

Performance Analysis of Cognitive Radio Networks with Average Interference Power Constraints

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Abstract—In this paper we study the system level performance of cognitive radio (CR) networks under average received interference power constraints. Under the assumption of uniform node placements and a simple power control scheme, we derive the closed-form expression for the cumulative distribution function (CDF) of the maximum allowable transmit power of the target CR transmitter. We further study two CR network scenarios: a CR based central access network and a CR assisted virtual multiple-input multiple-output (MIMO) network. The average uplink capacities of both networks are derived and analyzed, with an emphasis on understanding the effect of the numbers of primary users and CR users on the capacity. Numerical and simulation results demonstrate that the CR based central access network is more suitable for less-populated rural areas where a lower density of primary receivers is expected, while the CR assisted virtual MIMO network performs better in urban environments with a dense population of mobile CR users.

I. INTRODUCTION

The imbalance between the spectrum scarcity and low spectrum utilization [1] motivates the development of CR [2], [3]. CR aims to improve the spectrum utilization by allowing a secondary network to “borrow” and reuse the radio spectrum from a licensed (primary) network, under the condition that no harmful interference is caused to the incumbent primary services [4]–[6]. A CR user may coexist with the incumbent primary users either on a non-interfering basis [7]–[11] or an interference-tolerant basis [11]–[14]. In the former case, CR users can only operate in the unused frequency bands, also known as *spectrum holes* or *white spaces* [6]. On the other hand, the interference-tolerant case allows the CR users to operate on the frequency band assigned to the primary users as long as the total interference power received at any primary user remains below a certain threshold [11]–[14]. To denote the tolerable interference level or threshold at the primary receiver, the *interference temperature* concept has been introduced in [1]. In this paper, we will restrict our studies to interference-tolerant based CR networks.

The link level performance of an interference-tolerant based CR networks has been analyzed in [12]–[14], under either average interference power constraints or peak interference power constraints. In this paper, however, we focus on the CR network performance at the system level. The adopted system model here consists of multiple CR transmitters and multiple primary receivers coexisting in a single cell with a finite radius. We also consider path loss in the underlying

channels. Path loss preserves the topology information of the network and therefore allows us to study the impact of the network topology, mainly the densities of CR transmitters and primary receivers, on the resulting network capacity. With a simple power control protocol, the constraint on the transmit powers of CR transmitters will be studied first. Such a constraint will then be applied to analyze the capacity limits of two CR network scenarios subject to average interference power constraints. Furthermore, only the uplink performance is analyzed.

The remainder of this paper is organized as follows. In Section II, the transmit power constraints of the CR users are derived assuming a simple power control scheme. Section III describes a central access CR scenario and analyzes the uplink channel capacity. In Section IV, a CR assisted virtual MIMO communication system is addressed followed by an analysis of the uplink channel capacity. Finally, conclusions are drawn in Section V.

II. POWER CONTROL AND CR TRANSMIT POWER CONSTRAINTS

As illustrated in Figs. 1 and 2, multiple CR users are uniformly distributed in a circular cell with radius R . Additionally, we assume that there are N primary receivers, denoted as X_i ($i = 1, \dots, N$), also uniformly distributed in the cell. Within the CR network, we assume that the CR users transmit in orthogonal channels to avoid mutual interferences. In this paper, we use a time division multiple access (TDMA) scheme, which implies that only one target CR user is scheduled to transmit in a given time slot. Although such a TDMA scheme does not necessarily achieves optimum spectrum efficiency, it leads to a simple and practical power control scheme which is explained subsequently.

At any given time, the scheduled/target CR transmitter is the only interference source to the primary network. We refer the underlying channels from the target CR transmitter to the primary receivers as interference channels. The instantaneous channel power gains from the target CR transmitter to the i th primary receiver is denoted as h_i^I . In addition, let P be the transmit power of the target CR user. We assume that an average interference power constraint I_0 applies to the interference powers perceived at all the N primary receivers

in any time slot, i.e.,

$$PE \{h_i^I\} = P\bar{h}_i^I \leq I_0, \quad i = 1, \dots, N \quad (1)$$

where $E\{\cdot\}$ is the statistical average operator and I_0 is the maximum average interference power that a primary receiver can tolerate. For simplicity of analysis, we assume that the averaged interference channel gain \bar{h}_i^I within the given time slot can be described by the path loss given by [16]

$$\bar{h}_i^I = E \{h_i^I\} = \frac{(h_c h_p)^2}{(d_i)^\alpha} \quad (2)$$

where d_i is the distance between the target CR transmitter and the i th primary receiver, $\alpha = 4$ is the path loss exponent, h_c and h_p are the antenna heights of the CR transmitter and the primary receivers, respectively. In this paper, we assume that $h_c = h_p = 1.5$ m. From (1) and (2), we define

$$P_{\max} = \frac{I_0 (d_{\min})^4}{(h_c h_p)^2}. \quad (3)$$

as the maximum allowable transmit power, where $d_{\min} = \min\{d_i\}$ stands for the distance between the CR transmitter and the nearest primary receiver. Note that $d_{\min} \in [0, R + r]$ holds, where r is the distance between the target CR transmitter and the cell center. In the rest of this paper, we assume that the target CR transmitter has an accurate estimate of d_{\min} , which can be obtained by either sensing the beacon signal transmitted from the primary receivers or listening to a common control channel [15]. In what follows, we will first give the probability density function (PDF) $f_{P_{\max}}(x)$ of P_{\max} .

As shown in (3), the random variable (RV) P_{\max} is expressed as a function of another RV d_{\min} . To calculate $f_{P_{\max}}(x)$, let us first study the CDF $F_{d_{\min}}(d)$ of d_{\min} , which can easily be derived as

$$F_{d_{\min}}(d) = 1 - \left[\frac{S(d)}{\pi R^2} \right]^N, \quad 0 \leq d \leq R + r \quad (4)$$

where

$$S(d) = \begin{cases} \pi R^2 - \pi d^2, & d \in [0, R - r], \\ \pi R^2 - \pi d^2 + S_1 - S_2, & d \in (R - r, \sqrt{R^2 - r^2}], \\ \pi R^2 - S_2 - S_1, & d \in (\sqrt{R^2 - r^2}, \sqrt{R^2 + r^2}], \\ S_2 - S_1, & d \in (\sqrt{R^2 + r^2}, R + r]. \end{cases} \quad (5)$$

In (5), S_1 and S_2 are given by

$$S_1 = d^2(\theta_1 - \sin \theta_1 \cos \theta_1) \quad (6)$$

$$S_2 = R^2(\theta_2 - \sin \theta_2 \cos \theta_2) \quad (7)$$

respectively, with

$$\theta_1 = \cos^{-1} \left| \frac{d^2 + r^2 - R^2}{2dr} \right| \quad (8)$$

$$\theta_2 = \cos^{-1} \left| \frac{R^2 + r^2 - d^2}{2Rr} \right| \quad (9)$$

The values of θ_1 and θ_2 are taken in the interval $[0, \pi/2]$. From (3) and (4), it can easily be shown that the CDF

$F_{P_{\max}}(x)$ of P_{\max} is given by

$$F_{P_{\max}}(x) = F_{d_{\min}} \left[\left(\frac{h_c^2 h_p^2 x}{I_0} \right)^{1/4} \right], \quad 0 \leq x \leq P_{\lim} \quad (10)$$

where $P_{\lim} = I_0(R + r)^4 / (h_c h_p)^2$ represents the upper limit of P_{\max} when d_{\min} in (3) takes the largest value $R + r$. From (10), the PDF of P_{\max} is given by $f_{P_{\max}}(x) = dF_{P_{\max}}(x)/dx$.

Fig. 3 shows the theoretical PDF $f_{P_{\max}}(x)$ on a \log_{10} scale as a function of the normalized power x/P_{\lim} with $r/R = 0.5$ and different values of N . The results agree with our intuition that with the increase of the number of primary users N , P_{\max} is decreased. Simulation results obtained with the Monto Carlo method are also shown to justify the theoretical derivations in (4)–(10).

III. UPLINK CAPACITY OF A CR BASED CENTRAL ACCESS NETWORK

In this section, we consider a scenario where CR is used to establish a central access network having a BS and multiple CR users. The scenario is illustrated in Fig. 1. To communicate with the CR BS, the target CR user transmits at the maximum allowable power P_{\max} based on the TDMA scheme, as described in Section II. The channel from the CR transmitter to the CR BS is defined as the CR access channel. The underlying instantaneous channel power gain is denoted by h^A , which can be written as the product of three parts [16]

$$h^A = g_p^A g_s^A g_m^A = \frac{h_b^2 h_c^2}{r^4} g_s^A g_m^A \quad (11)$$

where g_p^A , g_s^A , and g_m^A represent the power gains of path loss, shadowing, and multipath fading, respectively. In (11), r is the distance between the CR BS and the target CR transmitter, h_b and h_c are the antenna heights of the CR BS and CR transmitter, respectively. In this paper, we assume that $h_b = 30$ m and $h_c = 1.5$ m. The shadowing factor g_s^A is taken as a log-normally distributed RV with a standard deviation of 8dB [16] and the RV g_m^A follows an exponential distribution which corresponds to Rayleigh amplitude fading [16]. It follows that the instantaneous uplink channel capacity is given by

$$C_{CA} = W \log_2 \left(1 + \frac{I_0}{I_N} \frac{h_b^2}{h_c^2 r^4} d_{\min}^4 g_s^A g_m^A \right). \quad (12)$$

where W is the signal bandwidth and I_N is the noise plus interference power at the CR BS. In (12), d_{\min} , g_s^A , and g_m^A are independent RVs whose PDFs are known in closed-forms. The ergodic capacity, i.e., the mean value of C_{CA} , is given by a three-fold integration taken over the above three RVs. Such an integration can then be evaluated numerically.

Fig. 4 shows the normalized ergodic capacity $E\{C_{CA}\}/W$ as a function of the primary user number N with $I_0/I_N = 1$, $R = 1000$ m, and different values of r/R . Corresponding Monto Carlo simulation results were also obtained by averaging over 10,000 realizations of the instantaneous capacities calculated from (12). The theoretical results obtained from the numerical integration agree well with the simulation results.

From Fig. 3, we have the following observations. Given the number of primary users N , the ergodic capacity of the uplink CR channel decreases quickly with the increasing r/R . Given r/R , the capacity decreases dramatically with the increasing N . Only with a small number of primary users N , a large capacity can be achieved. This demonstrates that the capacity provided by the CR based central access network is significantly restricted by the number of primary users. As a result, such a scenario is more suitable for less populated rural areas, where the density or the number of primary receivers tends to be small.

IV. UPLINK CAPACITY OF A CR ASSISTED VIRTUAL MIMO NETWORK

The second CR scenario we consider in this section is the so-called CR assisted virtual MIMO communication network. The purpose of utilizing CR here is to improve the radio access ability of a cellular system, e.g., UMTS. MIMO technologies promise significant capacity increases in spatially dispersive channels [17], [18]. However, it is still not feasible to implement a large number of antennas into small-size mobile terminals with sufficient decorrelation among antenna elements. Virtual MIMO communication [19] was proposed as an alternative which emulates a MIMO system by coordinating multiple single-antenna users to form a virtual antenna array (VAA).

This scenario is shown in Fig. 2. The cellular BS is equipped with an antenna array located at the center of a cell with radius R . The mobile terminals are dual-mode devices capable of operating in both cellular bands and CR bands simultaneously. We assume that there are M mobile CR users and N primary users, both uniformly distributed in the cell. The basic idea behind this scenario is to first utilize the ad-hoc CR network for helping a target mobile transmitter to cooperate (using the CR bands) with neighboring mobile terminals to form a VAA. The VAA will then communicate with the cellular BS antenna array in the cellular bands. Such a CR assisted virtual MIMO system is expected to greatly improve the spectrum efficiency of the cellular bands and consequently the cellular system capacity.

A. Virtual MIMO Capacity

Once the target transmitter is allocated with the CR bands, it first determines the maximum allowable transmit power P_{\max} as described in Section II. Then, the CR transmitter broadcasts in the allocated CR band and cooperates with neighboring users that happen to be inside a circle with radius \hat{R} centered on the CR transmitter. Consequently, the number of cooperating users is a random number. Let us denote the number of transmit antenna elements in the VAA as n_T . We arrange the antenna array so that the p th ($1 \leq p \leq n_T - 1$) antenna of the VAA is from the p th cooperating user and the n_T th antenna is from the target transmitter. The PDF of the RV n_T can be easily derived using basic combinatorial mathematics as a function of M (the number of CR users), \hat{R} , and R .

Two phases are needed to complete a uplink transmission. In the first phase, the symbols are transmitted from the target CR user to $n_T - 1$ cooperating CR users through the CR channels. The available CR bandwidth W is divided into $n_T - 1$ channels based on orthogonal frequency division. Also, the maximum allowable power P_{\max} of the target transmitter is allocated to each channel with equal power of $P_{\max}/(n_T - 1)$. Let us define the channels from the target user to cooperating users as cooperation channels. The p th cooperation channel gain is denoted as h_p^C , which is given by an equation similarly to (11). In the second phase, the cooperating users directly amplify the symbols received in the first phase and retransmit them on the cellular channel [20]. The cellular virtual MIMO channel matrix \mathbf{H} are modeled as the composite of a log-normal shadowing process with a standard deviation of 8 dB and an independent Rayleigh fading process with a standard deviation of $\sqrt{2}/2$ for the underlying real Gaussian process.

Follow similar deviations in [21], it can be show that the capacity of the virtual MIMO link is given by [21]

$$C_{VM} = \log_2 \left[\det \left(\mathbf{I}_{n_R} + \frac{E_s}{n_T} \mathbf{R}_{nn}^{-1} \mathbf{H} \mathbf{R}_{ss} \mathbf{H}^H \right) \right] \quad (13)$$

where \mathbf{I}_{n_R} denotes a $n_R \times n_R$ identity matrix, \mathbf{R}_{nn} is the covariance matrix of the noise vectors at the cellular BS given by

$$\mathbf{R}_{nn} = \Omega_0 \left(\mathbf{I}_{n_R} + \frac{E_s}{\Omega_0 n_T} \mathbf{H} \mathbf{R}_{\hat{n}\hat{n}} \mathbf{H}^H \right) = \Omega_0 \mathbf{G} \quad (14)$$

In (14), E_s/Ω_0 is the received SINR at the cellular BS and $\mathbf{R}_{\hat{n}\hat{n}}$ is the covariance matrix of the noise vectors at the VAA given by

$$\mathbf{R}_{\hat{n}\hat{n}} = \frac{I_N}{P_{\max}} \mathbf{diag} [(h_1^C)^{-1}, \dots, (h_{n_T-1}^C)^{-1}, 0] \quad (15)$$

where I_N is the received interference plus noise power at the cooperating CR users, $\mathbf{diag}[\mathbf{x}]$ returns a square matrix whose diagonal entries are taken from the vector \mathbf{x} while other entries are zero.

In most cases, when the channel \mathbf{H} is completely unknown to the transmitter, the vector \mathbf{s} may be chosen to be statistically non-preferential, i.e., $\mathbf{R}_{ss} = \mathbf{I}_{n_T}$ [17]. The normalized capacity of the virtual MIMO channel in the absence of channel knowledge at the transmitter can be obtained as

$$C_{VM} = \log_2 \left[\det \left(\mathbf{I}_{n_R} + \frac{E_s}{\Omega_0 n_T} \mathbf{G}^{-1} \mathbf{H} \mathbf{H}^H \right) \right]. \quad (16)$$

Note that the classical MIMO channel capacity is given by [17]

$$C_{MIMO} = \log_2 \left[\det \left(\mathbf{I}_{n_R} + \frac{E_s}{\Omega_0 n_T} \mathbf{H} \mathbf{H}^H \right) \right]. \quad (17)$$

The comparison of (16) and (17) demonstrates that the virtual MIMO channel capacity differs from the classical MIMO channel capacity by an additional matrix of \mathbf{G}^{-1} .

There are a number of parameters that would affect the instantaneous channel capacity C_{VM} . The system parameters include the cell radius R , the cooperation range \hat{R} , and the

value of I_0/I_N . Here, we assume that $R=1000$ m, $\hat{R}_R=20$ m, and $I_0/I_N = 1$. Other relevant parameters include the received SNR E_s/Ω_0 at the cellular BS, the maximum allowable CR transmit power P_{\max} , and the VAA antenna numbers n_T . It is important to mention that the random variables P_{\max} , n_T , and \mathbf{H} are independent. Taking the mean value of C_{VM} and C_{MIMO} over fading channels \mathbf{H} results in the normalized ergodic virtual MIMO channel capacity $E\{C_{VM}\}$ and real MIMO channel capacity $E\{C_{MIMO}\}$, respectively.

B. Numerical Results

With fixed values of the minimum distance d_{\min} and antenna pairs n_T , Fig. 5 shows the numerical results of $E\{C_{VM}\}$ as a function of the average received SNR. The corresponding results of $E\{C_{MIMO}\}$ are also shown for comparison. For any given multiple antenna pairs ($n_T = n_R > 1$), a relatively large d_{\min} ($d_{\min} = 400$ m as an example here) enables the resulting ergodic virtual MIMO channel capacity to approach closely to the corresponding real MIMO channel capacity, i.e., $E\{C_{VM}\} \approx E\{C_{MIMO}\}$. With a smaller d_{\min} (e.g., $d_{\min} = 100$ m), the virtual MIMO channel capacity is much less than the real MIMO capacity. The multiplexing gain of a MIMO system, also known as the gain in the number of degrees of freedom, can be obtained from the slope of capacity curves at the high SNR regime [17]. From Fig. 5, it is obvious that the multiplexing gain of the virtual MIMO system is reduced as d_{\min} decreases.

On the system level, both the minimum distance d_{\min} and antenna pairs n_T can be treated as random variables. The capacities averaged over d_{\min} and n_T are shown in Fig. 6, as a function of N with different values of M . With the increase of M , the virtual MIMO channel capacity increases and gradually approaches the real MIMO channel capacity. This is due to the fact that when more CR users are located in the cell, there is a higher probability that more CR users will be within the cooperation range. Consequently, the number of antennas for establishing the VAA will be increased, which will further increase the virtual MIMO channel capacity. Interestingly, we can see from Fig. 6 that the virtual MIMO channel capacity decreases slowly with the increasing N . This is different from the CR based central access network, where the capacity decreases dramatically with the increase of N , as shown in Fig. 4. On the other hand, a larger number of CR users M results in a higher virtual MIMO channel capacity. This motivates us to conclude that the CR assisted virtual MIMO network is more suitable for urban areas, where a high density of CR users exists, despite the fact that the number of primary receivers N might also be large.

V. CONCLUSIONS

In this paper, we have studied the system level capacities of interference-tolerant based CR networks under received average interference power constraints. A closed-form expression has been given for the CDF of the maximum allowable CR transmit power. Furthermore, the performance of two CR scenarios, namely the CR based central access network and CR

assisted virtual MIMO network, has been analyzed. Numerical results have shown that the performance of the CR based central access network is sensitive to the number of primary users N , while the performance of CR assisted virtual MIMO network is less sensitive to N but sensitive to the number of CR users M . We therefore conclude that the CR central access network is more suitable for less-populated rural areas, while the CR assisted virtual MIMO communication network performs better in urban environments.

REFERENCES

- [1] Federal Communication Commission, Spectrum policy task force report, Washington DC, FCC 02-155, 2 Nov. 2002.
- [2] J. Mitola and G. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Commun. Mag.*, vol. 6, no. 6, pp. 13–18, Aug. 1999.
- [3] H. H. Chen and M. Guizani, *Next Generation Wireless Systems and Networks*, Chichester: John Wiley & Sons, 2006.
- [4] Federal Communications Commission, Unlicensed operation in the TV broadcast bands, ET Docket No. 04-186, 2004.
- [5] Federal Communication Commission, Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies, NPRM & Order, ET Docket No. 03-108, FCC 03-322, 30 Dec. 2003.
- [6] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Select. Areas Commun.*, vol. 3, no. 2, pp. 201–220, Feb. 2005.
- [7] IEEE P802.22, Functional requirements for the 802.22 WRAN standard, IEEE 802.22-05/0007r48, 29 Nov. 2006.
- [8] S. A. Jafar and S. Srinivasa, "Capacity limits of cognitive radio with distributed and dynamic spectral activity," *Proc. IEEE ICC'06*, Istanbul, Turkey, June 2006, pp. 5742–5747.
- [9] T. A. Weiss and F. K. Jondral, "Spectrum pooling: an innovative strategy for the enhancement of spectrum efficiency," *IEEE Radio Commun.*, vol. 42, no. 3, pp. 8–14, Mar. 2004.
- [10] S. Mangold, A. Jarosch, C. Monney, "Operator assisted cognitive radio and dynamic spectrum assignment with dual beacons – detailed evaluation," *Proc. First Int. Conf. Commun. Systems Software and Middleware*, Delhi, India, Jan. 2006, pp. 1–6.
- [11] D. Ugarte and A. B. McDonald, "On the capacity of dynamic spectrum access enable networks," *Proc. IEEE DySPAN'05*, Baltimore, USA, Nov. 2005, pp. 630–633.
- [12] M. Gastpar, "On capacity under received-signal constraints," *42th Annual Allerton Conf. on Commun., Control, and Computing*, Oct. 2004, pp. 1322–1331.
- [13] M. Gastpar, "On capacity under receive and spatial spectrum-sharing constraints," *IEEE Trans. Inform. Theory*, vol. 53, no. 2, pp. 471–487, Feb. 2007.
- [14] A. Ghasemi and E. S. Sousa, "Capacity of fading channels under spectrum-sharing constraints," *Proc. IEEE ICC'06*, Istanbul, Turkey, June 2006, pp. 4373–4378.
- [15] S. Kim, H. Jeon, H. Lee, and J. S. Ma, "Robust Transmission Power and Position Estimation in Cognitive Radio," *Proc. Intl. Conf. Inform. Networking (ICOIN'07)*, Jan. 2007.
- [16] G. L. Stüber, *Principles of Mobile Communication*, 2nd Edition, Boston: Kluwer Academic Publishers, 2001.
- [17] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge: Cambridge University Press, 2003.
- [18] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Tech. Rep., AT&T Bell Labs*, June 1995.
- [19] M. Dohler, *Virtual Antenna Array*, Ph.D Thesis, King's College London, London, UK, 2003.
- [20] J. N. Laneman, G. W. Wornell, and D. N. C. Tse, "An efficient protocol for realizing cooperative diversity in wireless networks," *Proc. IEEE Intl. Symp. Inform. Theory*, June 2001, pp. 294.
- [21] F. R. Farrokhi, G. J. Foschini, A. Lozano, and R. A. Valenzuela, "Link-optimal space-time processing with multiple transmit and receive antennas," *IEEE Commun. Letters*, vol 5, no. 3, pp. 85–87, Mar 2001.

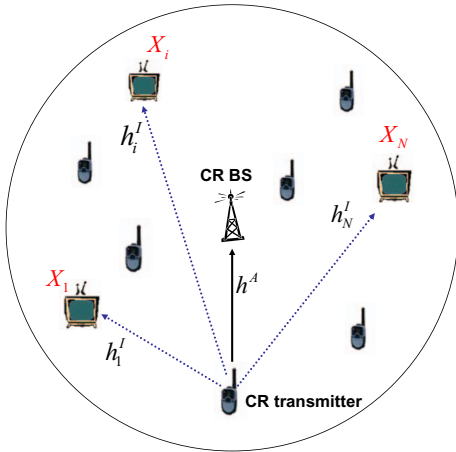


Fig. 1. CR based central access network.

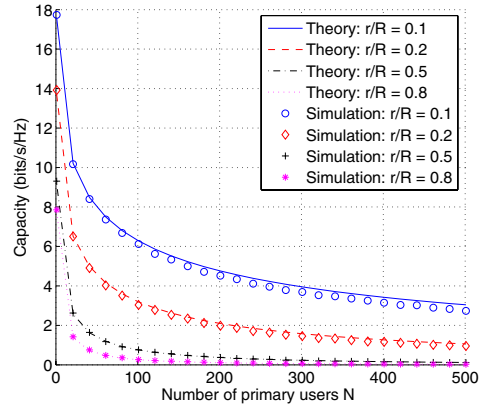


Fig. 4. The uplink normalized ergodic capacity of the CR based central access network as a function of N with different values of r/R ($I_0/N_0=1$, $R = 1000$ m).

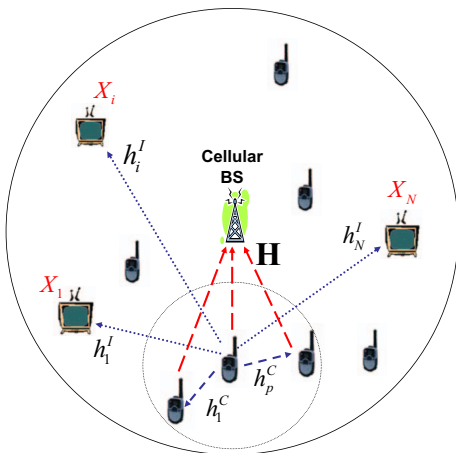


Fig. 2. CR assisted virtual MIMO communication network.

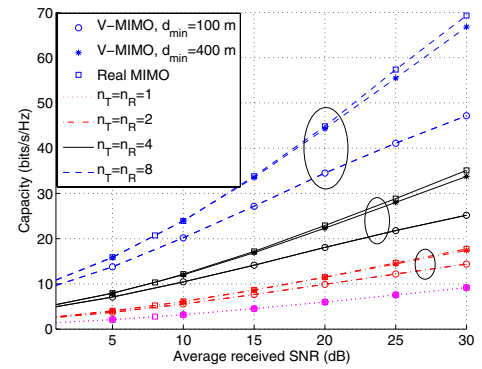


Fig. 5. The uplink normalized ergodic capacity of the CR assisted virtual MIMO network as a function of the average received E_s/N_0 ($I_0/N_0 = 1$).

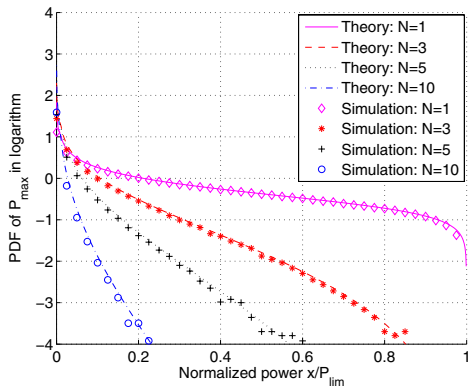


Fig. 3. The PDF of P_{\max} in logarithm $\log_{10}(f_{P_{\max}}(x))$ with $r/R = 0.5$ and different values of N .

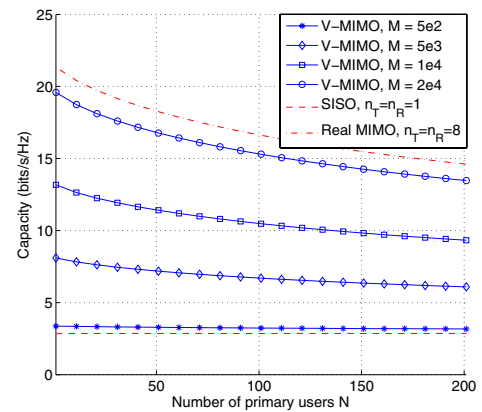


Fig. 6. The uplink normalized ergodic capacity of the CR assisted virtual MIMO network as a function of N with different values of M ($I_0/N_0 = 1$, $E_s/N_0 = 8$ dB, $\hat{R} = 20$ m, $R = 1000$ m).