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On the Sum Rate Capacity of MIMO Broadcast Channels in Cognitive Radio Networks with Interference Power Constraints

Hai-Lin Xiao · Shan Ouyang · Cheng-Xiang Wang

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Abstract This paper investigates the sum rate capacity of MIMO broadcast channels (MIMO-BCs) in cognitive radio networks. A suboptimal user-selection algorithm is proposed to achieve a large sum rate capacity with reduced complexity. This algorithm consists of two steps. First, zero-forcing beamforming is utilized as a downlink precoding technique that precancels inter-user interference. Second, singular value decomposition is applied to the channel matrices of all the secondary users and only consider the singular vectors corresponding to the maximum singular values. The proposed user-selection algorithm chooses singular vectors which are nearly orthogonal to each other and nearly orthogonal to the vector of primary users. With this algorithm, the sum rate capacity of MIMO-BCs in CR networks with interference power constraints and transmit power constraints is derived. We formulate the sum rate capacity as a multi-constraint optimization problem and develop an algorithm to solve the problem in its equivalent form. Finally, numerical simulations are conducted to corroborate our theoretical results in flat Rayleigh fading environments. It is shown that the proposed algorithms are capable of achieving a large sum rate capacity with a very low complexity.

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1 Introduction

The radio spectrum available for wireless services is extremely scarce due to the wide deployment of wireless services. Despite the fact that the radio spectrum allocation chart suggests that we are in danger of running out of spectrum, recent studies and measurements have shown that most of the allocated bands are underutilized [1]. The spectrum underutilization motivates the idea of reusing the spectrum bands in an opportunistic manner based on the concept of cognitive radio (CR) [2,3]. In CR networks (CRNs), secondary users (SUs) are allowed to access the spectrum that is originally allocated to primary users (PUs) when the spectrum is not used. To support this spectrum reuse functionality, SUs are required to sense the radio environment. Once PUs are found to be active, SUs are required to vacate the channel within a certain amount of time. Therefore, spectrum sensing is of significant importance in CRNs [4]. However, to precisely sense a vacant spectrum is not an easy task [5,6]. Alternatively, CRNs can also be designed to allow simultaneous transmissions of PUs and SUs. From a PU's perspective, a SU is allowed to transmit as long as the interference from the SU does not degrade the quality of service (QoS) of the PU to an unacceptable level. From the SU's perspective, the SU should control its transmit power properly in order to achieve a reasonably high transmission rate without causing too much interference to the PU. This transmission strategy is termed as spectrum sharing. One fundamental challenge of spectrum sharing is to ensure the required QoS of the PU, by keeping the interference caused to it limited. Therefore, in the design of CR systems it is crucial to take into consideration two conflicting objectives, namely, maximizing the throughput of the CR system and minimizing the interference at the primary receiver (Rx). In recent years, some work has been done to deal with this problem. For instance, in [7], the authors presented the capacity-achieving algorithms for the SU link in CRNs under both its own transmit power constraint and interference power constraint at the primary Rxs. Other works for joint power constraints and channel allocation in the CRNs can be found, e.g., in [8,9].

The current and future wireless networks are challenged by the user's increasing demand of high quality and high rate multimedia services [10]. Multiple-input multiple-output (MIMO) is envisioned as a key technology to meet this challenge [11]. In MIMO broadcast channels (MIMO-BCs), the base station (BS) equipped with multiple antennas communicates with several multiple-antenna users [12]. MIMO-BCs can achieve a high sum rate capacity on the downlink by coordinating the transmissions to multiple users. Recently, there has been a lot of interest in characterizing the sum rate capacity of MIMO-BCs [13]. It is well known that dirty paper coding (DPC) is able to achieve the maximum sum rate capacity of MIMO-BCs. However, achieving the theoretical limits promised by DPC faces many challenges. Several search-based nonlinear precoding techniques have been proposed to enhance the link quality and to approach the sum rate capacity [14]. However, these methods require high complexity at the BS. To reduce the complexity, several suboptimal schemes have been proposed. Generally, these schemes fall into two categories. One is based on zero-forcing dirty paper coding (ZFDPC). Using an LQ decomposition (with L-lower triangular matrix and Q-orthogonal square matrix) of the channel matrix and DPC, ZFDPC can effectively eliminate interuser interference [15]. The other is based on zero-forcing beamforming (ZFBF). The basic idea of ZFBF is that each user is assigned one column vector of the pseudoinverse of the downlink channel matrix [16]. Hence, it is also called "channel inversion". Due to its simplicity, ZFBF has attracted more interests. It achieves a good performance when selected subchannels have high gains and are nearly orthogonal to each other. As the number of users increases, it becomes easier to satisfy these requirements. However, the exhaustive search of selecting the best set of users is very complex. In this paper, we consider MIMO-BCs with a large number of users and propose efficient suboptimum algorithm (Algorithm 1, the details of the Algorithm 1 will be given later) that assigns the coordinates of transmission space to different users in order to achieve the best performance. It is assumed that the ZFBF is used at the CR-BS as the precoding scheme. Algorithm 1 starts by setting a threshold value. In particular, by applying singular value decomposition (SVD) to the channel matrices of all the users, we consider only the singular vectors corresponding to maximum singular values. Among these candidate singular vectors, Algorithm 1 chooses a set of users which are nearly orthogonal to each other.

Sum rate capacity of MIMO-BCs in CRNs is very useful in investigating the ultimate performance limits and thus finding potential applications of CR systems. In particular, for an interference tolerant CRN, it is necessary to analyze the sum rate capacity under interference power constraints of the PU-Rx and the transmit power constraints of the CR-BS. In [7], the authors derived the ergodic capacity and outage capacity under optimal power allocation for fading channels in CRNs. It is noted that the optimal design of the SU transmission strategy under interference power constraints at PU-Rxs has also been studied in [17] for multi-antenna CR-Txs in CRNs. In [9], two different CRNs, namely a central access CRNs and a CR assisted virtual MIMO communication networks, were analyzed and compared under a common framework and the capacities under average interference power constraints were derived. In this paper, we consider MIMO-BCs based CRNs co-existing with PUs. Two sets of constraints are considered: interference power constraints to the PUs and transmission power constraints to the SUs. For the sum rate capacity maximization problem, we transform multi-constraint optimizing problem into its equivalent form, and develop Algorithm 2 to solve the equivalent problem.

Notation: Lowercase bold letters denote vectors and uppercase bold letters denote matrices. We use $\|\|$ to denote the Euclidean norm of a vector, $(\cdot)^H$ to denote the conjugate transpose, $(\cdot)^+$ to denote the Moore-Penrose inverse, $\det(\cdot)$ and $E[\cdot]$ to denote determinant and expectation of a matrix, respectively. We use $\boldsymbol{I}_M$ to denote an $M \times M$ identity matrix.

2 System Model and Power Constraints

2.1 System Model

As illustrated in Fig. 1, we consider a spectrum sharing scenario where a CR-BS is allowed to use the spectrum licensed to one PU, as long as the interference power inflicted at the PU fulfils the predefined constraints. Although we limit ourselves to one PU for simplicity, the proposed algorithms can be extended to include multiple PUs. Since a single PU is enough to demonstrate the key aspects of spectrum sharing while avoiding unnecessary complications. The PU is equipped with one antenna. An M-antenna CR-BS communicates with K SU-Rxs, with each SU-Rx equipped with $N_k (k = 1, \ldots, K)$ antennas. The received vector at the kth SU-Rx can be written as

$$\boldsymbol{y_k} = \boldsymbol{H}_k \boldsymbol{x} + \boldsymbol{n}_k \tag{1}$$

where $\boldsymbol{x} \in C^{M \times 1}$is the transmitted signal, $\boldsymbol{n}_k \in C^{N_k \times 1} \sim CN(0, \boldsymbol{I}_k)$ is the noise vector at these SU-Rxs, and $\boldsymbol{H}_k \in C^{N_k \times M}$is the channel matrix from the CR-BS to the kth user,

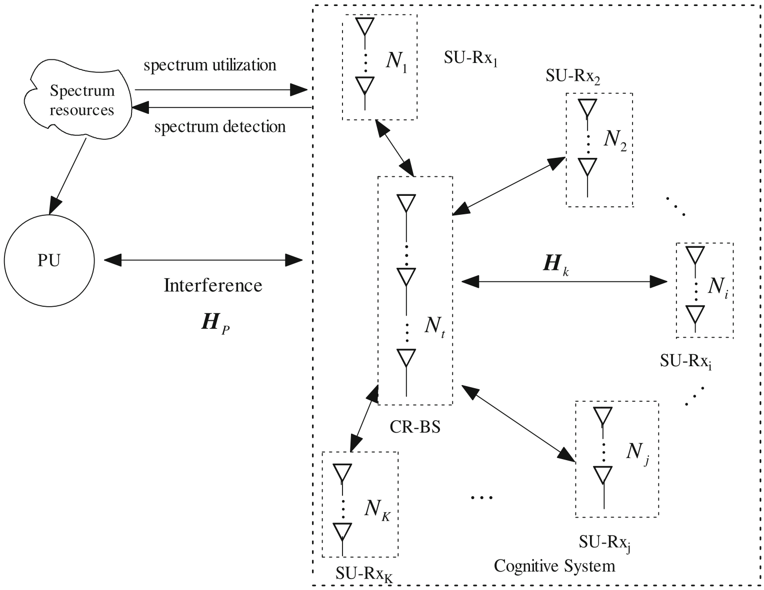

Fig. 1 System model for spectrum sharing in cognitive radio networks

which is assumed to be perfectly known at the SU-Rx's side and provided to the CR-BS via a noiseless feedback channel. However, each SU-Rx only has the channel state information (CSI) of its own channel and does not know the CSI of other SU-Rxs. We assume that the transmit antennas and users are sufficiently spaced apart such that the entries of $\boldsymbol{H}_k$, can be modelled as a set of independent and identically distributed (i.i.d) zero-mean circularly symmetric complex Gaussian random variables. Furthermore, there is no collaboration between SU-Rxs in decoding the signal.

As discussed earlier, the sum rate capacity-achieving strategy in a downlink environment requires applying DPC at the CR-BS. Due to its complexity, DPC is not practical in many applications. It is therefore desirable to utilize a precoding scheme with less complexity. Among the known precoding schemes, ZFBF has received considerable attention, as it uses a simple structure of channel matrix inversion. Using channel inversion, the capacity of fading channels with transmit power constraint has been derived in [13]. In this paper, we utilize the simple principle of ZFBF that nulls interference between cognitive data streams. Let the Moore-Penrose inverse of $\boldsymbol{H}_k$ be $\boldsymbol{H}_k^{+} = \boldsymbol{H}_k^H(\boldsymbol{H}_k\boldsymbol{H}_k^H)^{-1}$ and the columns of $\boldsymbol{H}_k^{+}$ be $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_k$, i.e., $\boldsymbol{H}_k^{+} = [\boldsymbol{w}_1, \ldots, \boldsymbol{w}_k]$. Let us denote $\boldsymbol{v}_k = (\boldsymbol{w}_k / \|\boldsymbol{w}_k\|)$. Using $\{\boldsymbol{v}_k\}$ as beamforming vectors for the selected SU-Rxs, we can construct the CR-BS signal $\boldsymbol{x}$ as

$$\boldsymbol{x} = \sum_{k=1}^{K} \sqrt{p_k}\boldsymbol{v}_k s_k \tag{2}$$

where the data streams for different CR-BS antennas are independent of each other, $E[|s_k|^2] = 1$, and p_k is the power from the CR-BS to the kth SU-Rx. The received signal vector at the SU-Rxs is

$$\boldsymbol{y} = \sum_{k=1}^{K} (\sqrt{p_k}\boldsymbol{H}_k\boldsymbol{v}_k s_k + \boldsymbol{n}_k) \tag{3}$$

where $\boldsymbol{H}_k$ is the channel matrix for SU-Rx k. In effect, using ZFBF decomposes the MIMO-BCs into K subchannels without cross channel interference.

2.2 Power Constraints

Throughout this paper, we refer the underlying channels from the CR-BS to PU as interference channels. The link between the CR-BS and PU is assumed to be a flat fading channel with instantaneous channel $\boldsymbol{H}_p \in C^{M\times 1}$ and additive white Gaussian noise (AWGN) $\boldsymbol{n}_0$. For simplicity, the noise $\boldsymbol{n}_0$ and channel matrix $\boldsymbol{H}_P$ are zero mean and unit variance complex Gaussian entries. Perfect CSI on $\boldsymbol{H}_P$ is also assumed to be available at the CR-BS. In this paper we focus on the interference power constraint to currently generated signal in each transmission block. Denoting the interference power values as Q_p, we define the corresponding constraints as

$$\sum_{i}^{M} P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) g_i \leq Q_p \tag{4}$$

$$\sum_{i=1}^{M} P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) \leq P_k \forall (\boldsymbol{H}_P, \boldsymbol{H}_k) \tag{5}$$

where the channel power gain $g_i(i = 1, 2, \ldots, M)$ corresponds to the multiple input single output (MISO) channels, which are assumed to be ergodic and stationary variables with probability density function (PDF) $f(g_i)$. Furthermore, CSI on g_i is assumed to be available at the CR-BS. $P_i(\boldsymbol{H}_P, \boldsymbol{H}_k)$ is the transmit power from the ith transmit antenna element of the CR-BS to the PU, Q_p denotes the interference threshold of the PU, and P_k denotes the sum power at the CR-BS. We further assume that g_i remains constant during a transmission block and changes independently from block to block. To acquire channel information from the PUs, as an example, the CR-BS needs to transmit pilot symbols so that the SUs and the PU can obtain respective estimates of channel matrices and g_i. In addition, such channel estimates are needed to be reliably transmitted back to the CR-BS via feedback channels.

In the CR setting, the problems (4) and (5) have not only a sum power constraint but also an interference power constraint. The multiple constraints render it difficult to formulate an efficiently solvable dual problem. In order to overcome the difficulty, we transform this multi-constrained optimization problem into its equivalent problem that has a single constraint with multiple auxiliary variables. According to Lagrange multipliers [18], the multi-constraint problem is transformed into its equivalent form

$$q_p\left(\sum_{i}^{M} P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) g_i - Q_p\right) + q_c\left(\sum_{i=1}^{M} P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) - P_k\right) \leq 0 \tag{6}$$

where q_p and q_c are the auxiliary variables related to the interference constraint and transmit power constraint, respectively.

3 The Proposed Algorithms and Sum Rate Capacity

Sum rate capacity analysis is very useful in investigating the ultimate performance limits and thus finding potential applications of CR systems. In this paper, we study the sum rate capacity of SU-Rxs. The sum rate capacity is defined as the maximum of the long-term achievable rate with arbitrary small probability of error, subject to constraints on the power. In order to obtain the sum rare capacity of SU-Rxs, we first develop an algorithm with low complexity that selects the best candidates of receive users and then, from those candidates, the set of users that satisfies the system conditions are scheduled. In general, there are two ways to perform user selection with the so-called water-filling algorithm. We may term these two ways as "forward selection" and "backward deletion". Assume that the subchannel gains are ordered. The forward selection scheme starts with the subchannel with the largest gain. The backward deletion scheme starts with the assumption that the power is allocated over all the subchannels. Let us consider the ZFBF problem with user k and channel matrix $\boldsymbol{H}_k$. Then, the beamforming vectors are always from $\boldsymbol{H}_k$ even though some users are not active. As mentioned earlier, to maximize the sum-rate capacity using the ZFBF, the selected singular vectors must be nearly orthogonal to each other and their corresponding singular values be sufficiently large. In particular, by applying the SVD to the channel matrices of all the users, we will only consider the singular vectors corresponding to maximum singular values. The corresponding maximum singular values can be compared with a threshold. This threshold is set by the CR-BS at the beginning of the transmission that it reduces the amount of feedback and the size of search space for selecting the coordinates. We propose the Algorithm 1 that chooses users are nearly orthogonal to each other and nearly orthogonal to the primary user. To satisfy the orthogonality criterion, in each iteration the Algorithm 1 selects the singular vector which is the most orthogonal to the previously selected coordinates. The first coordinate is chosen as the singular vector corresponding to the maximum singular value. The Algorithm 1 is given as follows:

Step 1. Let $\Omega = \{1, 2, \ldots, K\}$ denote the set of indices of all the k users and $J_{(n)} = \{j_{(1)}, \ldots, j_{(n)}\}$ denote the set of selected users after n steps. Using the SVD, each user computes the singular vectors and singular values of its channel matrix and sends back the singular values which are larger than a predetermined threshold ξ, along with their corresponding "right" singular vectors, to the CR-BS. The indices of these singular vectors form the following set $S_0 = \{(k, j)|\lambda_j(k) > \xi\}$.
Step 2.

2a) The CR-BS selects the index in S_0, corresponding to the maximum singular value.
2b) Let us define this index as (s_1, d_1), i.e., the singular vector of the s_1 user is $\boldsymbol{v}_{s_1,d_1}$. Let us also define $S_1 = S_0 - \{(s_1, d_1)\}$ and $\gamma_{k,j}(1) = Z(\boldsymbol{v}_{s_1,d_1}, \boldsymbol{v}_{k,j})$, $\forall(k, j) \in s_1$, where $Z(\boldsymbol{v}_{s_1,d_1}, \boldsymbol{v}_{k,j}) = \frac{\left|\boldsymbol{v}^H_{s_1,d_1}\boldsymbol{v}_{k,j}\right|^2}{\|\boldsymbol{v}_{s_1,d_1}\|^2\|\boldsymbol{v}_{k,j}\|^2}$. Note that as $\|\boldsymbol{v}_{k,j}\| = \|\boldsymbol{v}_{s_1,d_1}\| = 1$, $Z(\boldsymbol{v}_{s_1,d_1}, \boldsymbol{v}_{k,j}) = \left|\boldsymbol{v}^H_{s_1,d_1}\boldsymbol{v}_{k,j}\right|^2$ holds.
2c) Let us set $n = 1$ and $k = 1$. Then, find a user, $j_{(1),1}$, such that $j_{(1),1} = \arg\max_{i\in k} \|\boldsymbol{H}_{(1),i}\|^2$. Set $J_{(1),1} = \{j_{(1),1}\}$ and denote the achieved rate $R^{BF}(J_{(1),1})$.

Step 3.

3a) For $2 \leq k \leq K$, $(s_k, d_k) = \arg\min_{(k,j)\in s_{k-1}} \gamma_{k,j}(k-1)$, $S_k = S_{k-1} - \{(s_k, d_k)\}$, and $\gamma_{k,j}(k) = Z(\boldsymbol{v}_{s_k,d_k}, \boldsymbol{v}_{k,j}) + \gamma_{k,j}(k-1)$.
3b). $n = n + 1$
3c) Find the kth user, $j_{(n),k}$, such that $j_{(n),k} = \arg\max_{i\in\Omega-J_{(n-1),k}} R^{BF}(J_{(n-1)} \cup \{i\})$, where max $i \in \Omega - J_{(n-1),k}$ $R^{BF}(J_{(n-1)} \cup \{i\})$ is obtained by the water-filling algorithm.
3d) Let us set $J_{(n),k} = J_{(n-1),k} \cup \{j_{(n),k}\}$ and denote the achieved rate as $R^{BF}(J_{(n),k})$. If $R^{BF}(J_{(n),k}) \leq R^{BF}(J_{(n-1),k})$, break and set $n = n - 1$.

In the above algorithm, $\gamma_{k,j}(k-1) = \sum_{i=1}^{k-1} Z(\boldsymbol{v}_{s_i,d_i}, \boldsymbol{v}_{k,j})$ is used as the measure of the orthogonality between a candidate singular vector $\boldsymbol{v}_{k,j}$ and the set of previously selected singular vectors $\{\boldsymbol{v}_{s_i,d_i}\}_{i=1}^{k-1}$. Since these singular vectors are orthogonal to each other, with a good approximation, $\gamma_{k,j}(k-1)$can be interpreted as the squared magnitude of the projection of $\boldsymbol{v}_{k,j}$over the subspace spanned by $\{\boldsymbol{v}_{s_i,d_i}\}_{i=1}^{k-1}$. It is obvious that the smaller this projection is, the better the orthogonality between the $\boldsymbol{v}_{k,j}$and this subspace we have. The recursive structure of $\boldsymbol{v}_{k,j}(k)$facilitates its computation at each step of the algorithm. Let $J_{(n),k} = J_{(n-1)} \cup \{k\}$ hold and denote the newly formed composite channel matrix for user k at the nth iteration as $\boldsymbol{H}_{(n),k}$. At each iteration, this algorithm selects one user that, combined with previously selected users, yields the largest increase in the ZFBF sum rate capacity, where this sum rate capacity is calculated by using the water-filling algorithm. Note that as in the "forward selection" technique, at each iteration, the users selected in previous iterations are guaranteed to obtain a nonzero power.

As discussed above, the sum rate capacity-achieving strategy in CR MIMO-BCs applies the SVD and ZFBF at the CR-BS. The channels are assumed to be quasi-static fading, in which each channel $\boldsymbol{H}_k$ is drawn randomly at the start of each transmission frame and remains constant for the whole transmission frame, and changes independently to another realization in the start of the next frame. The frame itself is assumed to be long enough to allow communication at rates close to the capacity. The optimization objective of sum rate capacity can be achieved when the transmit power constraint and interference power constraint are both met. Therefore, the optimized sum rate capacity in this case represents the solution to the following problem

$$\begin{aligned} R_{\max}^{BF} &= \max_{\sum Tr(\boldsymbol{Q}_k)=\sum_{k=1}^{K} P_k} \log\det\left(\boldsymbol{I}_M + \sum_{k=1}^{K} \boldsymbol{H}_k^H \boldsymbol{Q}_k \boldsymbol{H}_k\right) \\ \sum_{i=1}^{M} P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) &= \sum_{k=1}^{K} tr(\boldsymbol{Q}_k) \le P_k \end{aligned} \tag{7}$$

where Q_k is the transmit covariance matrix for the kth SU-Rx user. The sum rate capacity of CR MIMO-BCs under interference power constraints and transmit power constraints can be represented as

$$\begin{aligned} R = \arg\max \Bigg\{ R_{\max}^{BF} + \sum_{j}^{K} \lambda_j \Bigg(q_p \Bigg(\sum_{i}^{M} P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) g_i - Q_p \Bigg) \\ + q_c \Bigg(\sum_{i=1}^{M} P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) - P_k \Bigg) \Bigg) \Bigg\} \end{aligned} \tag{8}$$

where $\lambda_j (\lambda_j \ge 0)$ is Lagrange multiplier. According to Kuhn-Tucker theorem [18], we can obtain

$$\nabla R_{\max}^{BF} + \sum_{j}^{K} \lambda_j \nabla \left\{ q_p \left(\sum_{i}^{M} P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) g_i - Q_p \right) + q_c \left(\sum_{i=1}^{M} P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) - P_k \right) \right\} = 0 \tag{9}$$

$$\sum_{j}^{K} \lambda_j \left\{ q_p \left(\sum_{i}^{M} P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) g_i - Q_p \right) + q_c \left(\sum_{i=1}^{M} P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) - P_k \right) \right\} = 0 \tag{10}$$

When the power constraints are satisfied with inequality (6), then $\lambda_1 = \lambda_2 = \cdots \lambda_K = 0$. This is because $\sum_i^M P_i(\boldsymbol{H}_P, \boldsymbol{H}_k)g_i - Q_p$ and $\sum_{i=1}^M P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) - P_k$ are both negative. It is easy to observe that all the selected users share the same $\lambda_j = 0$ and thus, the initial value of λ_j can be viewed as a water level in the water filling principle. However, in the Algorithm 2, the key difference is that the water-filling principle indicates that all users use different water levels. Hence, for any positive value of λ_j, we have

$$q_p\left(\sum_i^M P_i(\boldsymbol{H}_P, \boldsymbol{H}_k)g_i - Q_p\right) + q_c\left(\sum_{i=1}^M P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) - P_k\right) = 0 \tag{11}$$

The complementary slackness condition is satisfied. We find an efficiency solvable dual problem for (10) by using an iterative power allocation algorithm. We search for the optimal q_p and q_c through the Algorithm 2. In each iterative step, we update the vector $[q_p, q_c]$ ($q_p \geq 0$ and $q_c \geq 0$). The steps of the iterative power allocation algorithm (the Algorithm 2) are given in the following.

Step 1. 1. Initialization: $q_p^{(1)} = 0.005, q_c^{(1)} = 0.005, n = 1$
Step 2.

2a) Find the solution of (7) through the Algorithm 1.
2b) Update $q_p^{(n)}$ and $q_c^{(n)}$ by using $q_p^{(n+1)} = q_p^{(n)} + t\left(\sum_i^M P_i(\boldsymbol{H}_P, \boldsymbol{H}_k)g_i - Q_p\right)$ and $q_c^{(n+1)} = q_c^{(n)} + t\left(\sum_{i=1}^M P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) - P_k\right)$, respectively.
2c) Increase n by 1 and repeat Step 2.

3. Stop when $\left|q_p^{(n)}\left(\sum_i^M P_i(\boldsymbol{H}_P, \boldsymbol{H}_k)g_i - Q_p\right)\right| \leq \varepsilon$ and $\left|q_c^{(n)}\left(\sum_{i=1}^M P_i(\boldsymbol{H}_P, \boldsymbol{H}_k) - P_k\right)\right| \leq \varepsilon^+$ are satisfied simultaneously.

Here, t denotes the step size of the Algorithm 2 and $\varepsilon, \varepsilon^+$ ($\varepsilon, \varepsilon^+ > 0$) are small-valued positive number in practice to represent an infinitely small quantity.

4 Numerical Simulation Results

In this section, numerical simulation results are provided to examine the performance of our proposed algorithms (the Algorithm 1 and the Algorithm 2) with finite numbers of users and antennas. For simplicity, we assume that there is only one PU. The channel power gains can be described as exponentially distributed from the CR-BS to the PU. The elements of the channel matrix are assumed to be circularly symmetric complex Gaussian with zero mean and unity variance. The noise covariance matrix at the CR-BS is assumed to be an identity matrix, the sum power and interference power are defined in *dB* relative to the noise power, and Q_p is chosen to be 0 *dB*. The optimum threshold ξ (see [19] and references therein) and the coordinates were chosen randomly among the preselected singular vectors, unless they are specifically stated.

In Fig. 2, we examine the effectiveness of the Algorithm 2 using $N = 4$, $K = 1$, and $P_k = 10dB$. It is well known the optimal transmit signal covariance can be obtained through the water-filling principle. As observed from Fig. 2, after several iterations, the Algorithm 2 converges to the optimal solution obtained by the water-filling principle, and they achieve better performance than the optimal water-filling algorithm. This is because the number ε is a decreasing function of the step size and finds ε suboptimal points within a finite numbers of steps.

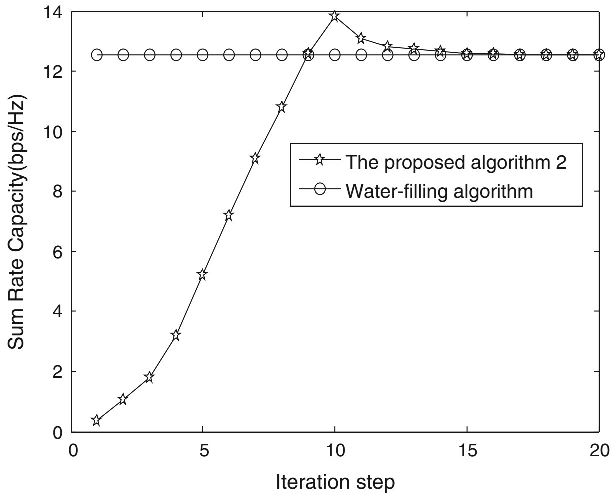

Fig. 2 Comparison of the optimal achievable rate capacity of MIMO-BCs obtained by Algorithm 2 and the water-filling algorithm with $M = 4$, $N = 4$, $K = 1$, and $P_k = 10\,\text{dB}$

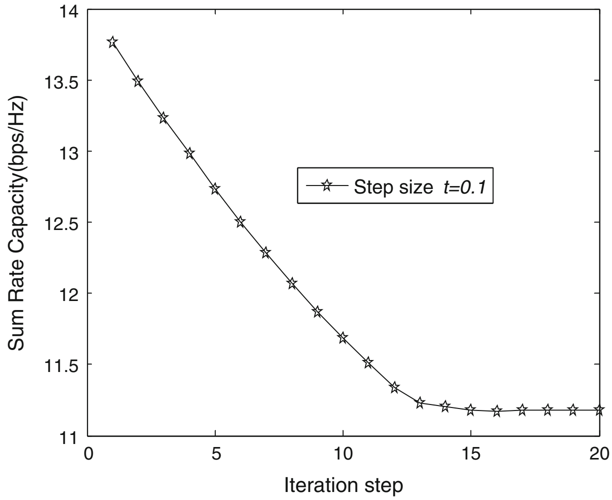

Fig. 3 The convergence behavior of Algorithm 2 with $M = 4$, $N = 4$, $K = 1$, and $P_k = 10\,\text{dB}$. The parameter t denotes the step size of Algorithm 2

Figure 3 plots the sum rate capacity versus the number of iterations of the Algorithm 2 for step sizes $t = 0.1$. As can be seen from the Fig. 3, the step size affects convergence speed of the algorithm 2. Numerical simulation results for the maximum sum rate capacity are also presented in Fig. 3 to examine the performance of our proposed algorithm 2 in practical networks with a finite number of users. For flat Rayleigh fading channels, the channel power gains g_i are mutually independent and exponentially distributed with unit mean.

Figures 4 and 5 plot the sum rate capacity versus the number of users under interference power constraints for different values of $\rho = P_k/Q_p$ in dB and different numbers of transmit and receive antennas. The transmitted power P_k of the CR-BS is fixed to 10 dB in all

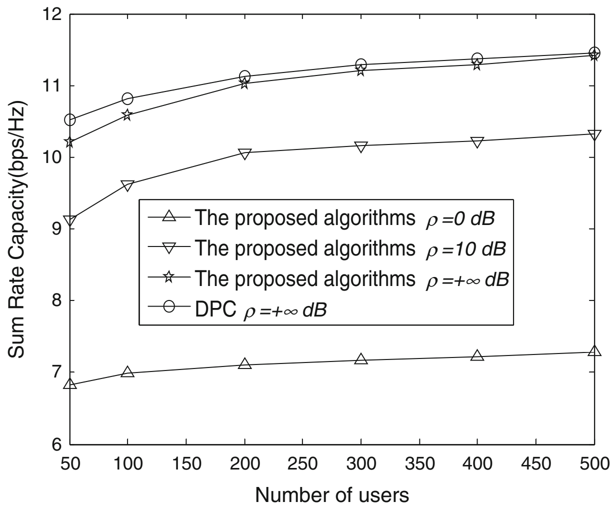

Fig. 4 Sum rate capacity versus the number of users ($M = N = 2$ and $P_k = 10\,\text{dB}$)

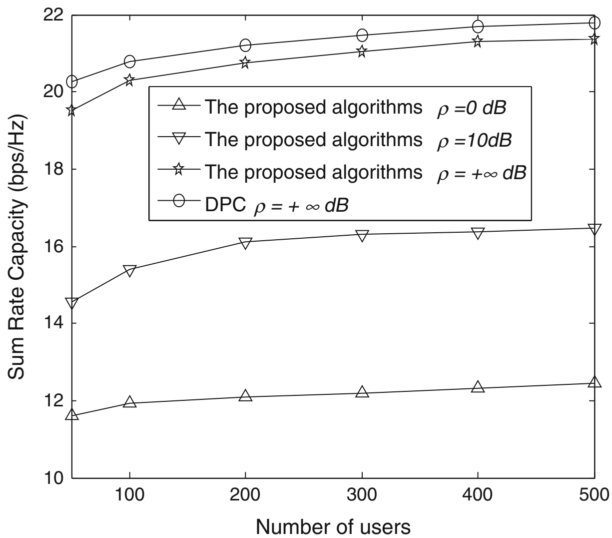

Fig. 5 Sum rate capacity versus the number of users ($M = N = 4$ and $P_k = 10\,\text{dB}$)

the curves. For comparison purposes, the plots of the optimum scheme with DPC in [19] are given as well. It can be observed that the sum rate capacity by the proposed algorithms increase with the numbers of transmit and receive antennas. The figures also show that the sum rate capacity plots converge to the case with no interference constraint as ρ increases. We can also see that as K becomes large, ZFBF combined with the SVD user selection can achieve a sum rate capacity that is close to that of DPC. For simultaneous transmissions, DPC is shown to be the optimal strategy in MIMO-BCs. However, DPC is presented by an optimization problem that can be solved by a sum-power iterative algorithm with high complexity. The complexity analysis of the proposed algorithms and DPC is given as follows.

1) As it has been seen previously, the user selection consists of two steps: PU orthogonal user selection and SUs orthogonal user selection. In the first step, for the selection strategy, we need K times inner product operations and $2K$ vector 2-norm calculations. In the second step, we need NK times inner product operations and $2NK$ vector 2-norm calculations. If there is only one PU, the complexity of DPC is presented by $M(N+1)K$ times inner product operations and $2M(N+1)K$ vector 2-norm calculations ($M > 1$).
2) To obtain the Beamforming vectors, at each user selection, DPC requires many matrix multiplication and inversions along with the SVD. However, using our proposed algorithm 1, only one matrix is needed to get the beam vectors.
3) DPC needs to perform precoding that uses concatenated coding with high complexity. For our scheme, because of the use of ZFBF, no interference pre-subtraction is required.

Hence, Compared with DPC, the complexity of the proposed algorithms is greatly reduced using the orthogonal user selection algorithm. Moreover, the use of the SVD enables us to prune the search space for the proposed algorithms, which further reduces its complexity.

5 Conclusions

In this paper, we have considered MIMO-BCs based CRNs co-existing with PUs. Two sets of constraints are considered: interference power constraints to the PUs and transmission power constraints to the SUs. The Algorithm 1 has been proposed for selecting a set of users. With Algorithm 1, the sum rate capacity of MIMO-BCs in CR networks has been derived. We have formulated the sum rate capacity as a multi-constraint optimization problem in its equivalent form and developed the Algorithm 2 to solve the problem. Numerical simulations have been conducted to corroborate our theoretical results in a flat Rayleigh fading environment. We have shown that the proposed algorithms are capable of achieving a near-optimal sum rate capacity with a very low complexity.

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