in CODE2 scheme



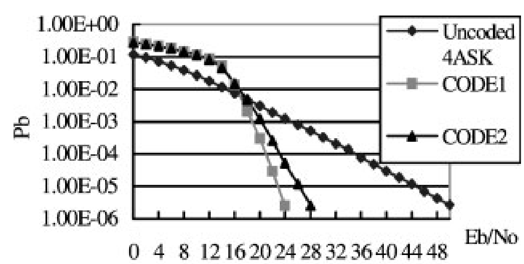
CODE1:C0(255,79,27);C1(255,191,8);C2(255,239,2)
CODE2:C0(255,45,43);C1(255,207,6); C2(255,255)

Fig. 13. Performance comparison of different rate design (Rayleigh channels, UP, MLC/MSD).



CODE1: C0(255,207,6);C1(255,171,11);C2(255,131,18)
CODE2: C0(255,215,5);C1(255,171,11);C2(255,123,19)

Fig. 14. Performance comparison of different rate design (Rayleigh channels, BP, MLC/MSD).



CODE1:C0(255,215,5);C1(255,87,26);C2(255,207,6)
CODE2:C0(255,215,5);C1(255,63,30);C2(255,231,3)

Fig. 15. Performance comparison of different rate design (Rayleigh channel, MP, MLC/MSD).

are distributed as the design values for AWGN channels.

Comparing Figures 13–15, we can see that only for BP strategy in Figure 14, the MLC/MSD scheme (in which the code rates on three levels are designed according to rate design values for AWGN channels) can be also applied to Rayleigh fading channels. That is to say, in Rayleigh fading channels, the performance degradation can be neglected in MLC/MSD scheme only for BP strategy employing the design

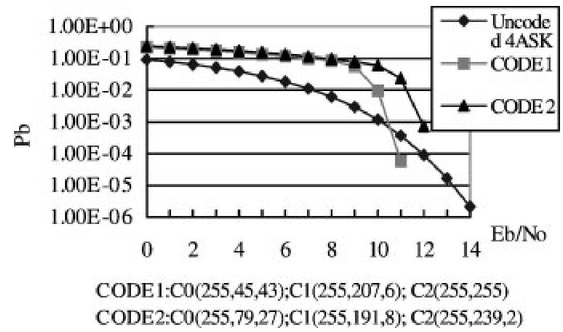


Fig. 16. Performance comparison of different rate design (AWGN channel, UP, MLC/MSD).

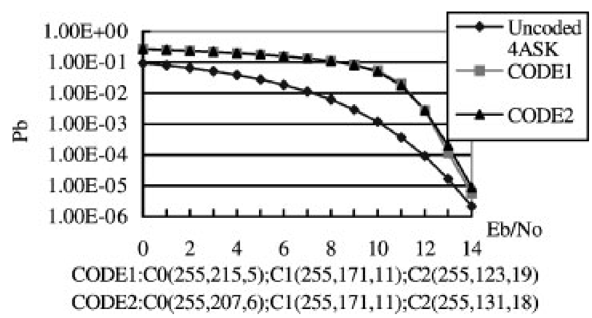


Fig. 17. Performance comaprison of different rate design (AWGN channel, BP, MLC/MSD).

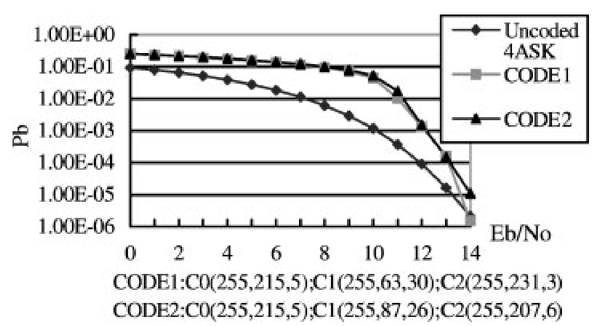


Fig. 18. Performance comparison of different rate design (AWGN channel, MP, MLC/MSD).

rates for AWGN channels. As Figures 13 and 15 show, the performance of MLC/MSD scheme with UP or MP scheme, employing the design rates for AWGN channels, will degrade greatly in Rayleigh fading channels.

The results in Figures 16–18 show the performance comparison of MLC/MSD schemes with different design rates in AWGN channels. The code rates of CODE1 scheme are all designed according to 'capacity rule' for AWGN channels, while the code rates of CODE2 scheme are all distributed as the designed values for Rayleigh fading channels.

Comparing the results, the same conclusion can be drawn: BP strategy is a kind of robust set-partitioning strategy in all the three when the channel characteristics change from AWGN to Rayleigh fading or *vice versa*. Figure 17 shows that the performance of CODE2 scheme is nearly the same as that of CODE1 scheme.

5.3.4. Comparison of MLC/MSD and MLC/PDL for AWGN channels

Simulation results of MLC schemes using two different decoding methods with UP, BP or MP strategy over AWGN channels are shown in Figures 19–21, respectively. In each scheme code, rates of component codes are designed according to 'capacity rule' shown in Table I. For performance comparison, the total rates of scheme are all chosen as: R = 2.5 bits/symbol. From the results, we observe the following:

(a) As shown in Figures 19–21, MSD is superior to PDL for MLC scheme with any set-partitioning strategy over AWGN channels.
(b) For UP and MP strategies, the performance of MLC /PDL and MLC/MSD is nearly the same at lower SNR when E_b/N_0 is lower than 11 dB; With the increase of SNR ($E_b/N_0 > 11$ dB), the performance difference will be larger. As shown in

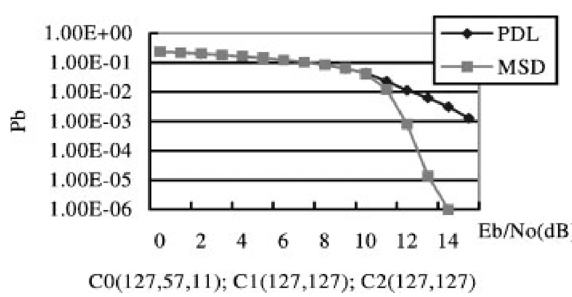


Fig. 19. Comparison of MLC/MSD and MLC/PDL with UP over AWGN channel.

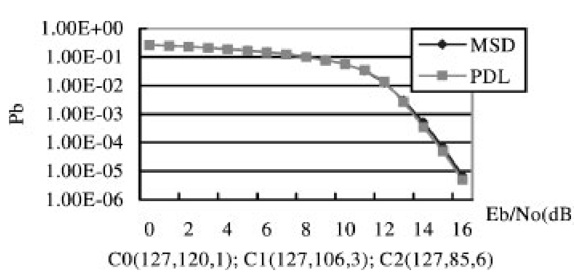


Fig. 20. Comparison of MLC/MSD and MLC/PDL with BP over AWGN channel.

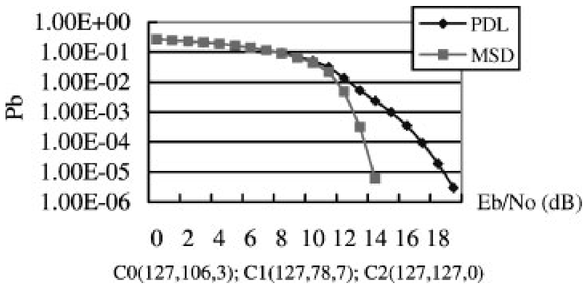


Fig. 21. Comparison of MLC/MSD and MLC/PDL with 8ASK, MP, over AWGN channel.

Figures 19 and 21, the power efficiency of MLC/MSD scheme is higher than MLC/PDL with 3–4 dB coding gain for UP strategy, while with 2–3 dB coding gain for MP strategy when $P_b = 10^{-3}$.

(c) As Figure 20 shows, the performance of MLC/PDL and MLC/MSD schemes with BP strategy is nearly the same at any SNR.

5.3.5. Comparison of MLC/MSD and MLC/PDL for Rayleigh fading channels

Another set of simulations was performed to provide a comparison of MLC/MSD and MLC/PDL for Rayleigh fading channels. For any set-partitioning strategy, code rates of component codes are designed based on 'capacity rule' listed in Table II. Numerical results are depicted in Figures 22–24. BCH codes with code lengths of 255 are chosen as component codes. The total rates of all schemes are all $R = 2$ bit/symbol. For comparison, the performance of uncoded 4ASK modulation scheme is also given. From the simulation results, we observe the following:

(a) For any set-partitioning strategy, MSD is superior to PDL for MLC scheme over Rayleigh fading channels. Therefore, the MLC/MSD scheme is

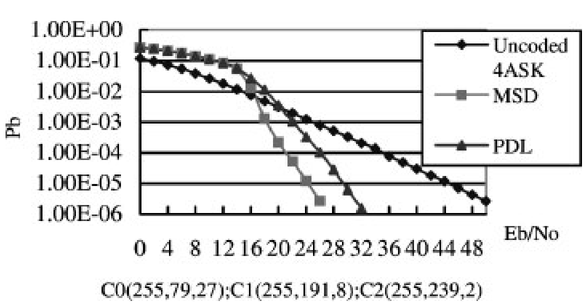


Fig. 22. Comparison of MLC/MSD and MLC/PDL with UP over Rayleigh channel.

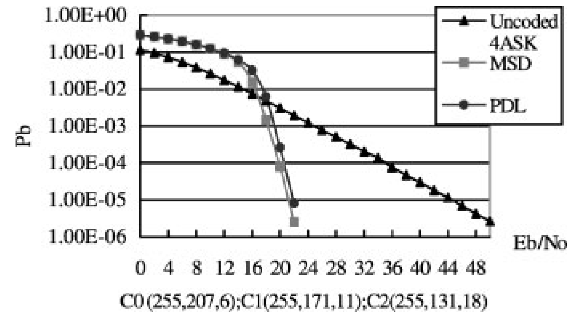


Fig. 23. Comparison of MLC/MSD and MLC/PDL with BP over Rayleigh channel.

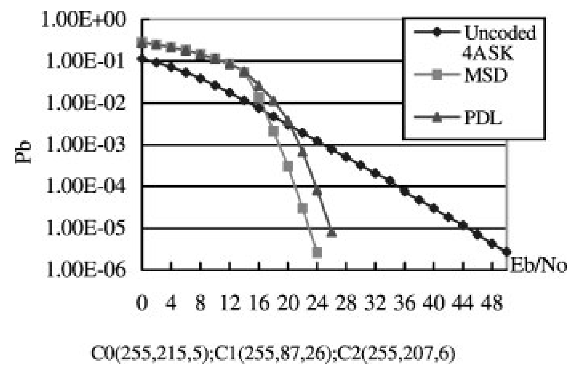


Fig. 24. Comparison of MLC/MSD and MLC/PDL with MP over Rayleigh channel.

proved to be an asymptotically optimum approach to coded modulation for both AWGN and Rayleigh fading channels. The condition for this optimality is that the rates of the component codes are chosen to be equal to the capacities of the equivalent channels.

(b) As shown in Figures 22 and 24, for UP and MP strategies, the power efficiency of MLC/MSD and MLC/PDL is nearly the same at lower SNR when E_b/N_0 is between 4 dB and 16 dB. At high SNR, the performance difference will be larger. When $P_b = 10^{-3}$, MLC/MSD scheme is superior to MLC/PDL with 2–4 dB coding gain for MP strategy, as shown in Figure 24.

(c) Figure 23 shows that PDL and MSD lead to approximately the same performance for MLC scheme. Therefore, PDL, the simple and pragmatic decoding method, can be used instead of MSD, the complex and iterative decoding method with long time delay, for MLC system over Rayleigh fading channels when BP strategy is employed. This conclusion has great importance and significance for multilevel coding schemes with more levels, e.g. MLC/MSD scheme with

64QAM, because the complexity and time delay of MSD will be decreased greatly.

6. Conclusions

A novel criterion that is 'capacity rule' plus 'mapping rule' for the design of the optimum MLC scheme over Rayleigh fading channels is proposed in this paper. Based on this theory, a few issues related to designing an optimum MLC system are investigated, which include a novel optimal design criterion proposed, different mapping strategies, different decoding methods of MLC/MSD and MLC/PDL and their performance comparison. From simulation results and discussions, we can draw some conclusions:

(a) In AWGN channels, when code rates of component codes in MLC system are designed based on 'capacity rule' and UP strategy is used, the system performance will be optimum. Therefore, 'capacity rule' plus 'UP' is the 'non-Euclidean metric' for designing an optimum MLC system in AWGN channel, which is identical with the Euclidean criterion of good codes for coded modulation schemes proposed by Ungerboeck for TCM.

(b) In Rayleigh fading channels, as long as BP strategy is used to 8ASK signal constellation and code rates of component codes are designed according to 'capacity rule', the best performance of MLC/MSD scheme can be obtained. Therefore, 'capacity rule' plus 'BP' can be a kind of novel and important 'non-Euclidean criterion' for designing an optimum MLC scheme in Rayleigh fading channels.

(c) For any set-partitioning strategy, MLC/MSD scheme is superior to MLC/PDL over AWGN and Rayleigh fading channels. Therefore, MSD is the sub-optimal decoding method for multilevel coding radio systems.

(d) As long as BP strategy is used, the performance of MLC/PDL is nearly the same as that of MLC/MSD scheme for both AWGN and Rayleith fading channels. Therefore, for mobile fading channels, PDL can be used as a more attractive and simple decoding method instead of MSD for MLC system. This conclusion has great significance for designing the MLC system with higher bandwidth efficiency, e.g. there are more than three levels in MLC radio systems.

(e) The performance of MLC scheme with different decoding methods is related to set-partitioning strategies. For UP and MP strategies, MSD method is strongly recommended to be used because the performance of MLC/MSD scheme is much better than that of MLC/PDL. For BP strategy, PDL is suggested to be used as a simple decoding method because the performance of MLC scheme with two decoding methods is nearly the same.

(f) The selection of set-partitioning strategies has great importance and significance to MLC schemes. In AWGN channels, when code rates of component codes in MLC/MSD system are designed based on 'capacity rule', and UP strategy is used, the system performance will be optimum. While in Rayleigh fading channels, BP is the best set-partitioning strategy as compared to UP and MP strategies.

(g) From the point of view of 'capacity rule', BP set-partitioning strategy is of good robustness in both kinds, i.e AWGN and Rayleigh fading channels to realize and design an optimum MLC system. Therefore, 'capacity rule' plus 'BP' can be a kind of new and important 'non-Euclidean criterion' for the optimal anti-interference MLC scheme suitable for mobile radio fading channels.

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