# Nonlinear Distortion Reduction for the Improvement of the BER Performance in OFDM Communication Systems

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*Abstract* — **Orthogonal frequency division multiplexing (OFDM) has been used widely in many communication systems. However, the major drawback of OFDM is the high peak-to-average-power ratio (PAPR) which results in the bit error rate (BER) performance degradation in nonlinear environments. In this paper, we introduce a new phase weighting method to reduce the nonlinear distortion of OFDM signals in the nonlinear high power amplifier, which is different from the well-known PAPR reduction method. Especially, the proposed method is based on the nonlinear distortion reduction criterion rather than the conventional PAPR reduction criterion so that we can improve the BER performance more effectively. The simulation results show that the BER performance of the OFDM system using the proposed method is better than that of the ordinary OFDM and other OFDM systems using the PAPR reduction criterion.**

*Keywords*: **OFDM, SPW, PAPR reduction, nonlinear distortion, inter-modulation distortion**

## I. INTRODUCTION

In recent years, OFDM has been used widely in digital transmissions and adopted in several communication systems, such as wireless local area network (WLAN) IEEE 802.11a [1], wireless metropolitan area network (WMAN) IEEE 802.16a [2], digital audio broadcasting (DAB), and digital video broadcasting (DVB-T) [3]. OFDM is also a potential candidate for the fourth generation (4G) mobile cellular systems. OFDM provides greater immunity to the frequency selective channel. Because a time domain OFDM signal is a summation of so many orthogonal sub-carrier waveforms, it has a very large fluctuation of the envelope. This phenomenon is called the high PAPR, which is the major drawback of an OFDM system. This high PAPR is a serious problem in a nonlinear environment like HPA. A nonlinear HPA produces the spectral regrowth of an OFDM signal in form of inter-modulation (IMD) among sub-carriers and out-of-band radiation. One technique to reduce the PAPR is the so-called clipping method [4, 5], but this generates another distortion. Block coding can lower down the PAPR below 3 dB [6-9]. However, the loss of code rate and sacrifice of bandwidth are very serious. Partial transmit sequence and selected mapping are known to be efficient [10-13]. These can improve the BER performance in a nonlinear environment but so many inverse fast Fourier transform stages and long processing times are required. In [14-16], the nonlinear distortion was analyzed to derive theoretically the BER performance of an OFDM system. The nonlinear distortion contributes to the calculation of the signal-to-distortion noise ratio in the BER performance analysis. In [15, 16], the authors analyzed the nonlinear distortion in the soft amplitude limiter, whose characteristic was derived from the ideal predistorter before HPA. This analysis can be used as an important criterion to improve the BER performance. In [17], the authors considered the nonlinear environment in which nonlinear characteristics can be expressed as a polynomial and the nonlinear distortion may work as IMD. The IMD is mainly triggered by odd order components in the polynomial expression.

In this paper, we propose a phase weighting technique based on the nonlinear distortion reduction criterion instead of the

PAPR reduction criterion. In nonlinear environments we show that the BER performance of the OFDM system with the proposed method is better than that of the ordinary OFDM and other OFDM systems using the PAPR reduction criterion. The simulation results show that the proposed method does not result in the minimum PAPR of the transmitted signal but it is always better than the PAPR reduction scheme with respect to the BER performance.

## II. OFDM SYSTEM IN NONLINEAR ENVIRONMENTS

OFDM can be implemented easily by FFT and IFFT. Let $X=[X_0,X_1,\ldots,X_{N-1}]^T$ denote the input data after the serial to parallel (S/P) converter. The complex base-band OFDM signal in the time domain is given by

$$x(t)=\frac{1}{N}\sum_{n=0}^{N-1}X_n e^{j2\pi\Delta ft},0\le t<NT \quad (1)$$

where $T$ is the data period, $NT$ is the OFDM symbol duration and $\Delta f=\frac{1}{NT}$ is the sub-carrier spacing. The "L-times over-sampled" time domain signal samples can be obtained from

$$x_k=\frac{1}{N}\sum_{n=0}^{N-1}X_n e^{j2\pi\frac{nk}{LN}} \qquad k=0,1,\ \ldots,LN-1. \quad (2)$$

Let us define the output of the P/S converter as $s_n$. The amplitude of $s_n$ is $r_n\overset{\Delta}{=}|s_n|$ and is a Rayleigh random variable with the probability density function (pdf)

$$f_{r_n}(r_n)=\frac{2r_n}{P_{in}}e^{-\frac{r_n^2}{P_{in}}} \quad (3)$$

where $P_{in}=2\sigma^2$ is the input power of the OFDM signal before the nonlinear device. The output of the nonlinear device can be

$$\tilde{s}_n=g(r_n)=A(r_n)e^{j\Phi(r_n)} \quad (4)$$

Some of the typical nonlinear devices are SAL amplifier, SSPA and TWTA. According to the Bussgang's theorem, the output can be

$$\tilde{s}_n=\alpha s_n+d_n, \qquad n=0,1,\ldots,N-1 \quad (5)$$

where the distortion term $d_n$ is uncorrelated with $s_n$. The attenuation factor $\alpha$ is

$$\alpha=\frac{E\left[s_n^*\tilde{s}_n\right]}{E\left[|s_n|^2\right]}=\frac{E[r_n g(r_n)]}{E\left[|r_n|^2\right]} \quad (6)$$

for SAL and is given by [15]

$$\alpha=1-e^{-\gamma^2}+\frac{\sqrt{\pi}\gamma}{2}erfc(\gamma) \quad (7)$$

where $\gamma\overset{\Delta}{=}\frac{A_{\max}}{P_{in}}$ is the clipping ratio.

The data at the $k$th subcarrier after the FFT is

$$\tilde{X}_k=\sum_{n=0}^{N-1}\tilde{s}_n e^{-j2\pi\frac{nk}{N}}=\alpha\underbrace{\sum_{n=0}^{N-1}s_n e^{-j2\pi\frac{nk}{N}}}_{X_k}+\underbrace{\sum_{n=0}^{N-1}d_n e^{-j2\pi\frac{nk}{N}}}_{D_k} \quad (8)$$

where $D_k, k=0,1,\ldots,N-1$, is the complex distortion term falling on the $k$th subcarrier. The nonlinear distortion to useful signal ratio is defined as

$$I_k=\frac{E[|D_k|]}{E[|\alpha X_k|]} \quad (9)$$

The minimization of this ratio is important for a weighting process to reduce the nonlinear distortion and to improve the BER performance. We assume that the nonlinear distortion is produced only by odd order components:

$$\tilde{s}=\sum_{i=0}^{\infty}a_{2i+1}S^{2i+1}. \quad (10)$$

Useful signal part as $U$ can be

$$U=a_1S \quad (11)$$

The nonlinear distortion part is

$$D=FFT(d)=FFT(a_3s^3+a_5s^5+\ldots) \quad (12)$$
$$=a_3S\otimes S\otimes S+a_5S\otimes S\otimes S\otimes S\otimes S+\ldots$$

So, the nonlinear distortion to useful signal ratio is

$$I_k=\frac{D_k}{U_k} \quad (13)$$

## III. PHASE WEIGHTING BASED ON THE PAPR REDUCTION AND NONLINEAR DISTORTION REDUCTION

In Fig.1, the input data vector after the S/P conversion is multiplied with the factor $b=[b_1,b_2,\ldots,b_M]^T$ . We can employ three kinds of subblock partitioning, namely adjacent, interleaved, and random patterns.

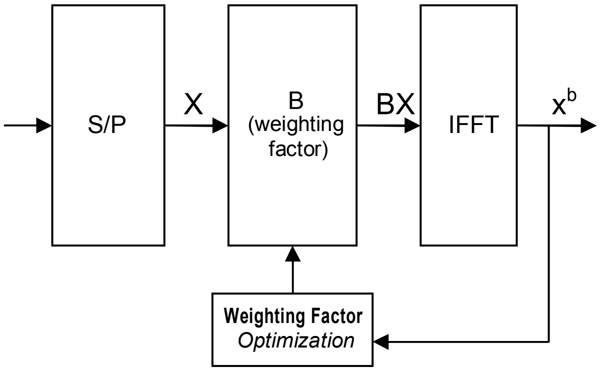

Fig. 1. The block diagram of the SPWmethod.

The multiplication of the weighting factor can be implemented by a matrix B ,

$$B = \left[ diag(\tilde{b}) \right]_{LN \times LN} \tag{14}$$

The weighting factor optimization block aims at finding out the optimized weighting factor in order to minimize the PAPR.

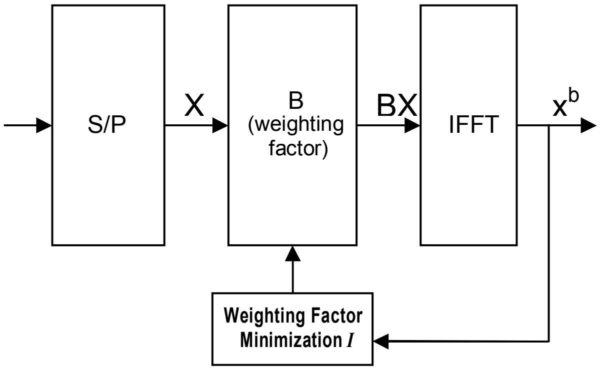

Fig. 2. The SPW technique to minimize the nonlinear distortion.

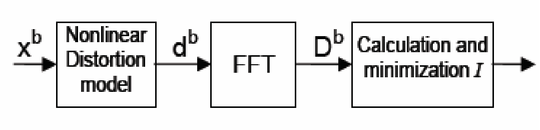

Fig. 3. Calculation of nonlinear distortion *I*.

The nonlinear distortion-to-useful signal ratio *I* is calculated through the iteration of the previous weighting factors, i.e.,

$$I = \left( \frac{D_0^b}{U_0}, \frac{D_1^b}{U_1}, \ldots, \frac{D_{N-1}^b}{U_{N-1}} \right)) \tag{15}$$

Therefore, the optimum weighting factor is found to minimize the maximum of *I*.

$$b^{opt} = \arg \min_{b} \max \left( \frac{D_0^b}{U_0}, \frac{D_1^b}{U_1}, \ldots, \frac{D_{N-1}^b}{U_{N-1}} \right) \tag{16}$$

## IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we compare the proposed method with the PAPR reduction method and ordinary OFDM. The OFDM system has 64 sub-carriers and the used signal constellation is QPSK. The number of sub-blocks for phase weighting is V. For simplicity, the weighting factors are either -1 or +1.

- We use the AM/AM and AM/PM characteristics of the SAL (soft amplitude limiter) model in [5]. In our simulations, $\gamma = 1.2$ holds. *I* is estimated from (13), (14), and (15) in the proposed method.
    - Gain: $G_0$=17dB,
    - Output IP3: $OIP_3$=34dBm
    - $P_{1dB}$: $P_{1dB}$=20dBm

We derive the 5th order polynomial model for MMIC AP112

$$y = a_1 x + a_3 x^3 + a_5 x^5 \tag{17}$$

If the input signal is $A\cos\omega_1 t$ and $A \to 0$, Then

$$a_1 = 10^{\frac{G_0}{20}} \tag{18}$$

where $G_0$ is the gain for small input signal. The coefficient $a_3$ is derived from the concept of $OIP_3$. we have

$$a_3 = -\frac{2}{3} 10^{\frac{3G_0}{20} - \frac{OIP_3}{10}} \tag{19}$$

$$a_5 = -\frac{8 a_1 (1 - 10^{0.05}) - 6 a_3 \beta 10^{0.05}}{5 \beta^2 10^{0.05}} \tag{20}$$

where $\beta = 2 \times 10^{\frac{P_{1dB}+1-G_0}{10}}$. The model for MMIC AP112 is

$$y = x - 0.1995\, x^3 - 0.9292\, x^5 \tag{21}$$

Fig. 4 shows the BER performance of OFDM systems. At BER=$10^{-7}$, the OFDM system having the SAL with the clipping ratio $\gamma = 1.2$ requires the SNR of 17.9 dB, while the OFDM system without the SAL only needs the SNR of 11.3 dB.

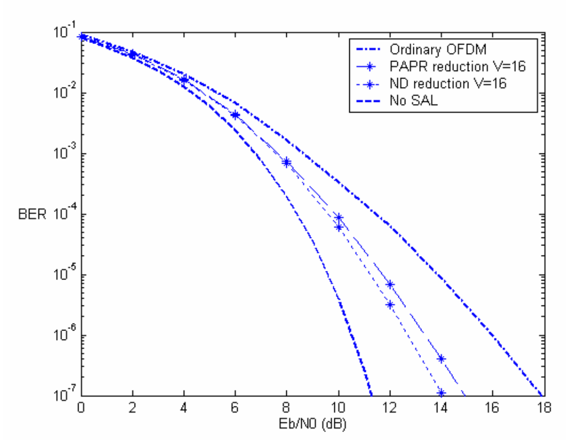

Fig. 4. BER performance at SAL (number of sub-blocks V=16).

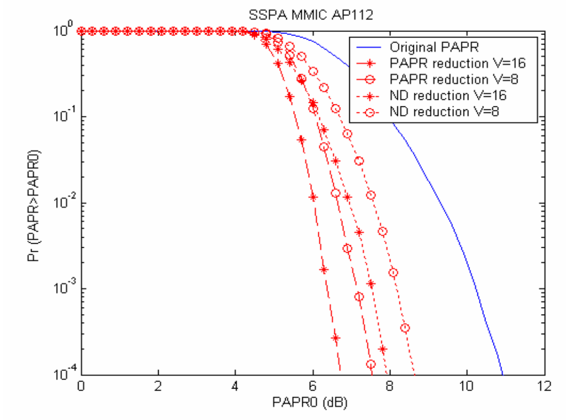

Fig. 5. Comparison of PAPRs at MMIC AP112.

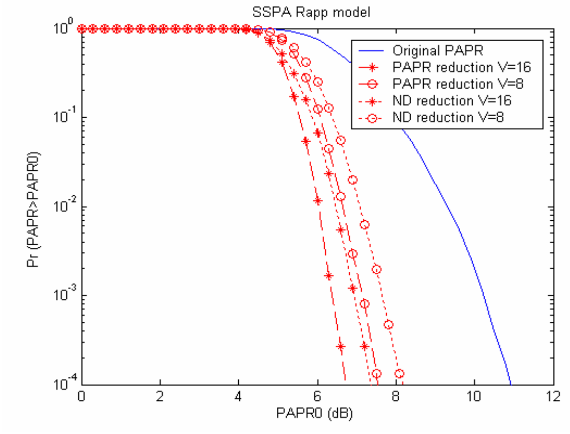

Fig. 6. Comparison of PAPR reduction in SSPA Rapp's model.

Fig. 5-8 show that the transmitted signal of the proposed method does not have the minimum PAPR. In three cases of MMIC AP112 amplifier, SSPA Rapp's model and TWTA Saleh's model, the PAPR threshold of the signal based on the ND reduction is about 1dB higher than that of the PAPR reduction scheme.

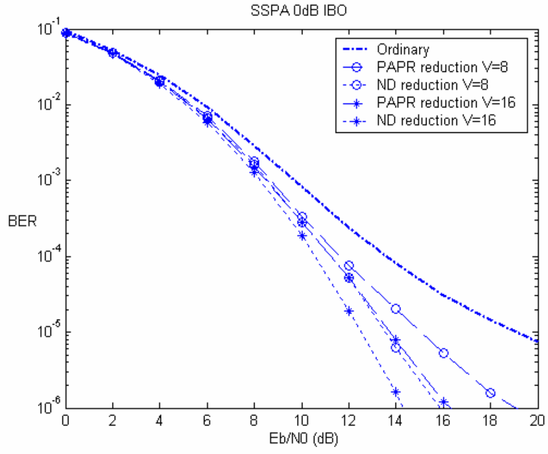

Fig.7. Comparison of BER performance in SSPA Rapp's model (IBO=0dB).

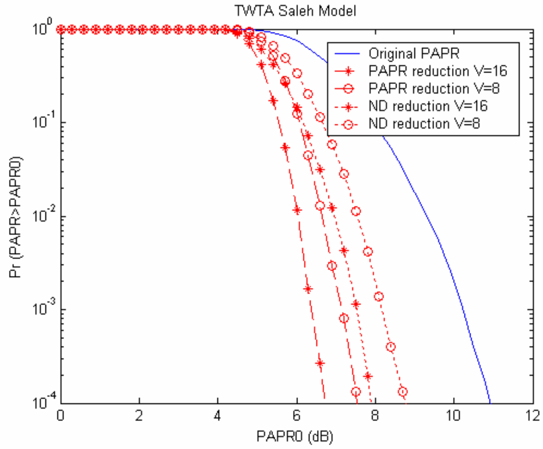

Fig.8. Comparison of PAPR in TWTA Saleh model.

However, the system based on the ND reduction is always better than the system based on the PAPR reduction. For SSPA Rapp's model and TWTA Saleh's model, nonlinear characteristics are expanded by Taylor series. The nonlinear distortion is caused by odd order components. In this paper, we consider the 3rd and 5th order components. In SSPA case, the ND reduction improves 2dB at $10^{-6}$ in BER performance in comparison with the PAPR reduction. Although only AM/AM characteristic of TWTA is concerned, the BER performance is remarkably improved. With IBO=8dB, the BER of ordinary OFDM is saturated at $2 \cdot 10^{-5}$, while is about $10^{-6}$ with the PAPR reduction. The BER of the ND reduction scheme is 2dB better than that of the PAPR reduction scheme at the BER of $10^{-6}$.

## V. CONCLUSION

The proposed technique employs nonlinear distortion reduction instead of the PAPR reduction. Although the transmitted signal of the OFDM system with the proposed method does not have the minimum PAPR, the proposed nonlinear distortion reduction method is always better than PAPR reduction scheme with respect to the BER performance. The BER performance is also improved in the nonlinear environments such as MMIC AP 112, SSPA Rapp's model and TWTA Saleh's model. The simulation results show that the 3rd order is adequate to estimate the nonlinear distortion in the ND reduction scheme.

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