

Spectral and Energy Efficiency Analysis for Cognitive Radio Networks

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Abstract—Cognitive radio (CR) is considered one of the prominent techniques for improving the utilization of the radio spectrum. A CR network (i.e., secondary network) opportunistically shares the radio resources with a licensed network (i.e., primary network). In this work, the spectral-energy efficiency trade-off for CR networks is analyzed at both link and system levels against varying signal-to-noise ratio (SNR) values. At the link level, we analyze the required energy to achieve a specific spectral efficiency for a CR channel under two different types of power constraint in different fading environments. In this aspect, besides the transmit power constraint, interference constraint at the primary receiver (PR) is also considered to protect the PR from a harmful interference. Whereas at the system level, we study the spectral and energy efficiency for a CR network that shares the spectrum with an indoor network. Adopting the extreme-value theory, we are able to derive the average spectral and energy efficiency of the CR network. It is shown that the spectral efficiency depends upon the number of the PRs, the interference threshold, and how far the secondary receivers (SRs) are located. We characterize the impact of the multi-user diversity gain of both kinds of users on the spectral and energy efficiency of the CR network. Our analysis also proves that the interference channels (i.e., channels

between the secondary transmitter and PRs) have no impact on the minimum energy efficiency.

Index Terms—Cognitive radio networks, spectral efficiency, energy efficiency, extreme-value theory, multi-user diversity gain.

I. INTRODUCTION

THE continuous evolution of wireless communication with more sophisticated technologies has had a massive impact on changing how people on the globe can communicate with each other in all aspects of life including business operations, individuals, and society. There is an increasing number of smart phones and laptops every year. All of them are demanding advanced multimedia and high data rate services. More and more people crave better Internet access on the move resulting in a boundary-less global information world. One way to meet the continuously increasing demand for high-speed data is to secure new spectrum bands. However, achieving this is a very difficult task as the spectrum is a rare resource. Hence, the radio spectrums are congested and there are limited new spectrum bands available for wireless uses. Despite this fact, the federal communications commission (FCC) has reported that a significant amount of the radio spectrum is underutilized during the day [1]. This ignited the research activities to improve the usage of the highly sought-after radio spectrum and as a result, the cognitive radio (CR) concept has been proposed [2], [3].

CR is an innovative radio technique that aims to utilize the radio spectrum more efficiently by intelligently exploiting the licensed spectrum. Hence, a CR network, i.e., secondary network, shares radio spectrum owned by a licensed network, i.e., primary network. The secondary network is authorized of dynamically and autonomously adapting its radio operating parameters to coexist with the primary network, providing that the performance primary network is protected or above a certain level of quality. CR networks can be classified under two categories, namely interference-free and interference-tolerant CR networks [4]. In the former CR networks, secondary transmitters (STs) can only use those spectrums which are not occupied by primary receivers (PRs) [5], [6]. Whereas in the latter CR networks, the STs can share the spectrum as long as they do not cause any outage to the primary network operation and the interference to PRs is kept below a threshold [7]. Therefore, it is essential that interference-tolerant CR systems acquire the interference level, in real-time feedback, from the PRs. To this end, some modification on the primary system is unavoidable. In this paper, we focus on the spectral and energy efficiency for the interference-tolerant CR networks. The spectral efficiency is defined as the number of bits per second transmitted over

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a given bandwidth (in bps/Hz) while the energy efficiency is defined as the required energy per bit (in joules/bit) for reliable communication, normalized to the background noise level.

There are various studies that analyzed the spectral efficiency of CR networks at the link level [8]–[11] as well as at the system level [12]–[14]. For link-level CR networks, the spectral efficiency for additive white Gaussian noise (AWGN) channels was derived in [8] under an assumption of average power constraint. In [9], the spectral efficiency of a CR channel was analyzed against various fading channel distributions. The authors of [10] analyzed the spectral efficiency of a CR channel under different power allocation policies. In [11], ergodic and outage capacities of CR channels were evaluated under both peak and average interference power constraints. The spectral efficiency for both the link level and system level cooperative CR networks was studied in [12]. However, the power control of STs did not consider the interference threshold that the PRs can tolerate. In [13], the authors derived the average throughput of a system level CR network and studied its asymptotic behavior. However, the analysis was limited to a single PR. The spectral efficiency for hybrid CR networks was studied in [14] under average interference power constraint. Hybrid CR allows a network to be simultaneously both primary and secondary networks, thus gaining the advantages of both networks. Hybrid CR networks can be adopted in cellular networks to explore additional bands and enhance the spectral efficiency. It is noticeable that all the aforementioned studies focused on analyzing the spectral efficiency but neglected to study the spectral-energy efficiency trade-off which is an increasingly important area nowadays.

Energy-efficient communication, or as it is globally well-known as Green Radio [15], has been attracting more and more attention in various societies. Studying the energy consumption is crucial to find efficient strategies that minimize the carbon footprint from wireless networks and its impact on the environment [16]–[18]. Jointly attaining both enhanced energy efficiency and spectral efficiency is unfortunately a challenging problem to solve. Often, achieving enhancement of one of them means sacrificing the other. Therefore, it is important to study different trade-offs between the two performance indicators to decide the minimum energy consumption that is required to achieve the target spectral efficiency, or vice versa [20]–[22]. Two analytical tools used to analyze the spectral-energy efficiency trade-off for any given wireless networks were proposed in [19] and [23] in low and high signal-to-noise ratio (SNR) regimes, respectively. These tools were used to analyze the energy consumption in different network scenarios [24]–[30]. Using the low-SNR tool, the interplay of the spectral and energy efficiency was studied for single-user multiple-input multiple-output (MIMO) channels [24], single-user relay channels [25], [26], and multi-user relay channels [27]. The authors of [28], [29] used the high-SNR tool to analyze the energy efficiency of MIMO channels. The works of [24]–[28], [30] can only be applied to primary networks where the transmit power and spectral efficiency depend on the characteristics of the PRs' channels only. In CR networks, however, the network performance is characterized by both the primary and secondary receivers' channels. In [30], we investigated the spectral-energy efficiency trade-off in an interference-

tolerant link-level CR network under an assumption of average transmit power constraint. To extend this work further, we will compare the outcomes of [30] with a different CR power policy. Furthermore, to the best of our knowledge, no other research has considered similar analysis in a system-level CR network with multiple primary and secondary receivers. Therefore, our novel contributions are summarized as follows:

- 1) We extend the work of [30] to compare the spectral-energy efficiency trade-off in the low and high SNR regimes when transmitting a signal under average power constraint with transmitting a signal under peak power constraint while keeping the interference on primary receiver below an acceptance level for both.
- 2) We propose a CR-based cellular network where a secondary network shares a spectrum that belongs to an indoor system. The spectral efficiency for the proposed network with multiple primary and secondary users is analyzed using extreme value theory. The analysis will highlight the impact of the multi-user diversity gain of both the primary and secondary users on the achievable spectral efficiency.
- 3) A general analytical framework to evaluate the energy-spectral efficiency trade-off of CR-based cellular network is established for all SNR values using peak-power interference constraint. The framework takes into account the numbers of primary and secondary receivers, transmit power, and interference threshold.

The remainder of this paper is organized as follows. Section II introduces two analytical tools to study the spectral-energy efficiency trade-off in low and high SNR regimes. Section III describes the link-level model of the proposed study and analyzes the resulting link-level spectral-energy efficiency trade-off, along with numerical results and discussions. The system-level spectral efficiency and energy efficiency of multiple cognitive links are subsequently derived in Section IV. Finally, conclusions are drawn in Section V.

II. SPECTRAL-ENERGY EFFICIENCY TRADE-OFF UNDER LOW AND HIGH SNR REGIMES

In this section, we summarize two tools that analyze the consumed energy per transmitted bit for a given spectral efficiency in low and high SNR regimes [23]. The spectral efficiency, C , here refers to the number of bits per second transmitted over a given bandwidth (in bps/Hz). The energy efficiency, $\frac{E_b}{N_0}$, is defined as the required energy per bit (in joules/bit) normalized to the background noise power N_0 for reliable communications.

A. Low SNR Regime

In the low SNR regime, $\left(\frac{E_b}{N_0}\right)$ can be approximated as an affine function with respect to the spectral efficiency and can be expressed by [19]

$$\left(\frac{E_b}{N_0}\right)\Big|_{\text{dB}} = \left(\frac{E_b}{N_0}\right)_{\min} + \frac{3}{S_0}C \quad (1)$$

where $\left(\frac{E_b}{N_0}\right)_{\min}$ is the minimum energy efficiency required for transmitting information reliably over a channel and it is given by

$$\left(\frac{E_b}{N_0}\right)_{\min} = \lim_{\text{SNR} \rightarrow 0} \frac{\text{SNR}}{\bar{C}(\text{SNR})}. \quad (2)$$

Here, $\bar{C}(\text{SNR})$ is the spectral efficiency as a function of SNR. In (1), S_0 is the wideband slope of the spectral efficiency, defined as the increase of bits per second per hertz per 3 dB (bps/Hz/ (3 dB)), and can be expressed by [19]

$$S_0 = \frac{2\dot{\bar{C}}(0)}{-\ddot{\bar{C}}(0)} \quad (3)$$

where $\dot{\bar{C}}(0)$ and $\ddot{\bar{C}}(0)$ are the first and second derivative, respectively, when $\text{SNR} = 0$.

B. High SNR Regime

In the high SNR regime, the required energy efficiency to obtain a specific spectral efficiency can be expressed by [23]

$$\left(\frac{E_b}{N_0}\right)_{\text{dB}} \approx \frac{C}{S_\infty} 10 \log_{10} 2 - 10 \log(C) + \left(\frac{E_b}{N_0}\right)_{\text{penalty}} 10 \log_{10} 2 \quad (4)$$

where S_∞ is the slope of the spectral efficiency in the high SNR regime in bps/Hz/(3 dB) and is given by [23]

$$S_\infty = \lim_{\text{SNR} \rightarrow \infty} \text{SNR} \dot{\bar{C}}(\text{SNR}). \quad (5)$$

In (4), $\left(\frac{E_b}{N_0}\right)_{\text{penalty}}$ is the horizontal penalty which represents the power offset in dB with respect to a reference channel having the same high SNR regime slope but with an unfaded channel (i.e., AWGN) and it is calculated by [23]

$$\left(\frac{E_b}{N_0}\right)_{\text{penalty}} = \lim_{\text{SNR} \rightarrow \infty} \left(\log_2(\text{SNR}) - \frac{\bar{C}(\text{SNR})}{S_\infty} \right). \quad (6)$$

III. LINK-LEVEL SPECTRAL-ENERGY EFFICIENCY TRADE-OFF

In this section, we consider a link-level CR channel. It consists of an interference-tolerant secondary transmitter-receiver pair that shares a spectrum with a primary transmitter-receiver pair [9]–[11]. We assume a point-to-point flat fading channel that is corrupted by AWGN. All nodes in this model are assumed to be equipped with a single antenna. The channel between the ST and secondary receiver (SR) is defined as the cognitive channel, while the channel between the ST and the PR is defined as the interference channel. The cognitive and interference channel gains are denoted by g_c and g_i , respectively. They are random variables drawn from an arbitrary continuous distribution with an expected value of unity and they are mutually independent. The ST is assumed to have perfect

knowledge of the instantaneous channel status information (CSI) for the cognitive and interference channels. It is further assumed that the interference from the primary transmitter (PT) to the SR is considered as background noise [9], [10]. There are two constraints that the ST has to take into the account before transmitting a signal to the SR. The first constraint is the allowable received peak interference power at the PR. This constraint is essential in CR networks to avoid harmful interference at the PR. The second constraint is the available transmit power that the ST has. In this paper, we consider two types of power constraint which are the average and peak transmit power constraints.

A. Fading Channels With Average Transmit Power Constraint

If we consider a CR channel under the average transmit power and peak interference power constraints, the spectral efficiency in this case can be calculated by [10]

$$C = \max_{\gamma_s(g_c, g_i) \geq 0} \mathbb{E} \left[\log_2 \left(1 + \frac{g_c \gamma_s(g_c, g_i)}{N_0} \right) \right] \quad (7)$$

$$\text{s.t.} \quad \mathbb{E}[\gamma_s(g_c, g_i)] \leq \gamma_{\text{avg}} \quad (8)$$

$$g_i \gamma_s(g_c, g_i) \leq Q \quad (9)$$

where $\gamma_s(g_c, g_i)$, γ_{avg} , and Q are the instantaneous transmit power, allowed average transmit power, and peak received interference power that the PR can tolerate, respectively, and $\mathbb{E}[\cdot]$ denotes the statistical expectation. The optimum power allocation can then be expressed by

$$\gamma_s^*(g_c, g_i) = \min \left\{ \left(\frac{1}{\gamma_0} - \frac{N_0}{g_c} \right)^+, \frac{Q}{g_i} \right\} \quad (10)$$

where γ_0 is the water-filling cutoff value that can be calculated from the constraint in (8) and $(x)^+ = \max\{0, x\}$. Numerical optimization is required to get the optimum value of γ_0 [30]. Unlike in the primary network, where only the CSI of the PR is required at the PT, the CSIs of both the PR and SR are needed at the ST as inputs for the power allocation algorithm. Depending upon the gains of the two types of channel, the CR transmission resides in different modes. No communication is allowed as long as the CR channel gain is below the cutoff value, i.e., $g_c \leq \gamma_0$. The classical water filling algorithm can be adopted if $\left(\frac{1}{\gamma_0} - \frac{N_0}{g_c}\right) \leq \frac{Q}{g_i}$. If $\left(\frac{1}{\gamma_0} - \frac{N_0}{g_c}\right) > \frac{Q}{g_i}$, the transmit power is equal to $\frac{Q}{g_i}$.

Theorem 1: Under average transmit and peak interference power constraints, the minimum energy efficiency required for reliable information over the cognitive channel is given by

$$\left(\frac{E_b}{N_0}\right)_{\min} = \frac{\ln 2}{g_{c(\max)}} \quad (11)$$

Proof: See Appendix A. ■

where $g_{c(\max)}$ is the supremum of a random variable g_c .

Unsurprisingly, the minimum energy is only affected by the cognitive channel while the interference channel has no influence on it. If the cognitive channel, for instance, is an AWGN

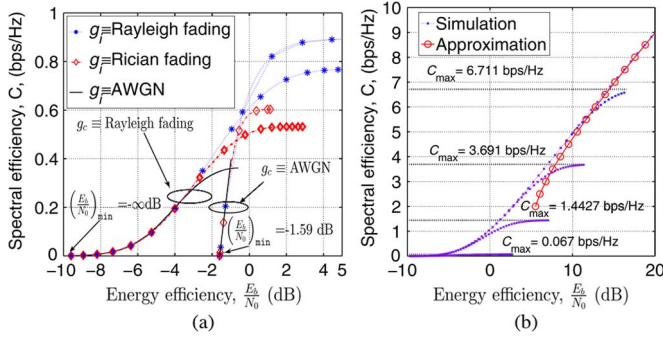


Fig. 1. (a) Per-link spectral-energy efficiency trade-off with different fading distributions of the interference channel in the low SNR regime ($Q = -5$, Rician factor $K = 5$). (b) Per-link spectral-energy efficiency trade-off for different interference threshold values in the high SNR regime (g_c and $g_i \equiv$ exponential distribution).

channel, $\left(\frac{E_b}{N_0}\right)_{\min}$ will be equal to -1.59 dB. For the Rayleigh fading channel, $\left(\frac{E_b}{N_0}\right)_{\min} = 0$ ($-\infty$ dB) as the fading channel gain is unbounded, i.e., $g_{c(\max)} = \infty$. Fig. 1(a) presents the spectral-energy efficiency trade-off when the cognitive channel is under Rayleigh and AWGN channels against different interference channel fading distributions. Here, Q is assumed to be -5 dB. We can see that $\left(\frac{E_b}{N_0}\right)_{\min}$ depends only on the fading statistics of the cognitive channel regardless of the distribution of the interference channel, which verifies *Theorem 1*. Better energy efficiency can be obtained when the cognitive channel follows a Rayleigh distribution due to additional gain in the fading distribution.

In the high SNR regime, Fig. 1(b) shows the spectral-energy efficiency trade-off when g_c and g_i are both changing according to Rayleigh fading distributions. It is worth noting that the SNR regime in which the cognitive channel resides is not only decided by the transmit power but also by the interference threshold Q . Regardless Q and the fading distribution of the cognitive and interference channels, the slope of the spectral efficiency goes to 0 as $\left(\frac{E_b}{N_0}\right)$ grows. The reason is that for CR channel the spectral efficiency is limited by interference threshold of the primary channel, i.e., even without Gaussian noise it achieves a bounded spectral efficiency C_{\max} . If the cognitive and interference channels follow Rayleigh distributions, then C_{\max} is equal to

$$C_{\max} = \frac{\log_2\left(\frac{Q}{N_0}\right)}{\left(1 - \frac{N_0}{Q}\right)}. \quad (12)$$

The detailed derivation for (12) is given in Appendix B. Hence, C_{\max} is characterized by Q and it is independent of the fading distribution of the cognitive and interference channels. However, we can notice that If Q is high enough, the spectral-energy efficiency trade-off can be approximated by

$$\left(\frac{E_b}{N_0}\right)_{\text{dB}} \approx C \times 10 \log_{10} 2 - 10 \log(C) + 2.5067, \quad \forall C < C_{\max}. \quad (13)$$

Expression (13) is similar to spectral-energy efficiency trade-off approximation that can be applied to the single primary channel

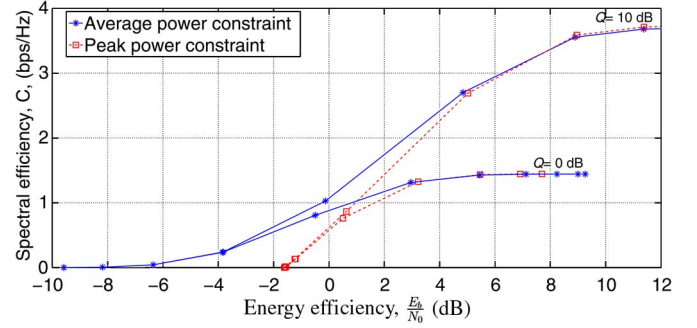


Fig. 2. Per-link spectral-energy efficiency trade-off for CR channel with average and peak power constraints (Rician factor $K = 5$).

[23]. The only difference is that C_{\max} has no limited value for the primary channel while it is limited for the cognitive channel.

B. Fading Channels With Peak Transmit Power Constraint

The optimum power allocation in this case is equal to

$$\gamma_s^*(g_c, g_i) = \min \left\{ \gamma_{\text{pk}}, \frac{Q}{g_i} \right\} \quad (14)$$

where γ_{pk} is the peak transmit power of the ST. Unlike (10), only g_i is required as input for power policy. That makes it more straight forward as it only requires the CSI of the interference channel as an input. The minimum energy efficiency can be calculated by

$$\left(\frac{E_b}{N_0}\right)_{\min} = \lim_{\gamma_{\text{pk}} \rightarrow 0} \frac{E[\gamma_s^*(g_c, g_i)]}{N_0 E \left[\log_2 \left(1 + \frac{\gamma_s^*(g_c, g_i) g_c}{N_0} \right) \right]}. \quad (15)$$

In the low SNR regime, $E[\gamma_s^*(g_c, g_i)] = \gamma_{\text{pk}}$ and when we take this into consideration, $\left(\frac{E_b}{N_0}\right)_{\min}$ is always equal -1.59 dB for all types of cognitive channel fading distribution.

Fig. 2 compares spectral-energy efficiency trade-off when the ST transmits a signal under the average and peak power constraints. In the low SNR regime, transmitting a signal with average power constraint provides better energy efficiency than transmitting a signal with peak power constraint. Moreover, reliable communication is no possible for $\left(\frac{E_b}{N_0}\right) < -1.59$ dB when transmitting a signal under peak power constraint. This is due to that fact that the power policy with peak power constraint does not benefit from the available energy at those moment in which the cognitive channel fading is exceedingly high. In the high SNR regime, both power policies behave similarly and approach the same maximum spectral efficiency C_{\max} since the transmit power for both policies is controlled by $\frac{Q}{g_i}$. Therefore, (13) can yet be applied in the high SNR regime if Q is high enough.

IV. SYSTEM-LEVEL SPECTRAL-ENERGY EFFICIENCY TRADE-OFF

In this section, we will study the spectral and energy efficiency for a CR-based cellular network. The intention here is not to build a complete cellular network using the concept of CR, but rather to enhance the spectral efficiency of the cellular networks for a short period of time by sharing a spectrum that

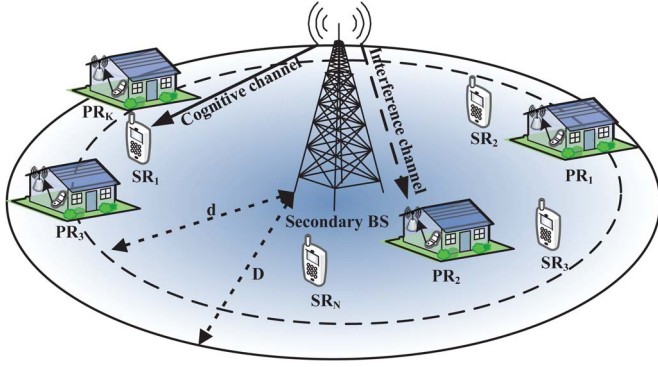


Fig. 3. System model of CR-based cellular network with secondary BS, multiple PRs, and multiple SRs.

belongs to another licensed network. We assume that a CR network consists of a single ST, i.e., macro BS, which transmits signals to multiple SRs. The CR network shares a spectrum owned by an indoor primary network. The primary network also consists of multiple PRs, i.e., primary indoor access points (APs). The SRs and PRs are indexed by $n \in \mathcal{N} = \{1, \dots, N\}$ and $k \in \mathcal{K} = \{1, \dots, K\}$, respectively. The SRs and PRs are uniformly distributed in a cell of radius d and a cell of radius D ($d \leq D$), respectively, as shown in Fig. 3. The downlink transmissions of the CR network are considered and assumed to occur in the uplink transmission of the primary network. There are many advantages for sharing the spectrum of the uplink transmissions of an indoor network. First, since the primary network is assumed to be an indoor one, the mutual interference between the primary and secondary networks will be scaled down because of the penetration losses. Secondly, as the PRs, are all fixed in position, this offers an opportunity to easily detect them by the ST. Hence, the STs can detect the pilot channel broadcast from indoor PRs and decide how many PRs with which they are surrounded [31]. The ST can then rely on channel reciprocity and estimate the channel coefficient of the interference channel using injected pilots in the uplink channel of the PRs. Finally, it is also possible that the interference channel status information (ICSI) is sent from all PRs along with their identities and collected by a certain central unit. In fact, using a separate wireline control channel that broadcasts the interference measured over a broadband connection is a very practical solution. Before the secondary network can utilize the spectrum, it must register itself with the central unit first to be updated regarding the ICSI. However, the PRs do not necessarily need to identify each registered ST. The ICSI can inform the STs regarding the status of the worst aggregate interference that a PR suffers. The STs can also use ICSI as an alternative way to estimate the channel status to that PR and regulate their transmit power accordingly. In this work, we assume that there is only one registered CR network with a single secondary cell.

A. The Distribution of the Channel Gain

The cognitive channel power gain between the ST and the n_{th} SR is denoted by $g_c(n)$, while the interference channel gain between the ST and the k_{th} PR is denoted by $g_i(k)$.

The cognitive and interference channels experience pathloss, shadowing, and multi-path fading. The focus in this section will be on the cognitive channel. However, the same analysis can be applied to the interference channel. The pathloss is a function of the distance r between the ST and the n_{th} SR, and can be expressed by

$$g_p(n) = Ar^\beta(n) \quad (16)$$

with β representing the pathloss exponent. The propagation coefficient A includes parameters related to antenna height, antenna gain, path-loss frequency dependence, and, in the case of the interference channel, the indoor loss. The combined channel gain $g_c(n)$ is given by

$$g_c(n) = \frac{g_m(n)g_s(n)}{g_p(n)} \quad (17)$$

where $g_s(n)$ and $g_m(n)$ represent the power gain of the shadowing and multi-path fading of the n_{th} SR, respectively. We use the composite channel model for both shadowing and fast fading [32]. This model is the result of the multiplication of the log-normal distribution with the Nakagami distribution. Thus, the composite channel gain can be approximated by log-normal distribution [32, pp. 132], i.e.,

$$f_h(x) = \frac{\xi}{\sqrt{2\pi}\sigma x} \exp \left\{ -\frac{(10 \log_{10}(x) - \mu)^2}{2\sigma^2} \right\} \quad (18)$$

where $\xi = \frac{10}{\log(10)}$. The mean μ and variance σ^2 can be given by

$$\mu = \left(\sum_{k=1}^{m-1} \frac{1}{k} - \ln(m) \right) + \mu_\Omega \quad (19)$$

$$\sigma^2 = \sum_{k=0}^{\infty} \frac{1}{(m+k)^2} + \sigma_\Omega^2 \quad (20)$$

respectively, where μ_Ω and σ_Ω^2 are the mean and variance of the shadowing, respectively, while m represents the Nakagami fading factor. If we recall (17) and (18), the distribution of $g_c(n)$ can be expressed by

$$f_{g_c}(g) = \frac{\text{Be}^{\left(\frac{2}{a}(\mu - \xi \log g)\right)}}{g} \text{erfc} \left(\frac{a(L_{\max} + \xi \log g - \mu) - 2\sigma^2}{a\sqrt{2\sigma^2}} \right) - \frac{\text{Be}^{\left(\frac{2}{a}(\mu - \xi \log g)\right)}}{g} \text{erfc} \left(\frac{a(L_{\min} + \xi \log g - \mu) - 2\sigma^2}{a\sqrt{2\sigma^2}} \right) \quad (21)$$

where L_{\max} and L_{\min} are the maximum and minimum path-loss values in dB, respectively, and they depend on the cell radius d and minimum distance to the ST, i.e., d_{\min} . In (21), $a = \xi\beta$ and B is a constant given by

$$B = \frac{\xi e^{\left(\frac{2\sigma^2}{a^2}\right)}}{aA^2(d^2 - d_{\min}^2)}. \quad (22)$$

The detailed derivation of (21) is given in Appendix C.

B. Interference Constraint and CR Power Control

To keep a certain level of performance for the primary network, the aggregate interference I_k at any PR must always be below a predefined threshold, i.e., Q . The interference I_k that a PR experiences consists of the aggregate interference I_p from all transmit nodes in the primary network as well as the interference $I_c = \gamma_s g_i(k)$ from the CR network, where γ_s is the ST transmit power. In other words, the aggregate interference at a PR can be calculated by

$$I_k = I_p + I_c \leq Q. \quad (23)$$

It is assumed that the adjacent indoor APs are using orthogonal radio resources to avoid strong interference among them. Moreover, the interference between non-adjacent indoor APs, i.e., I_p , can be negligible or considered as background noise. This is because signals that come from a primary user should travel through at least two walls to reach the other primary APs [33]. However, $I_c = \gamma_s g_i(k)$ is dominant compared with I_p because typically macro BS transmits a signal with high power and then its signal could be high enough to propagate through the walls of the building where the PR is deployed and generate interference. Given there are many PRs, it is important to ensure that ST transmit power γ_s should always be tightly controlled to avoid harmful interference on a PR which has the maximum channel gain toward ST. This will inevitably protect the other PRs and (23) remains true for all PRs. Therefore, the transmit power of ST is controlled according to

$$\gamma_s = \begin{cases} \frac{Q}{\max_{k \in \mathcal{K}} g_i(k)}, & \max_{k \in \mathcal{K}} g_i(k) > \frac{Q}{\gamma_{pk}} \\ \gamma_{pk}, & \max_{k \in \mathcal{K}} g_i(k) \leq \frac{Q}{\gamma_{pk}} \end{cases} \quad (24)$$

where γ_{pk} is the peak transmit power. The above power control is similar to (14) which demonstrates the power control under peak power constraint. It is more suitable for the proposed system to use the power policy with peak power constraint rather than power policy with average power constraint for many reasons. Firstly, in typical cellular networks, the BS has a limited maximum power that it can transmit with. The power control with average power constraint does not take into account this limitation. Secondly, to get as much benefit as possible of CR network, the SRs would usually be close to the ST, and so they would be within high SNR regime. This means that any gain of power control under average power constraint is minor. Finally, the power control with peak power constraint is more straight forward as it requires g_i only as an input rather than g_i and g_c . To this end, the ST can request the worst ICSI, which belong to the surrounding PRs, from the central unit. The ST then uses this ICSI to estimate the channel status and regulate the transmit power. Alternatively, or concurrently, the ST can use injected uplink pilots to estimate the channel status assuming that the channel is reciprocal.

C. Spectral Efficiency Analysis

The ST schedules SRs in orthogonal mode to avoid the intra-cell interference. In this work, time division multiple access

(TDMA) is assumed by which the ST chooses an SR whose CSI implies the largest channel gain among all other SRs. In this case, the received signal to interference plus noise ratio, $\hat{\gamma}$, for the scheduled SR is equal to

$$\hat{\gamma} = \frac{\gamma_s g_c(n^*)}{I} = \begin{cases} \frac{Q}{I} \frac{X}{Y}, & Y > \frac{Q}{\gamma_{pk}} \\ \frac{\gamma_{pk}}{I} X, & Y \leq \frac{Q}{\gamma_{pk}} \end{cases} \quad (25)$$

where X and Y are random variables that represent $\max_{n \in \mathcal{N}} \{g_c(n)\}$ and $\max_{k \in \mathcal{K}} \{g_i(k)\}$, respectively, and n^* is the index for an SR who has the maximum value of the channel gain. In (25), I is interference plus noise power. The spectral efficiency can then be evaluated by

$$C_{\text{sys}} = \int \log_2(1 + \hat{\gamma}) f(\hat{\gamma}) d\hat{\gamma} \quad (26)$$

where $f(\hat{\gamma})$ is the probability density function (PDF) of $\hat{\gamma}$. By adopting the extreme value theory [34], $f(\hat{\gamma})$ converges to

$$\begin{aligned} f(\hat{\gamma}) \rightarrow & \frac{K_1 \hat{\gamma}^{-1.5}}{2} \exp(-K_1 \hat{\gamma}^{-0.5} - K_2) \\ & - \frac{K_1 K \hat{\gamma}^{-1}}{2(K \hat{\gamma}^{0.5} + 1)} \exp(-K_1 \hat{\gamma}^{-0.5} - K K_1) \\ & + \frac{K}{2(K + \hat{\gamma}^{-0.5})^2 \hat{\gamma}^{\frac{3}{2}}} (1 - \exp(-K_1 \hat{\gamma}^{-0.5} - K K_1)). \end{aligned} \quad (27)$$

See Appendix D for the detailed derivation. In (27),

$$K = \left(\frac{I \delta_p}{Q \delta_c} \right)^{0.5} \quad (28)$$

$$K_1 = \left(\frac{\gamma_{pk} \delta_c}{I} \right)^{0.5} \quad (29)$$

$$K_2 = \left(\frac{\gamma_{pk} \delta_p}{Q} \right)^{0.5} \quad (30)$$

where δ_c and δ_p are the scale parameters for the cognitive and interference channels, respectively. A numerical integration of (26) provides convenient spectral efficiency evaluation.

D. Spectral-Energy Efficiency Trade-Off

The average energy efficiency is given by $\left(\frac{E_b}{N_0} \right)_{\text{sys}} = \frac{\bar{\gamma}_{\text{avg}}}{N_0 C_{\text{sys}}}$, where $\bar{\gamma}_{\text{avg}}$ is the average transmit power. From (24), we have

$$\begin{aligned} \bar{\gamma}_{\text{avg}} = E[\gamma_s] &= \int \gamma_s(y) f_Y(y) dy \\ &= \int_{\frac{Q}{\gamma_{pk}}}^{\infty} \frac{Q}{2} \delta_p^{0.5} y^{-2.5} \exp - \left(\frac{\delta_p}{y} \right)^{0.5} dy \\ &\quad + \gamma_{pk} \exp \left[- \left(\frac{\delta_p \gamma_{pk}}{Q} \right)^{0.5} \right] = P_1 + P_2 \end{aligned} \quad (31)$$

where $f_Y(y)$ is the PDF of Fréchet distribution (refer to (55) in Appendix D). The transmit power can be modeled as a

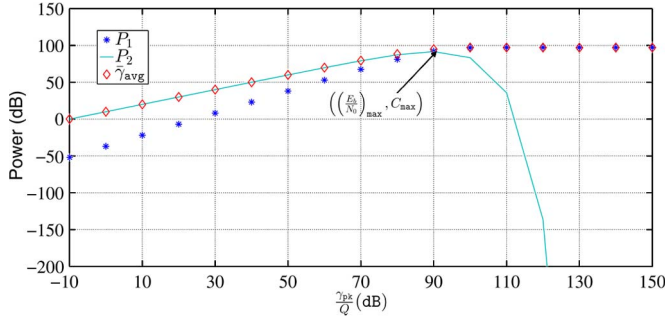


Fig. 4. The average transmit power as a function of $\frac{\gamma_{pk}}{Q}$ ($N = 20$, $Q = 0$ dB, $D = 1000$ m, $\frac{I}{N_0} = 1$, $A_{dB} = 128$ dB).

summation of two power components, i.e., P_1 and P_2 . Fig. 4 shows how P_1 , P_2 , and $\bar{\gamma}_{avg}$ change as a function of $\frac{\gamma_{pk}}{Q}$. We can notice that P_1 is a monotonously increasing function while P_2 is a waterfall curve with respect to $\frac{\gamma_{pk}}{Q}$. Hence, $\bar{\gamma}_{avg}$ is dominated by P_2 until a point, after which $\bar{\gamma}_{avg}$ is relatively constant and dominated by P_1 . This gives the conclusion that $\bar{\gamma}_{avg}$ can be approximated by

$$\bar{\gamma}_{avg} \approx \max(P_1, P_2). \quad (32)$$

If we allow the ST to transmit to the best user who has the maximum channel gain, then the average energy efficiency is given by

$$\left(\frac{E_b}{N_0}\right)_{sys} = \begin{cases} \frac{\gamma_{pk} \exp\left[-\left(\frac{\delta_p \gamma_{pk}}{Q}\right)^{0.5}\right]}{N_0 C_{sys}}, & C_{sys} < C_{max} \\ \left(\frac{E_b}{N_0}\right)_{max}, & \text{otherwise} \end{cases} \quad (33)$$

where C_{max} and $\left(\frac{E_b}{N_0}\right)_{max}$ are the maximum spectral efficiency and energy efficiency that the CR network can reach, respectively. They are given by

$$C_{max} = \int \log(1 + \gamma) \frac{K}{2(K + \gamma^{-0.5})^2 \gamma^{\frac{3}{2}}} d\gamma \quad (34)$$

$$\left(\frac{E_b}{N_0}\right)_{max} = \frac{P_1}{N_0 C_{max}} = \frac{\int_{\frac{Q}{\gamma_{pk}}}^{\infty} \frac{Q}{2} \delta_p^{0.5} x^{-2.5} \exp\left[-\left(\frac{\delta_p}{x}\right)^{0.5}\right] dx}{N_0 C_{max}}. \quad (35)$$

See Appendix E for the detailed derivation of (34). We can notice that C_{max} and $\left(\frac{E_b}{N_0}\right)_{max}$ can be considered as new parameters that characterize the CR network. In (33), γ_{pk} can be numerically calculated as a function of C_{sys} from (26) and (27).

So far, the theoretical framework, which numerically evaluates the spectral-energy efficiency trade-off, is established for system-level CR networks. In the following two sub-sections, we will focus on low and high SNR regimes for the proposed network.

1) *Low SNR Regime*: If we recall (2), $\left(\frac{E_b}{N_0}\right)_{min}$ is giving by

$$\left(\frac{E_b}{N_0}\right)_{min} = \lim_{\bar{\gamma}_{avg} \rightarrow 0} \frac{\bar{\gamma}_{avg}}{N_0 E \left[\log_2 \left(1 + \frac{\bar{\gamma}_{avg} X}{I} \right) \right]} \quad (36)$$

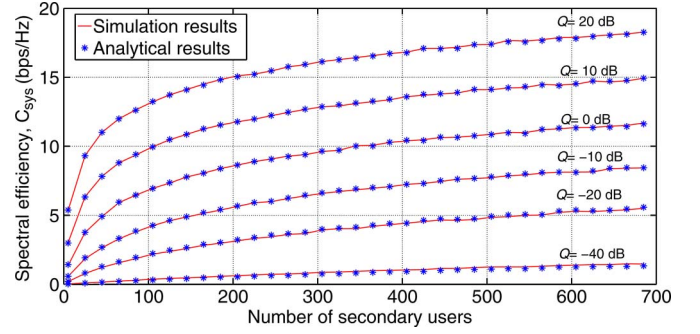


Fig. 5. The average network spectral efficiency of the CR network as a function of the number of the secondary users ($\gamma_{pk} = 43$ dB, $K = 20$, $D = 1000$ m, $\frac{I}{N_0} = 1$, $A_{dB} = 128$ dB).

where the expectation is with respect to the random variable X . Applying L'Hopital's Rule into (36), $\left(\frac{E_b}{N_0}\right)_{min}$ can then be calculated by

$$\left(\frac{E_b}{N_0}\right)_{min} = \frac{I \ln 2}{N_0 E \left[\max_{n \in \mathcal{N}} \{g_c(n)\} \right]}. \quad (37)$$

We can conclude that in a very noisy region, the minimum energy efficiency in the CR network is characterized by the cognitive channels of the best SR. Hence, the interference channels have no impact on $\left(\frac{E_b}{N_0}\right)_{min}$. The slope of the spectral efficiency versus $\frac{E_b}{N_0}$ is given by

$$S_0 = \frac{2E^2 \left[\max_{n \in \mathcal{N}} \{g_c(n)\} \right]}{E \left[\max_{n \in \mathcal{N}} \{g_c(n)\}^2 \right]} = \frac{2}{k(X)} \quad (38)$$

with $k(X)$ is the kurtosis of X .

2) *High SNR Regime*: In the high SNR regime, the available transmit power and Q are important. If Q is set to be high, the spectral and energy efficiency converges to C_{max} and $\left(\frac{E_b}{N_0}\right)_{max}$, respectively. Using (5), the slope of spectral efficiency for the cognitive network in this case is equal to 1. The horizontal penalty with respect to the AWGN channel, $\left(\frac{E_b}{N_0}\right)_{penalty}$, is given by

$$\left(\frac{E_b}{N_0}\right)_{penalty} = E \left[\log_2 \left(\frac{1}{X} \right) \right]. \quad (39)$$

See Appendix F for the detailed derivation.

E. Numerical Results and Discussions

This section presents the simulation results of the spectral and energy efficiency for a multi-user CR network. The simulation is based on the Monte Carlo method, which consists of 10^6 channel realizations. The analysis is carried out with the following parameters: $\frac{I}{N_0} = 1$, $\beta = 3.7$, the indoor loss is 8 dB, and $A = 120$ dB (128 dB in the case of the interference channel).

Fig. 5 shows the spectral efficiency of the CR network as a function of the number of SRs. We assume a reasonable

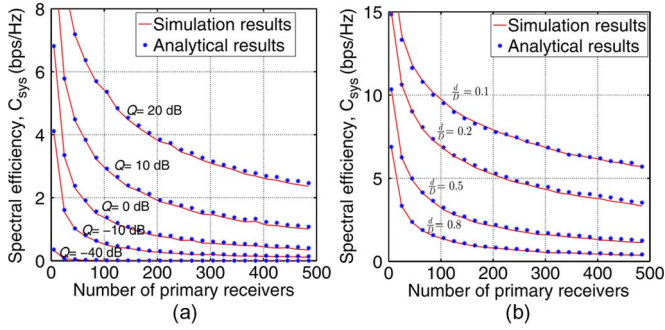


Fig. 6. The average network spectral efficiency of the secondary network as a function of the primary receivers (a) with different values of Q (b) with different values of d , ($\gamma_{pk} = 43$ dB, $N = 20$, $D = 1000$ m, $\frac{I}{N_0} = 1$, $A_{dB} = 128$ dB).

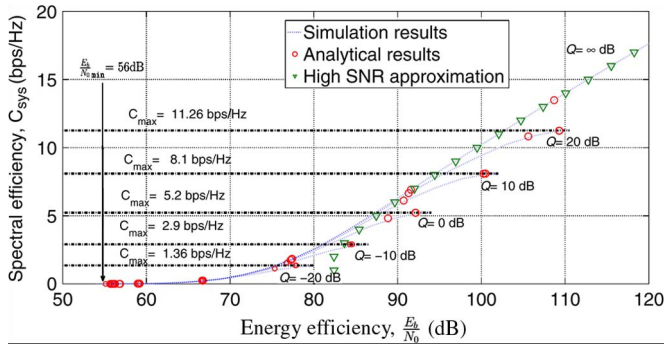


Fig. 7. Per-network spectral-energy efficiency trade-off for CR network with different values of Q ($N = 30$, $K = 20$, $D = 1000$ m, $\frac{I}{N_0} = 1$, $A_{dB} = 128$ dB).

transmit power that can be used in a typical cellular network. The number of PRs is assumed to be 20. We can notice from this figure that the maximum spectral efficiency is still achievable even with a reasonable transmit power and does not need to be unlimited. The spectral efficiency of the CR network improves with the increase of the number of the SRs due to the additional gain that comes from the multi-user diversity. The theoretical results obtained from the numerical integration of (34) agree with the simulation results.

The increase in the spectral efficiency is, however, sensitive to the interference threshold and the number of the PRs. Fig. 6(a) shows the impact of the number of the PRs on the spectral efficiency of the CR network. The number of the SRs is assumed to be 20 in this example. As shown, the spectral efficiency decreases quickly with the increase of the number of PRs. It indicates that adopting the spectrum sharing with another licensed network is unsuitable if the density of the PRs is relatively very high. However, the spectral efficiency can be improved by relaxing the interference threshold of the primary network, as it is shown in Fig. 6(a). The spectral efficiency can also be improved if the CR network considers only the SRs that are within a short distance to the ST. Hence, Fig. 6(b) shows the spectral efficiency of the CR network as a function of the number of the PRs with different values of d for a given D , assuming $Q = 0$ dB. Clearly, for the given number of the PRs, the spectral efficiency increases dramatically with the decreasing of d .

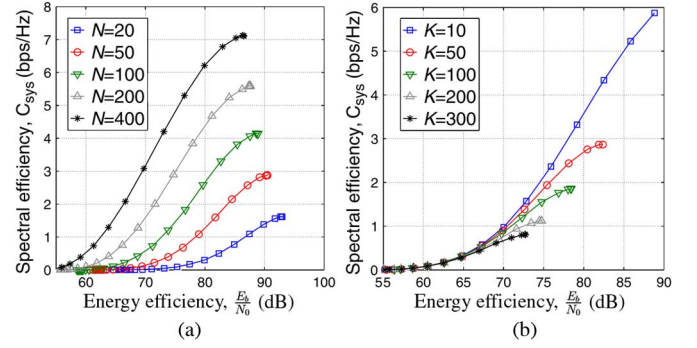


Fig. 8. Per-network spectral-energy efficiency trade-off as a function of (a) the number of secondary receivers ($K = 50$, $D = 1000$ m, $\frac{I}{N_0} = 1$, $A_{dB} = 128$ dB) and (b) the number of primary receivers ($N = 50$, $D = 1000$ m, $\frac{I}{N_0} = 1$, $A_{dB} = 128$ dB).

Fig. 7 shows the spectral-energy efficiency trade-off for the CR network. The numbers of SRs and the PRs are assumed to be 30 and 20, respectively. As we can see, for a given Q , the spectral-energy efficiency trade-off curve tends to a point that corresponds to the maximum spectral efficiency, i.e., C_{max} . It also indicates that the high SNR asymptotic tool is valid only if the CR network is working below its maximum spectral efficiency. The minimum energy efficiency, defined in (37), is the same for all curves.

The impact of the multiuser diversity gain on the spectral-energy trade-off is pointed out in Fig. 8(a) and (b). Thus, Fig. 8(a) illustrates the spectral-energy efficiency trade-off as a function of the number of the SRs. We can notice that, for a given spectral efficiency, increasing the SRs improves the energy efficiency. This improvement is because the horizontal penalty spectral-energy efficiency trade-off and $\left(\frac{E_b}{N_0}\right)_{min}$ are characterized by the cognitive channels only. The impact of the number of the PRs on the spectral-energy efficiency trade-off, however, has a different aspect. Thus, from Fig. 8(b), the number of PRs has impact only on the maximum spectral efficiency but not on the horizontal penalty nor $\left(\frac{E_b}{N_0}\right)_{min}$.

V. CONCLUSION

This paper has investigated the spectral and energy efficiency in interference-tolerant CR networks. The initial analysis has studied the spectral-energy efficiency trade-off for a link-level CR network under transmit power and interference constraints. In the low SNR regime, transmitting a signal with average power constraint provides better energy efficiency than transmitting a signal with peak power constraint. In addition to that, the interference channel has no impact on $\left(\frac{E_b}{N_0}\right)_{min}$ required for reliable communications. In the high SNR regime, however, transmitting signals with either power constraint gives the same energy efficiency.

We have also proposed a CR-based cellular network in which a secondary network shares a spectrum belonging to an indoor system. This paper has also demonstrated that with CR technology, cellular operators can share their spectrum opportunistically with each other to increase the performance

of their network. One way to do so is to share a spectrum in the uplink phase of an indoor system. This is indeed an opportunity to make the implementation of the CR-based cellular network more feasible in the near future without the necessity of modification at the end user's handset. An example of a practical application of CR and its integration with existing technology, is the use of carrier aggregation applied with CR to allow greater spectrum accessibility whilst remaining efficient by using the underutilized spectrum. The challenge is how to practically estimate the interference channels by the STs. Relying on channel reciprocity or broadcasting ICSIs can give some insight to solving this issue.

The spectral and energy efficiency of the proposed network have been analyzed. By adopting the extreme value theory, we have derived the spectral efficiency of the system-level CR network under optimal power allocation. We have studied the impact of multi-user diversity gain in both the primary and secondary receivers on the spectral and energy efficiency. The spectral efficiency of the CR network is relatively large when the number of primary receivers is small. The spectral efficiency, however, diminishes rapidly with the increase in the number of primary receivers. This degradation can be compensated by relaxing the interference threshold or by increasing the number of SRs that are within a short distance from the ST.

APPENDIX A

PROOF OF THEOREM 1

The minimum energy efficiency $\left(\frac{E_b}{N_o}\right)_{\min}$ occurs when γ_{avg} approaches zero, i.e.,

$$\left(\frac{E_b}{N_o}\right)_{\min} = \lim_{\gamma_{\text{avg}} \rightarrow 0} \frac{E[\gamma_s^*(g_c, g_i)]}{N_o E\left[\log_2\left(1 + \frac{\gamma_s^*(g_c, g_i)g_c}{N_o}\right)\right]} \quad (40)$$

where the expectation in the dominator and nominator is with respect to two random variables, i.e. g_i and g_c . We can notice from (10) that γ_{avg} vanishes when γ_0 approaches $g_{c(\max)}$. Then, (40) can be re-written as (41), shown at the bottom of the page. Applying L'Hôpital's Rule into (41) and following Leibniz integral rule, $\left(\frac{E_b}{N_o}\right)_{\min}$ is then calculated by

$$\begin{aligned} \left(\frac{E_b}{N_o}\right)_{\min} &= \lim_{\gamma_0 \rightarrow g_{c(\max)}} \frac{\frac{Q}{\gamma_0} \int_{\gamma_0}^{g_{c(\max)}} \int_0^{\frac{1}{\gamma_0} - \frac{N_o}{g_c}} f(g_c) f(g_i) dg_c dg_i}{\frac{1}{\gamma_0 \ln 2} \int_{\gamma_0}^{g_{c(\max)}} \int_0^{\frac{1}{\gamma_0} - \frac{N_o}{g_c}} f(g_c) f(g_i) dg_c dg_i} \\ &\quad \times \frac{\left(\frac{1}{\gamma_0}\right) \int_{\gamma_0}^{g_{c(\max)}} \int_0^{\frac{1}{\gamma_0} - \frac{N_o}{g_c}} f(g_c) f(g_i) dg_c dg_i}{\frac{1}{\gamma_0 \ln 2} \int_{\gamma_0}^{g_{c(\max)}} \int_0^{\frac{1}{\gamma_0} - \frac{N_o}{g_c}} f(g_c) f(g_i) dg_c dg_i} \end{aligned} \quad (42)$$

Expression (11) can then be obtained after applying $\gamma_0 \rightarrow g_{c(\max)}$.

APPENDIX B DERIVATION OF (12)

Let us first rewrite (7) as

$$C = \int_0^\infty \int_0^\infty \log_2 \left(1 + \frac{g_c \gamma_s^*(g_c, g_i)}{N_o}\right) f(g_c) f(g_i) dg_c dg_i. \quad (43)$$

In the high SNR regime, for sufficiently large transmit power, $\gamma_s(g_c, g_i)^*$ will be equal to $\frac{Q}{g_i}$. Let $x = \frac{g_c}{g_i}$ and $y = g_i$. In this case,

$$f(x) = \int_0^\infty y f_{g_c}(xy) f_{g_i}(y) dy. \quad (44)$$

If both the cognitive and interference channels follow Rayleigh distribution (i.e., both g_c and g_i would be exponentially distributed with unit-mean), then

$$f(x) = \frac{1}{(1+x)^2}. \quad (45)$$

Therefore,

$$C = \int_0^\infty \log_2 \left(1 + \frac{xQ}{N_o}\right) \frac{1}{(1+x)^2} dx. \quad (46)$$

By applying integration by parts method to (46), (12) can be obtained.

APPENDIX C DERIVATION OF (21)

The distance from the ST to a given SR is an independent and identical random variable following a uniform distribution. Thus, the probability that $R \leq r$ holds is given by

$$F_d(r) = \begin{cases} \frac{2r}{d^2 - d_{\min}^2} & d_{\min} \leq r \leq d \\ 0 & \text{otherwise} \end{cases} \quad (47)$$

where d_{\min} ($0 \leq d_{\min} \leq d$) is the minimum distance between the ST and a SR. The cumulative distribution function (CDF) of pathloss in the dB scale can be calculated by

$$F_L(y) = \Pr\{a \log(Ar) \leq y\} = \int_{d_{\min}}^{\frac{e^{\frac{y}{a}}}{A}} \frac{2r}{d^2 - d_{\min}^2} dr. \quad (48)$$

The PDF, i.e., $\frac{dF_L(r)}{dr}$, is then given by

$$f_L(y) = \frac{2e^{\left(\frac{2y}{a}\right)}}{aA^2(d^2 - d_{\min}^2)} \quad (49)$$

$$\begin{aligned} \left(\frac{E_b}{N_o}\right)_{\min} &= \lim_{\gamma_0 \rightarrow g_{c(\max)}} \frac{\int_{\gamma_0}^{g_{c(\max)}} \left(\int_0^{\frac{1}{\gamma_0} - \frac{N_o}{g_c}} \left(\frac{1}{\gamma_0} - \frac{N_o}{g_c} \right) + \int_{\frac{1}{\gamma_0} - \frac{N_o}{g_c}}^\infty \frac{Q}{g_i} \right) f(g_c) f(g_i) dg_c dg_i}{N_o \int_0^{g_{c(\max)}} \left(\int_0^{\frac{1}{\gamma_0} - \frac{N_o}{g_c}} \log_2 \left(\frac{g_c}{\gamma_0 N_o} \right) + \int_{\frac{1}{\gamma_0} - \frac{N_o}{g_c}}^\infty \log_2 \left(1 + \frac{g_c Q}{g_i N_o} \right) \right) f(g_c) f(g_i) dg_c dg_i} \end{aligned} \quad (41)$$

where $a = \xi\beta$. The channel gain $g_c(n) = \frac{S(n)}{g_p(n)}$, where $S(n) = g_m(n)g_s(n)$ is a random variable derived from (18) to represent the combined shadowing and fading channel status. The CDF of $g_c(n)$ can be expressed by (50), shown at the bottom of the page. By differentiating (50) with respect to the random variable g and by applying Leibniz's rule, the PDF of $g_c(n)$ can be calculated by (51), shown at the bottom of the page, where we have used $\frac{d}{dx}\text{erfc}(x) = \frac{2}{\sqrt{\pi}}e^{-x^2}$ and $w = (\xi \log(g) + y - \mu)$. By some manipulations, (21) can be obtained.

APPENDIX D DERIVATION OF (27)

To derive an explicit expression for the PDF of the γ , we have to first find the distribution of $X = \max_{n=1,\dots,N} \{g_c(N)\}$ and $Y = \max_{k=1,\dots,K} \{g_i(K)\}$. Since $g_c(1), g_c(2), \dots, g_c(n)$ and $g_i(1), g_i(2), \dots, g_i(k)$ are independent random variables that are drawn from a common distribution, i.e., (21), then the CDF of the maximum for any size of the samples is equal to $[F_{g_c}(g)]^N$ and $[F_{g_i}(g)]^K$, respectively [35]. Using these two new distributions may not provide understandable results. However, the distribution of the maximum function can be tracked using the extreme-value theory, as we will see in the following Lemma.

Lemma 1: Let z_1, z_2, \dots, z_n be independent and identically distributed (i.i.d.) random variables drawn from a common CDF $F_Z(z)$. By setting $Z = \max_{n=1,\dots,N} \{z_n\}$, there exist a sequence of constants $\bar{\lambda}, \bar{\delta}, \bar{\zeta}$ and some non-degenerate distribution function $\bar{H}_Z(\bar{\lambda}, \bar{\delta}, \bar{\zeta})$ such that $f(z)$ converges to the distribution $\bar{H}_Z(\bar{\lambda}, \bar{\delta}, \bar{\zeta})$. The distribution $\bar{H}_Z(\bar{\lambda}, \bar{\delta}, \bar{\zeta})$ is called generalized extreme value distribution (GEVD) [35] and it is equal to

$$\bar{H}_Z(\bar{\lambda}, \bar{\delta}, \bar{\zeta}) = \exp - \left[1 + \bar{\zeta} \left(\frac{z - \bar{\lambda}}{\bar{\delta}} \right) \right]^{-\frac{1}{\bar{\zeta}}} \quad (52)$$

where $\bar{\lambda}, \bar{\delta}$, and $\bar{\zeta}$ are the location, scale, and shape parameters, respectively. The GEVD inherently contains the three well-known extreme value distributions, i.e., Gumbel, Weibull, and Fréchet distributions. If $\bar{\zeta} > 0$, then the distribution converges

to Weibull. If $\bar{\zeta} < 0$, it converges to Fréchet distribution. If $\bar{\zeta} = 0$, then it converges to Gumbel distribution. It has been found that the following condition is sufficient to determine if $F_X(x)$ and $F_Y(y)$ belong to type II or Fréchet distribution domain of attraction [34, Theorem 1.6.1]

$$\lim_{t \rightarrow \infty} \frac{tf(t)}{1 - F(t)} = \alpha > 0. \quad (53)$$

Therefore,

$$F_X(x) \rightarrow \exp - \left(\frac{x - \lambda_c}{\delta_c} \right)^\beta \quad (54)$$

$$F_Y(y) \rightarrow \exp - \left(\frac{y - \lambda_p}{\delta_p} \right)^\beta \quad (55)$$

where $\lambda_c = \bar{\lambda}_c - \frac{\bar{\delta}_c}{\bar{\zeta}_c}$, $\delta_c = \frac{\bar{\delta}_c}{\bar{\zeta}_c}$, $\lambda_p = \bar{\lambda}_p - \frac{\bar{\delta}_p}{\bar{\zeta}_p}$, $\beta = -\frac{1}{\bar{\zeta}}$, and $\delta_p = \frac{\bar{\delta}_p}{\bar{\zeta}_p}$. By applying Theorem 9.5 of [35], δ_c and δ_p can be calculated by $\delta_c = F_{g_c}^{-1} \left(1 - \frac{1}{N} \right)$ and $\delta_p = F_{g_i}^{-1} \left(1 - \frac{1}{K} \right)$, respectively. Here, λ_c and λ_p both approach 0, and β always equals -0.5 for all cases. Due to the complicated distribution of channel gain, finding closed form expressions for δ_c and δ_p is difficult. In this paper, however, we adopt the maximum likelihood method to estimate the parameters of (52), and consequently δ_c and δ_p [35]. Other methods to estimate these parameters can be found in [36]. Now, the conditional distribution of γ , i.e., $\Pr\{\gamma \leq \gamma_0\}$, can be expressed by (56), shown at the top of the next page. Expression (27) is obtained by differentiating (56) with respect to γ .

APPENDIX E DERIVATION OF (34)

In (56), if we assume $\bar{\gamma} \rightarrow \infty$, then the term $\exp\{-(K_1\bar{\gamma}^{-\beta})\}$ vanishes. Therefore, the PDF of the random variable γ is given by

$$f(\gamma) \approx \frac{K}{2(K + \gamma^{-0.5})^2 \gamma^{\frac{3}{2}}} \quad (57)$$

which is then used to calculate the C_{\max} , as expressed by (34).

$$F_{g_c}(g) = \int_{L_{\min}}^{L_{\max}} \frac{2e^{\left(\frac{2y}{a}\right)}}{aA^2(d^2 - d_{\min}^2)} \left\{ \frac{1}{2} + \frac{1}{2}\text{erfc} \left(\frac{\xi \log(ge^{\xi^{-1}y}) - \mu}{\sqrt{2\sigma^2}} \right) \right\} dy \quad (50)$$

$$\begin{aligned} f_{g_c}(g) &= \frac{2\xi}{g\sqrt{\pi}aA^2(d^2 - d_{\min}^2)\sqrt{2\sigma^2}} e^{\left(\frac{2}{a}(\mu - \xi \log g)\right)} \int_{L_{\min} + \xi \log g - \mu}^{L_{\max} + \xi \log g - \mu} e^{\left(\frac{2w}{a}\right)} e^{-\left(\frac{w^2}{2\sigma^2}\right)} dw \\ &= B \frac{e^{\left(\frac{2}{a}(\mu - \xi \log g)\right)}}{g} \left[\text{erfc} \left(\frac{aw - 2\sigma^2}{a\sqrt{2\sigma^2}} \right) \right]_{L_{\min} + \xi \log g - \mu}^{L_{\max} + \xi \log g - \mu} \end{aligned} \quad (51)$$

$$\begin{aligned}
F(\gamma) &= \exp \left\{ - \left(\frac{\gamma_{pk} \delta_c}{I\gamma} \right)^\beta - \left(\frac{\delta_p \gamma_{pk}}{Q} \right)^\beta \right\} + \beta \delta_p^\beta \int_{\frac{Q}{\gamma_{pk}}}^{\infty} y^{-(\beta+1)} \exp \left\{ - \left(\frac{Q\delta_c}{I\gamma y} \right)^\beta - \left(\frac{\delta_p}{y} \right)^\beta \right\} dy \\
&= \exp \{ - (K_1 \gamma^{-\beta}) - K_2 \} + \frac{K \gamma^\beta}{(K \gamma^\beta + 1)} (1 - \exp [- (K_1 \gamma^{-\beta} + K K_1)])
\end{aligned} \tag{56}$$

APPENDIX F DERIVATION OF (39)

From (6), we have

$$\left(\frac{E_b}{N_0} \right)_{\text{penalty}} = \lim_{\text{SNR} \rightarrow \infty} \left(\log_2(\text{SNR}) - \frac{E[\log_2(1 + \text{SNR}X)]}{S_\infty} \right) \tag{58}$$

where the expectation is with respect to the random variable X . Knowing that $\int_0^\infty f(X) dX = 1$, (58) can be re-written as

$$\left(\frac{E_b}{N_0} \right)_{\text{penalty}} = \int_0^\infty \log_2 \left[\lim_{\text{SNR} \rightarrow \infty} \frac{\text{SNR}}{(1 + \text{SNR}X)} \right] f(X) dX. \tag{59}$$

Applying L'Hôpital's rule into (59) leads to

$$\left(\frac{E_b}{N_0} \right)_{\text{penalty}} = \int_0^\infty \log_2 \left(\frac{1}{X} \right) f(X) dX \tag{60}$$

which is the expected value of $\log_2 \left(\frac{1}{X} \right)$ and (39) is then obtained.

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