

Predicting Burst Error Statistics of Digital Wireless Systems with HARQ

(Invited Paper)

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Abstract—Hybrid Automatic Retransmission reQuest (HARQ) is an effective technique to improve the reliability of wireless communication systems by detecting, correcting, and retransmitting the erroneous packets. Packet-level error sequences obtained from physical layer wireless communication systems are important for the design and performance evaluation of high layer protocols, e.g., HARQ. In this paper, we utilize the open source Vienna long-term evolution (LTE) simulator to study the impact of HARQ on the burst error statistics of packet-level error sequences. Moreover, we propose a generative model that can generate packet-level error sequences with predicted burst error statistics similar to those of error sequences obtained from wireless systems with HARQ. Simulation results demonstrate that the proposed generative model is accurate and efficient in predicting the behavior of HARQ in terms of a set of burst error statistics rather than predicting the packet error rate (PER) only.

Index Terms—Burst error statistics, error models, generative models, link error prediction, HARQ.

I. INTRODUCTION

LTE systems employ HARQ in order to effectively detect and correct errors occurred in wireless channels and hence enhance the throughput performance of the system. HARQ consists of error detection, Forward Error Correction (FEC), and the well-known ARQ, e.g., with an N -channel Stop-And-Wait (SAW) protocol [1]. There are three types of HARQ, namely packet combining or Chase Combining (CC), full Incremental Redundancy (IR), and partial IR [2]. By using the Cyclic Redundancy Check (CRC), erroneous packets can be detected and a request for a retransmission is sent to the transmitter. The retransmission can be a duplicate packet or just some redundancy bits that are combined with the erroneous packet so that it can easily be corrected by the FEC. The LTE system utilizes the full IR HARQ with 1/3 turbo encoder [3]. Full IR HARQ decreases the coding gain in each retransmission by retransmitting only redundancy bits, which will be combined with the stored erroneous packet in the receiver buffer. The retransmission continues when needed until the packet is successfully decoded at the receiver or the maximum number of retransmissions is reached. In the latter case, the packet is discarded and is recorded as a packet error.

Models that can address the behavior of packet error sequences in terms of their occurrence and distribution are called packet-level error models. They have two types, namely descriptive [4], [5] and generative [6]–[15]. Descriptive error models analyze error sequences obtained from a real system or an equivalent computer based system. Generative models are mechanisms to create error sequences with burst error statistics similar to those of error sequences obtained from real systems. The need for generative models is due to the fact that they can significantly reduce the time required to obtain error sequences from real systems. Receiving thousands of packets in the LTE system requires several hours. However, generative models create a packet error sequence with the same length as the one obtained directly from the LTE system in seconds. Furthermore, using a generative model allows us to produce error sequences with different lengths according to the application requirement. Very long generated error sequences are essential to design and evaluate some physical layer components, e.g., interleaving and channel coding schemes, and higher layer protocols [16]–[18].

In this paper, we obtain packet error sequences utilizing the Vienna LTE simulator [19], [20]. We first statistically analyze the obtained error sequences without considering HARQ. Then, we attach HARQ to the physical layer and investigate the statistical properties of the resulting error sequences. Moreover, we propose a prediction generative model that is capable of creating packet error sequences with similar burst error statistics to those obtained from the LTE Vienna simulator having HARQ from error sequences that do not take into account the effect of HARQ. In other words, the newly developed prediction generative model can predict the performance of HARQ in terms of statistical metrics called burst error statistics. In the literature [21]–[24], researchers have predicted the performance of HARQ in terms of PER only, which does not require generating error sequences. However, we here predict more statistics of the HARQ performance that require error sequences generation.

The remainder of this paper is organized as follows. Section II introduces definitions related to packet error sequences and some related burst error statistics. Section III explains the

novel prediction generative model. The used LTE simulator together with the simulation results and discussions are shown in Section IV. Section V concludes the paper.

II. TERMS AND PARAMETERS FOR PACKET ERROR SEQUENCES

An error sequence of a digital wireless channel can be obtained by comparing the digital output sequence with the input sequence. We consider a packet error sequence here as a sequence of “0”s and “1”s. If the packet is received correctly, either from the first transmission or after the retransmission, we denote it as “0” in the error sequence. Otherwise, the packet is discarded and denoted by “1”. In the following, we define some terms related to packet error sequences, as given in [12], [13]. A *gap* is defined as a string of consecutive zeros between two ones, having a length equal to the number of zeros [25], [26]. An *error cluster* (EC) is a region where the errors occur consecutively and has a length equal to the number of ones [7]. An error burst is a combination of error clusters and gaps of length less than η . However, an error-free burst is an all-zero sequence with a length of at least η packets. It should be noted that a packet error sequence consists of successive error clusters and gaps, or successive error bursts and error-free bursts [14].

For the purpose of analyzing error sequences, many burst error statistics have been introduced in the literature [12], [13]. These statistics can quantify the structural differences of error sequences. Therefore, they are essential in evaluating the performance of the HARQ. In the following, we list some burst error statistics for packet error sequences.

- 1) $G(m_g)$: the gap distribution (GD), which is defined as the cumulative distribution function (CDF) of gap lengths m_g . This statistic gives some indication of the randomness of the channel [26].
- 2) $P(0^{m_0}|1)$: the error-free run distribution (EFRD), which is the probability that an error packet is followed by at least m_0 error-free packets [7]. The EFRD can be calculated from the GD [26]. Obviously, $P(0^{m_0}|1)$ is a monotonically decreasing function of m_0 such that $P(0^0|1) = 1$ and $P(0^{m_0}|1) \rightarrow 0$ as $m_0 \rightarrow \infty$. This statistic is very useful in determining the minimum error-free burst length η .
- 3) $P(1^{m_c}|0)$: the error cluster distribution (ECD), which is the probability that a correct packet is followed by m_c or more consecutive packets in error [7]. This statistic distinguishes between the bursty channels and random channels.
- 4) $P(m, n)$: the block error probability distribution (BEPD), which is the probability that at least m out of n packets are in error. This statistic is important for determining the performance of block coding schemes and hybrid automatic repeat request (HARQ) protocols [25].
- 5) $\rho(\Delta k)$: the packet error correlation function (PECF), which is the conditional probability that the Δk th packet following an error packet is also in error [12]. The PECF is important to assess the burstiness of the channel.

III. THE PREDICTION GENERATIVE MODEL

In order to develop the prediction generative model, we first have to understand the effect of HARQ on error sequences, which are obtained without HARQ. Therefore, we obtain error sequences with disabled and enabled HARQ from the LTE simulator and compare between them. Fig. 1 shows the effect of HARQ on an extract of an error sequence. The HARQ breaks the error clusters reducing the error correlation and producing new error rates inside the error cluster blocks of lengths \mathbf{EC}_i ($\mathbf{EC}_1, \dots, \mathbf{EC}_k$), k is the number of error clusters of the original error sequence that does not include HARQ. This property of having many shorter error clusters after including the HARQ, guides us to develop a prediction generative model that can predict the performance of HARQ. In other words, the prediction generative model generates error sequences that cover the physical layer with HARQ from error sequences obtained without considering the HARQ. In this way, we can catch the statistical behavior of the HARQ.

To characterize the effect of adding HARQ to the physical layer, the following steps are considered:

1. we extract the k error clusters from the original error sequence which does not include HARQ.
2. we work out the error cluster lengths recorder \mathbf{EC}_i and gap lengths recorder \mathbf{G}_j ($\mathbf{G}_1, \dots, \mathbf{G}_{k+1}$).
3. we now consider the effect of HARQ. That means some errors are corrected in each \mathbf{EC}_i . Therefore, each original error cluster converts to new smaller error clusters, and new gaps are formed as previously mentioned. In other words, the error rate inside each \mathbf{EC}_i reduces from 1 to a value less than 1. We call the new error cluster lengths $\mathbf{EC}_{i,h}$ ($h = 1, \dots, l_i$), and new gap lengths $\mathbf{EG}_{i,u}$ ($u = 1, \dots, l_i + 1$). Here, l_i indicates the number of new error clusters in each \mathbf{EC}_i .

The aforementioned error rate distribution follows a Gaussian distribution $\mathcal{N}(\mu, \sigma^2)$. We have tested that for different channels, such as the Typical Urban (TU), Pedestrian A (PedA), and Pedestrian B (PedB) channels. Fig. 2 demonstrates the new error rates distribution for PedB. The mean value μ of the Gaussian distribution is the Packet Error Rate (PER) at the specific SNR, such that

$$\mathcal{N}(\mu, \sigma^2) = \mathcal{N}(\text{PER}, \sigma^2). \quad (1)$$

The PER can be calculated by using one of the PER prediction methods in [21]–[24]. These methods do not produce error sequences but predict the PER curves by other mathematical means. For example, we use the Exponential Effective SIR Mapping (EESM) algorithm in [21] to work out the required PER. The basic idea of EESM is to calculate the instantaneous effective SNR δ_{eff} of the AWGN channel that yields the right PER at the given SNR value δ in the LTE wireless channel, such that

$$\text{PER}(\delta) = \text{PER}_{\text{AWGN}}(\delta_{eff}) \quad (2)$$

where

$$\delta_{eff} = -\beta \ln\left(\frac{1}{N} \sum_{k=1}^N e^{\left(\frac{-\delta_k}{\beta}\right)}\right) \quad (3)$$

and N is the number of subcarriers, β is an optimization parameter depending on the code rate, the modulation, and the block size. Note that this method requires the prior knowledge of PER curve of the system with AWGN channel.

Fig. 3 also shows that the number of new error clusters l_i within \mathbf{EC}_i after the retransmission follows the exponential distribution. The mean value of the exponential distribution is:

$$E = \frac{\sum_{i=1}^k \mathbf{EC}_i}{k \times \text{SNR}}. \quad (4)$$

Here, $\text{SNR} \leq 10\text{dB}$ in our examples in order to guarantee sufficient number of error clusters (k) with sufficient error cluster lengths (\mathbf{EC}_i).

As we have recognized the distributions of both the new error rates and the error cluster lengths after the retransmission, we are able to produce any quantity of these error rates and error cluster lengths to construct generated error clusters combined with gaps that together occupy lengths equivalent to \mathbf{EC}_i of the system with HARQ. We rename these lengths as $\mathbf{EC}_{i,HARQ}$. In other words, we produce $\mathbf{EC}_{i,HARQ}$ to replace \mathbf{EC}_i of the original error sequence as of the effect of including the HARQ (see Fig. 1).

The procedure of constructing the $\mathbf{EC}_{i,HARQ}$ is as follows.

1. we randomly choose an error cluster length \mathbf{EC}_i .
2. we choose the number of new error clusters l_i (obtained from the exponential distribution) that can fit within the chosen \mathbf{EC}_i according to a specified error rate (obtained from the Gaussian distribution).
3. we fill in random gaps between these error clusters to complete the required length \mathbf{EC}_i , giving that the first and last parts within \mathbf{EC}_i are gaps. Hence, the structure of the $\mathbf{EC}_{i,HARQ}$ is completed.

To finalize the process of generating a new error sequence that takes into account the effect of HARQ, we replace each \mathbf{EC}_i , which contains ones only, with one of $\mathbf{EC}_{i,HARQ}$, which now contains zeros and ones. Consequently, the gap and error cluster distributions are now completely different from those obtained from the original error sequence.

IV. SIMULATION RESULTS AND DISCUSSIONS

The open source Vienna LTE simulator was used to obtain reference error sequences with the existence and absence of HARQ. The Vienna LTE simulator [20] includes Adaptive Modulation and Coding (AMC), MIMO transmissions, downlink transmission scheme based on Orthogonal Frequency Division Multiple Access (OFDMA). The LTE simulator can also be utilized at both link level and system level.

In this paper, we use the link level simulator which consists of one transmitting eNodeB, one receiver User Equipment (UE), a downlink channel model over which only the

Downlink Shared Channel (DL-SCH) is utilized, signaling information, and an error-free uplink feedback channel with zero delay.

In the transmitter, the user data are prepared as Transport Blocks (TBs). A CRC is derived and appended to each TB. Then, each TB is encoded using a turbo encoder, interleaved, and rate-matched with a target rate depending on the received Channel Quality Indicator (CQI). After these processes, the blocks are modulated and then mapped to the transmission antennas. The LTE simulator utilizes several channel models based on ITU and 3GPP standards.

In the receiver, each UE receives the signals transmitted by the eNodeB and performs signal processing in order to extract the useful transmitted data. Signal demapping, demodulation, and decoding are executed. The CRC of the received packets is calculated in order to check whether the packets are received in errors or correctly. If the received packet is correct, an acknowledgment signal is sent back to the transmitter. Otherwise, the transmitter will send further information, with the help of the HARQ scheme, in order to assist the receiver in correcting the errors. The received packet is dropped once the retransmission fails to correct the errors. The receiver is also capable of estimating the channel from the received data using the reference signals. From the channel estimation, the quality of the channel is evaluated and feedback information is sent in order to help the transmitter cope with the channel impairments by adjusting the transmission parameters.

In the LTE simulator, we use the Pedestrian B (pedB) wireless channel with an average SNR of 10 dB. The user speed is set at 5km/h, the bandwidth is 1.4MHz, and the channel quality indicator (CQI) is set to be 7, 8, and 9. This means that the modulation scheme used is 16QAM with coding rates 0.37, 0.48, and 0.60. Other modulation schemes are also tested. The MIMO configuration is 2×1 . In this paper, we show only the results having CQI of 8. The number of transmitted packets is 500000. For our used parameters, the method of EESM gives us that the PER is 0.38 at 10 dB. The variance in (1) is chosen to be very small, such as 0.02, this value has not been exceeded in our examples. The mean value of the exponential distribution is calculated as 6.

Figs. 4–7 show the burst error statistics with and without HARQ. It is apparent that the error occurrence is severe without HARQ. For example, Fig. 4 is the probability that an error packet is followed by a specific minimum error-free packets. It is apparent that the gap lengths become longer when the HARQ is added and hence the probability becomes higher. Fig. 5 demonstrates the error cluster distribution, the length of error clusters is severely affected when the HARQ is introduced. Moreover, many error clusters are canceled. For Fig. 6 the number of errors counted in blocks of 10 is very large without the HARQ. A dramatic decrease in the number of errors counted in blocks occurs when using the HARQ. Fig. 7 is the most impressive as it is an indicator of the channel burstiness. It shows that the error correlation decreases after the HARQ.

Figs. 4–7 also illustrate the predicted burst error statistics of the newly generated error sequence when the HARQ is activated. The burst error statistics of the generated error sequence have a satisfactory fit with those burst error statistics of an error sequence obtained directly from the LTE simulator having the HARQ included (descriptive model).

It is worth mentioning that receiving 500000 packets in the LTE simulator requires approximately 64 hours using a processor with speed of 2.27 GHz. However, our generated error sequence with the same length as the one obtained directly from the LTE system takes 1.38 s.

V. CONCLUSIONS

We have proposed a prediction generative model that is capable of predicting the statistical behavior of HARQ systems in terms of a set of packet-level burst error statistics utilizing the predicted PER. The burst error statistics are useful in evaluating and designing some digital components in the physical layer and higher layer protocols. We have used the Vienna LTE open source simulator to obtain packet error sequences. The proposed prediction generative model can generate packet error sequences with burst error statistics very similar to those of error sequences obtained directly from the LTE system with HARQ. Importantly, the proposed prediction generative model is not only accurate but also efficient as it considerably reduces the time consumed for generating HARQ error sequences. Such error sequences are consumables for higher layer checks and performance evaluations.

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REFERENCES

[1] M. Döttling, J. Michel, and B. Raaf, "Hybrid ARQ and adaptive modulation and coding schemes for high speed downlink packet access," *Proc. IEEE PIMRC'02*, Lisboa, Portugal, Sep. 2002, pp. 1073–1077.
 [2] B. Kian Chung, A. Doufexi, and S. Armour, "Performance evaluation of hybrid ARQ schemes of 3GPP LTE OFDMA system," *Proc. IEEE PIMRC'07*, Athens, Greece, Sep. 2007, pp. 1–5.

[3] J. C. Ikuno, M. Wrulich, and M. Rupp, "Performance and modeling of LTE H-ARQ," *Proc. WSA'09*, Berlin, Germany, Sep. 2007.
 [4] P. M. Crespo, R. M. Pelz, and J. Cosmas, "Channel error profiles for DECT," *IEE Proc.-Commun.*, vol. 141, no. 6, pp. 413–420, Dec. 1994.
 [5] M. Laner, P. Svoboda, and M. Rupp, "Measurement aided model design for WCDMA link error statistics," *Proc. ICC'11*, Kyoto, Japan, Jun. 2011.
 [6] L. N. Kanal and A. R. K. Sastry, "Models for channels with memory and their applications to error control," *Proc. of the IEEE*, vol. 66, no. 7, pp. 724–744, July 1978.
 [7] B. D. Fritchman, "A binary channel characterization using partitioned Markov chains," *IEEE Trans. Inf. Theory*, vol. 13, no. 2, pp. 221–227, Apr. 1967.
 [8] J. Garcia-Frias and P. M. Crespo, "Hidden Markov models for burst error characterization in indoor radio channels," *IEEE Trans. Veh. Technol.*, vol. 46, no. 6, pp. 1006–1020, Nov. 1997.
 [9] W. Zhu and J. Garcia-Frias, "Stochastic context-free grammars and hidden Markov models for modeling of bursty channels," *IEEE Trans. Veh. Technol.*, vol. 53, no. 3, pp. 666–676, May 2004.
 [10] E. Costamagna, L. Favalli, P. Gamba, and P. Savazzi, "Block-error probabilities for mobile radio channels derived from chaos equations," *IEEE Comm. Letters*, vol. 3, no. 3, pp. 66–68, March 1999.
 [11] A. Köpke, A. Willig, and H. Karl, "Chaotic maps as parsimonious bit error models of wireless channels," in *Proc. IEEE INFOCOM'03*, San Francisco, USA, Mar. 30–Apr. 3, 2003, pp. 513–523.
 [12] C.-X. Wang and W. Xu, "A new class of generative models for burst-error characterization in digital wireless channels," *IEEE Trans. Comm.*, vol. 55, no. 3, pp. 453–462, Mar. 2007.
 [13] C.-X. Wang and W. Xu, "Packet-level error models for digital wireless channels," *Proc. IEEE ICC'05*, Seoul, Korea, May 2005, pp. 2184–2189.
 [14] O. S. Salih, C.-X. Wang, and D. I. Laurenson, "Double embedded processes based hidden Markov models for binary digital wireless channels," in *Proc. ISWCS'08*, Reykjavik, Iceland, Oct. 2008, pp. 219–223.
 [15] G. Hasslinger, and O. Hohlfeld, "Analysis of packet errors in Gilbert-Elliott channels," *Proc. ICC'11*, San Francisco, California, USA, Jun. 2011, pp. 216–220.
 [16] L. Badia, N. Baldo, M. Levorato, and M. Zorzi, "A Markov framework for error control techniques based on selective retransmission in video transmission over wireless channels," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 3, pp. 488–500, Apr. 2010.
 [17] M. Zorzi, R. R. Rao, and L. B. Milstein, "ARQ error control for fading mobile radio channels," *IEEE Trans. Veh. Technol.*, vol. 46, no. 2, pp. 445–455, May 1997.
 [18] G. Mazzini, "Queue system with smart ARQ on Markov channel for energy efficiency," in *Proc. IWCMC'05*, Hawaii, USA, June 2005, pp. 546–551.
 [19] C. Mehlhrer, M. Wrulich, J. C. Ikuno, D. Bosanska, and M. Rupp, "Simulating the long term evolution physical layer," *Proc. EUSIPCO'09*, Glasgow, UK, Aug. 2009, pp. 1471–1478.
 [20] C. Mehlhrer, J. C. Ikuno, M. Simko, S. Schwarz, M. Wrulich, and M. Rupp, "The Vienna LTE simulators - enabling reproducibility in wireless communications research," *EURASIP Journal on Advances in Signal Processing*, vol. 2011, no. 1, pp. 1–13, 2011.
 [21] Y. W. Blankenship, P.J. Sartori, B.K. Classon, V. Desai, and K.L. Baum, "Link error prediction methods for multicarrier systems," *Proc. VTC'04-Fall*, Los Angeles, USA, Sep. 2004, pp. 4175–4179.
 [22] B. Classon, P. Sartori, Y. Blankenship, K. Baum, R. Love, and Y. Sun, "Efficient OFDM-HARQ system evaluation using a recursive EESM link error prediction," *Proc. WCNC'06*, Las Vegas, NV, USA, Apr. 2006, pp. 1860–1865.
 [23] J. Kim, A. Ashikhmin, A. V. Wijngaarden, E. Soljanin, and N. Gopalakrishnan, "On efficient link error prediction based on convex metrics," *Proc. VTC'04-Fall*, Los Angeles, USA, Sep. 2004, pp. 4190–4194.
 [24] J. C. Ikuno, C. Mehlhrer, and M. Rupp, "A novel link error prediction model for OFDM systems with HARQ," *Proc. ICC'11*, Kyoto, Japan, Jun. 2011, pp. 1–5.
 [25] F. Swarts and H. C. Ferreira, "Markov characterization of digital fading mobile VHF channels," *IEEE Trans. Veh. Technol.*, vol. 43, no. 4, pp. 977–985, Nov. 1994.
 [26] S. Tsai, "Markov characterization of the HF channel," *IEEE Trans. Commun. Technol.*, vol. 17, no. 1, pp. 24–32, Feb. 1969.

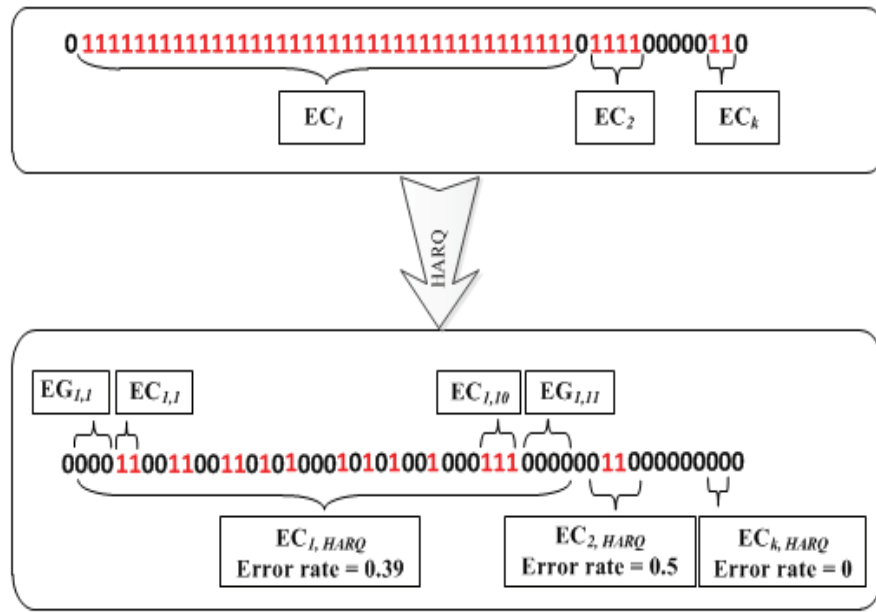


Fig. 1. An error sequence extract to show the effect of adding the HARQ .

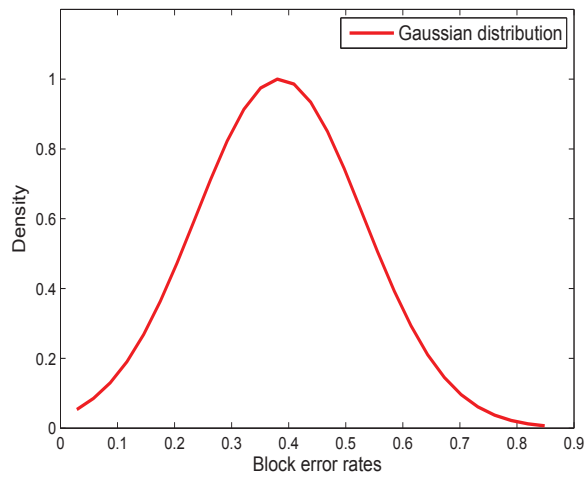


Fig. 2. The density of error rates inside EC_i after using the HARQ.

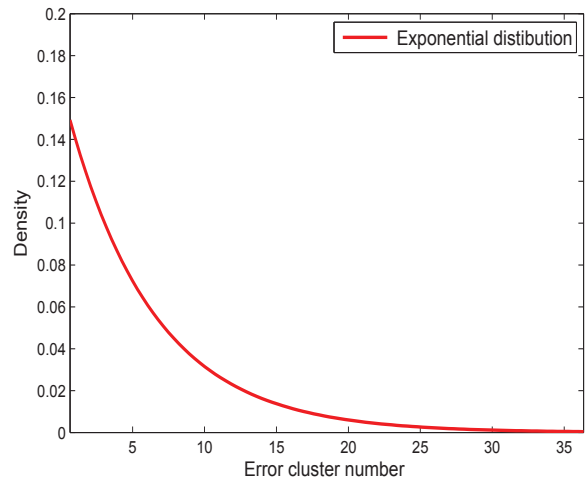


Fig. 3. The density of the number of error clusters inside the EC_i after using the HARQ.

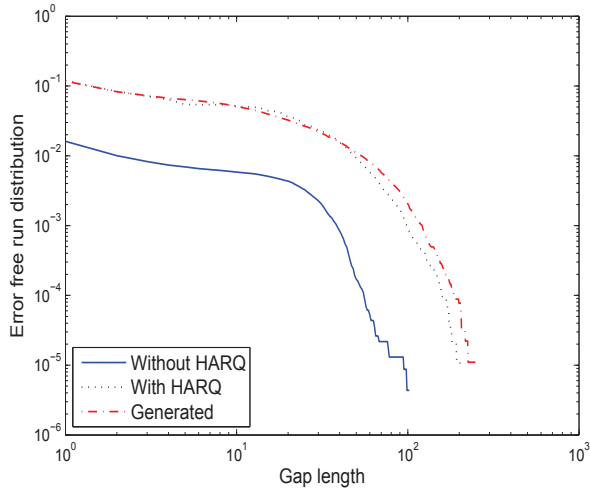


Fig. 4. EFRDs of LTE error sequences with and without HARQ and the HARQ prediction generative model.

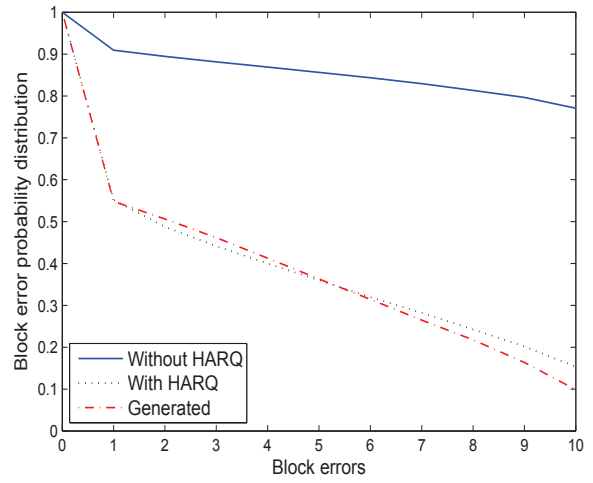


Fig. 6. BEPDs of LTE error sequences with and without HARQ and the HARQ prediction generative model. ($n = 10$).

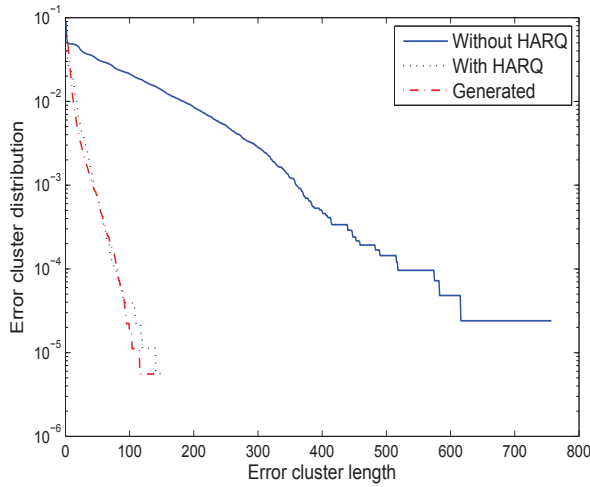


Fig. 5. ECDs of LTE error sequences with and without HARQ and the HARQ prediction generative model.

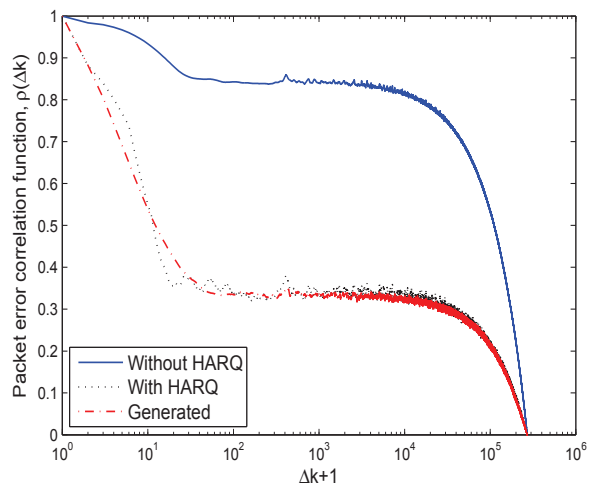


Fig. 7. PECFs of LTE error sequences with and without HARQ and the HARQ prediction generative model.