Comparison of Multilevel Coded Modulation with Different Decoding Methods over AWGN Channels*

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ABSTRACT

According to "capacity rule", the performance of multilevel coding (MLC) schemes with 8ASK modulation and three set partitioning strategies over AWGN channels is investigated. Two different decoding methods, which are multistage decoding(MSD) and parallel decoding on level(PDL), are used. In each scheme BCH codes with code lengths of 127 are used as component codes. Numerical results indicate that MSD is a sub-optimal decoding method of MLC for AWGN channels. For Ungerboeck partitioning(UP) and Mixed partitioning(MP) strategy, MSD method is strongly recommended to use for MLC system, while for Block partitioning(BP) strategy, PDL is suggested to use as a simple decoding method compared with MSD.

INTRODUCTION

A sub-optimal decoding technique called multi-stage decoding (MSD) was introduced in [1] for the decoding of multilevel codes. This decoding procedure is done stage by stage and is accomplished by decoding the component codes one at a time. The reliability of MLC system can be improved greatly by using MSD method which is to decode each component code individually starting from the lowest level and using decisions of previous decoding stages. Because of the advantage of MSD, many publications have concertrated on it[2-4].

Another decoding method for MLC proposed by P. Schramm in 1997 is parallel decoding on levels (PDL) or Independent decoding on levels (IDL)[5]. The complexity and time delay of this decoding method is lower than MSD and it has robustness to different channels[6].

In this paper, we are focusing on the comparison of these two decoding methods for MLC. Based on the calculation for capacities of equivalent channels[6-8], the performance of MLC/MSD and MLC/PDL schemes with three set partitioning strategies in AWGN channels is investigated, in which BCH codes with code lengths of 127 are chosen as component codes, and 8ASK signal constellation is used.

CAPACITY RULE

Fig.1 shows the structure of a multilevel coding system. Since the mapping M is bijective and hence lossless in the sense of information theory, the mutual information $I(Y;A)$ between the transmitted signal point $a \in \mathbf{A}$ and the received signal $y \in Y$ equals the mutual information

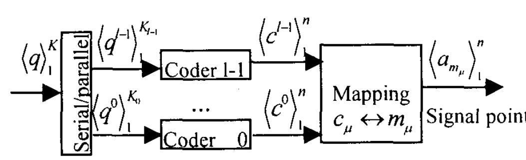

Fig. 1 Multilevel coding scheme

$I(Y; X^0, X^1, \ldots X^{l-1})$ between the address vector $X \in \{0,1\}^l$ and the received signal point:

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$$I(\mathrm{Y};\mathrm{A}) = I(\mathrm{Y};\mathrm{X}^0,\mathrm{X}^1,\ldots \mathrm{X}^{l-1}) \tag{1}$$

We denote the random variables corresponding to the transmitted and received signal point, to the binary address vector, and to its components by capital letters.

Applying the chain rule to the mutual information yields[9]

$$I(\mathrm{Y};\mathrm{X}^0,\mathrm{X}^1,\ldots \mathrm{X}^{l-1}) = I(\mathrm{Y};\mathrm{X}^0) + I(\mathrm{Y};\mathrm{X}^1 \mid \mathrm{X}^0) + \ldots + I(\mathrm{Y};\mathrm{X}^{l-1} \mid \mathrm{X}^0\mathrm{X}^1\ldots \mathrm{X}^{l-2}) \tag{2}$$

This equation may be interpreted in the following way: the transmission of vectors with binary digits $\mathrm{X}^i, i = 0,1,\ldots l-1$, over the physical channel can be virtually separated into the parallel transmission of the digits x^i over ℓ equivalent channels. The equivalent channel i consists of the equivalent mapper i, provided that the digits $\mathrm{X}^0\ldots\mathrm{X}^{i-1}$ and the noisy channel are known. The binary symbol x^i is multiply represented in the signal set of the equivalent mapper i for $i < \ell - 2$. The capacities of the equivalent channels for MLC/MSD scheme are proposed and derived by [10,11] which directly lead to the capacity rule or the rate rule design. Given a 2 -ary digital modulation scheme, choose the rate R^i at the individual coding level i of a MLC scheme to equal the capacity C^i of the equivalent channel i :

$$R^i = C^i \qquad i = 0,1,\ldots,l-1 \tag{3}$$

The basis of the capacity rule is to characterize the transmission properties of the equivalent channels by its capacities. Operating at the capacity limit of MLC scheme, the capacity rule provides the maximum individual rates to be transmitted with arbitrarily low error probability. Thus, the design of MLC system with an optimum trade-off between power and bandwidth efficiency has to be based on the capacity rule.

The capacities of the equivalent channels can be calculated very efficiently by using the following form of the chain rule for mutual information:

$$I(\mathrm{Y};\mathrm{X}^i\ldots\mathrm{X}^{l-1}/\mathrm{X}^0\ldots\mathrm{X}^{i-1}) = I(\mathrm{Y};\mathrm{X}^i/\mathrm{X}^0\ldots\mathrm{X}^{i-1}) + I(\mathrm{Y};\mathrm{X}^{i+1}\ldots\mathrm{X}^{l-1}/\mathrm{X}^0\ldots\mathrm{X}^i) \tag{4}$$

The capacity C^i for given a-priori probabilities of signal points yields:

$$\mathrm{C}^i = I(\mathrm{Y};\mathrm{X}^i/\mathrm{X}^0\ldots\mathrm{X}^{i-1}) = I(\mathrm{Y};\mathrm{X}^i\ldots\mathrm{X}^{l-1}/\mathrm{X}^0\ldots\mathrm{X}^{i-1}) - I(\mathrm{Y};\mathrm{X}^{i+1}\ldots\mathrm{X}^{l-1}/\mathrm{X}^0\ldots\mathrm{X}^i) \tag{5}$$

The mutual information $I(\mathrm{Y};\mathrm{X}^i\ldots\mathrm{X}^{l-1}/\mathrm{X}^0\ldots\mathrm{X}^{i-1})$ is calculated by averaging with respect to all possible combinations of $x^0,\ldots, x^{i-1}$:

$$I(\mathrm{Y};\mathrm{X}^i\ldots\mathrm{X}^{l-1}/\mathrm{X}^0\ldots\mathrm{X}^{i-1}) = E_{x^0\ldots x^{i-1}}\{I(\mathrm{Y};\mathrm{X}^i\ldots\mathrm{X}^{l-1}/\mathrm{X}^0\ldots\mathrm{X}^{i-1})\} \tag{6}$$

Thus, C^i is given by:

$$\begin{cases} C^i = E_{x^0\ldots x^{i-1}}\{C(\mathrm{A}(x^0\ldots x^{i-1}))\} - E_{x^0\ldots x^i}\{C(\mathrm{A}(x^0\ldots x^i))\}, & i=1,\ldots,l-1 \\ C^0 = C(\mathrm{A}) - E_{x^0}\{C(\mathrm{A}(x^0))\} & i=0 \end{cases} \tag{7}$$

DIFFERENT RATES OF THREE MAPPING STRATEGIES

There are three kinds of mapping (or set partitioning) strategies for the signal constellation. Traditional Ungerboeck partitioning (UP)[12] is aimed at maximizing the intra subset minimum Euclidean distance. As an inverse strategy, we call block partitioning (BP). This scheme minimizes the intra subset minimum Euclidean distance. Last strategy is a kind of combination of UP and BP strategy called mixed partitioning (MP). BP [10] is shown in Fig.2. MP results from a combination. In this letter, it is defined in this way:

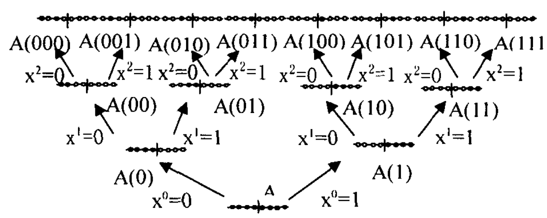


Fig.2 BP of 8ASK signal set

Table 1

Different Rates of MLC/MSD Schemes with 8ASK Modulation Based on Capacity Rule(R=2.5bits/symbol)

	UP	BP	MP
AWGN Channel	$C_0=R_0=0.5$ $C_1=R_1=1$ $C_2=R_2=1$	$C_0=R_0=0.95$ $C_1=R_1=0.85$ $C_2=R_2=0.7$	$C_0=R_0=0.875$ $C_1=R_1=0.625$ $C_2=R_2=1$

BP-UP-UP which means the first partitioning step is done by the rule of BP and followed by UP and UP. From the results of capacity for MLC/MSD scheme with three mapping strategies and 8ASK modulation, the rate design value of MLC/MSD over different channels are obtained. Table 1 lists the rate design values over AWGN channels when R is 2.5bits/symbol.

COMPARISON OF DECODING METHODS FOR MULTILEVEL CODING

For AWGN channels, optimal decoding of multilevel codes can be performed by a maximum-likelihood(ML) decoder that finds the best input sequence that maximizes the probability of receiving the observed sequence. But this decoder has to work with a very huge complexity. In this case the ratio between performance and decoding complexity is poor even for very simple codes in each level. Thus, good sub-optimal decoding techniques are needed to obtain the good trade-off between performance and complexity.

A. Multistage Decoding for Multilevel Codes (MLC/MSD)

MSD is proposed by Imai in his original work[1]. The component codes C^i are successively decoded by the corresponding decoders D_i, see Fig.3. At stage i, decoder D_i processes depending on not only the block $y=(y[1],\ldots,y[N])$, $y[\mu]\in Y$, of received signal points, but also decisions $\hat{x}^j$, $j=0,\ldots,i-1$, of previous decoding stages j. The use of previous decoding decisions accomplishes the selection of the current subsets of the equivalent mapper i for the different time instants $\mu=1,\ldots,N$.

Actually, the staged decoding according to the chain rule in (2) would require the transmitted symbol x^j instead of the estimate $\hat{x}^j$. But if we assume error-free decisions

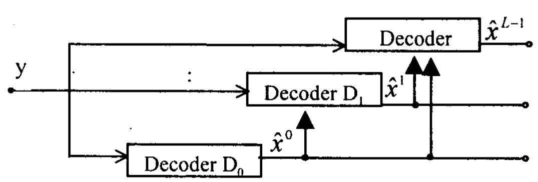


Fig.3 Multistage decoder(MSD)

$\hat{x}^j=x^j$ of decoder D_j, MSD can be interpreted as an implementation of the chain rule. Clearly, in practice, erroneous decisions occur and errors propagate from low levels to higher ones. But it is shown later that error propagation in MSD does not significantly influence the performance of the total scheme.

Obviously, multistage decoding is not identical to ML decoding, although each level can and should be ML decoded. Therefore, we will lose performance compared to the super-decoder. However, the decoding complexity is significantly reduced because now the complexity is the sum of decoding complexity of each level instead of the product. Of course, additional delay is imposed on the decoding process, because the single decoders cannot work in parallel.

B. Parallel Decoding for Multilevel Codes(MLC/PDL)

The use of estimates on lower levels may be unsuitable in practice, e.g. due to memory requirements. In this case, the codes on the levels could be decoded independently, i.e. without feedback of estimates. Therefore, an alternative decoding strategy for multilevel coded transmission is parallel decoding of the individual levels(PDL)[2]. Thereby, in contrast to the MSD approach, decoder D_i makes no use of decisions of other levels $i\neq j$. All decoders D_i, $i=0,1,\ldots,l-1$, are working in parallel. The PDL approach is sketched in Fig.4. For

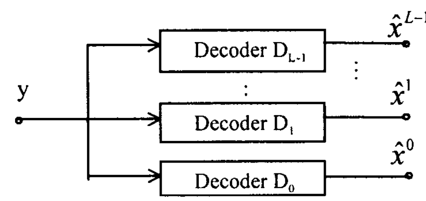


Fig.4 Parallel decoding of levels(PDL)

MLC/PDL the transmission of each address symbol x^i, $i=0,1,\ldots,l-1$, over the equivalent channel i is based on the entire signal constellation, since there is no preselection of signal points at higher levels due to decoding decisions of other levels. Of course, information is lost by not using estimates from lower levels. Thus, the sum of the capacities C_{sum} of all levels is less than(or equal to) the total capacity of the signal set, i.e.

$$C_{sum} = \sum_i C_i \leq C_{set} \quad (8)$$

To be more accurate, the concept of the equivalent channel and its characterizing pdf has to be adopted appropriately for an MLC/PDL scheme. While in the case of MLC/MSD the signal set of the equivalent mapper i is time variant for $i>0$ depending on the binary digits x^j of lower levels j, $j=0,\ldots,i-1$, the equivalent mapper i for the MLC/PDL scheme is time invariant for all $i=0,\ldots,l-1$. Since the decoding at one level is done independently of other levels, the equivalent mappers for MLC/PDL comprise the entire signal set **A** in every case. In the signal set of equivalent mapper i the binary symbol b^i is multiply represented by all signal points with address digit $x^i=b^i$, $b^i \in \{0,1\}$.

An advantage of the PDL decoding approach is certainly that error propagation from low to higher levels can be avoided since the levels are decoded independently. Additionally, PDL is favorable in terms of decoding delay since the individual decoders are working in parallel instead of serial in the staged decoding approach MSD.

RESULTS AND DICUSSIONS

According to the discussion of channel capacity in the previous section and the code rates in Table 1, the performance comparison of MLC/MSD and MLC/PDL over AWGN channels was performed by means of simulation. The presented results are bit error rates (BERs or Pb) as a function of E_b/N_0, where E_b denotes average energy per information bit. In order to allow a fair comparison, some parameters of the schemes are identical for all cases: the modulation scheme is 8ASK, BCH codes with code lengths of 127 and different code rates are chosen as component codes on three levels.

Fig.5 shows the performance comparison of MLC schemes using two different decoding methods with UP, BP and MP over AWGN channels. In each scheme code rates of component codes are designed according to "capacity rule" shown in Table 1. For performance comparison, the total rates of scheme are all chosen as: 2.5bits/symbol. From the results, we can see:

(a) For any mapping strategy, MSD is superior to PDL for MLC scheme over AWGN channels. That is to say, MSD is a sub-optimal decoding method of MLC scheme over AWGN channels.

(b) For UP and MP strategies, the performance of MLC /PDL and MLC/MSD is nearly the same at lower signal-to-noise ratio(SNR) when E_b/No is lower than 11dB; With the increase of SNR (E_b/No>11dB), the performance difference will be larger. As shown in Fig.5, the power efficiency of MLC/MSD scheme is higher than MLC/PDL by 3~4dB coding gain for UP strategy, while by 2~3dB coding gain for MP strategy when $P_b=10^{-3}$.

(c)The performance of MLC/PDL and MLC/MSD schemes with BP strategy is nearly the same at any SNR. Therefore, PDL is suggested as the good decoding method because of its less complexity and time delay for BP mapping strategy.

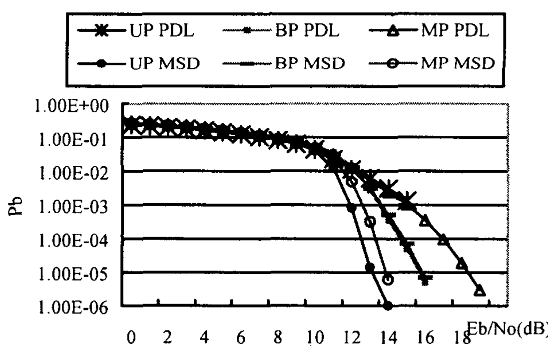
(d)For MSD decoding, the performance of MLC scheme with UP mapping strategy is optimal compared with BP and MP at the same bandwidth efficiency. The performance of MP is superior to that of BP.

(e)For PDL decoding, BP is the best mapping strategy compared with UP and MP from the view of performance. And MP is a little better than UP. Therefore, decoding method and mapping strategy are two important parameters of an optimal MLC scheme over AWGN channels.

CONCLUSIONS

From simulation results and discussions, some conclusions can be got:

(a) For any set partitioning strategy, MLC/MSD scheme is superior to MLC/PDL over AWGN channels. In each scheme the code rates of component codes with code lengths of 127 are all designed based on "capacity rule". Therefore, MSD is the sub-optimal decoding method for multilevel coding system. And the performance of MLC/MSD scheme with UP mapping strategy is optimal over AWGN channels.



UP: C0(127,57,11),C1(127,127),C2(127,127)
BP: C0(127,120,1),C1(127,106,3),C2(127,85,6)
MP: C0(127,106,3),C1(127,78,7),C2(127,127)

Fig.5 Comparison of MLC/PDL and MLC/MSD for different mapping strategies with 8ASK over AWGN

(b)As long as BP strategy is used, the performance of MLC/PDL is nearly the same with that of MLC/MSD scheme for both AWGN and Rayleith fading channels.

Therefore, PDL can be used as a more attractive and simple decoding method instead of MSD for MLC system. This conclusion has great significance for designing the MLC system with higher bandwidth efficiency, e.g. there is more than three levels in MLC system.

(c)The performance of MLC scheme with different decoding methods is related to set partitioning strategies. For UP and MP strategy, MSD method is strongly recommended to use because the performance of MLC/MSD scheme is much better than that of MLC/PDL. For BP strategy, PDL is suggested to use as a simple decoding method because the performance of MLC scheme with two decoding methods is nearly the same.

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