Performance Analysis and Resource Allocation of Heterogeneous Cognitive Gaussian Relay Channels

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***Abstract*—Motivated by the deployment of cognitive radio (CR) based relays in cellular networks, this paper studies the fundamental limits of heterogeneous cognitive Gaussian relay channels (HCGRCs). Unlike conventional relay channels, in HCGRC a source transmits to a relay in a licensed band, while the relay transmits to a destination in an unlicensed cognitive spectrum band. The licensed and unlicensed bands are characterized by different power, bandwidth and reliability constraints. Taking an information-theoretic perspective, the fundamental properties of the HCGRC are analyzed thoroughly in terms of capacity, spectral efficiency (SE), and energy efficiency (EE). With regard to each metric, we derive the optimal resource allocation strategy and discuss the impacts of CR spectrum reliability and relay location on the metric. We find that in HCGRC, improving the SE and EE are not necessarily conflicting objectives. Instead, both metrics can be optimized simultaneously with proper resource allocation.**

I. Introduction

Due to the proliferation of wireless communication systems in the past decades, available wireless spectrum resource is becoming increasingly scarce. Meanwhile, the allocated spectrum was found to be highly underutilized [1]. The problems of 'spectrum shortage' and 'spectrum underutilization' motivated the concept of cognitive radio (CR) [2], which enables opportunistic access of underutilized frequency bands to improve the overall spectrum utilization. Incumbents and CR users using the same frequency band are differentiated by their priorities to access the spectrum, hence they are also called primary users (PUs) and secondary users (SUs), respectively.

The radio resource (RR) available to incumbents and CR systems are called licensed RR and cognitive (or, secondary) RR, respectively. The licensed RR is typically featured with a relatively small bandwidth, high transmit power, and high reliability. On the contrary, the cognitive RR is characterized by its potentially broad bandwidth, low transmit power, and low reliability. It is obvious that these two types of RRs are complementary in nature and demand different approaches for system design and optimization.

Most of the existing works on CR assumed that the CR network only use the cognitive RR, i.e., an isomorphic/pure CR network [3]–[9]. The performance of isomorphic CR network, however, is fundamentally opportunistic and hence unreliable. To this regard, only a few articles have considered heterogeneous/hybrid CR networks that utilize both the licensed and cognitive RRs [10]–[14]. It was found that heterogeneous CR networks have the potential to outperform incumbent and isomorphic CR networks by exploiting the complementary natures of licensed and cognitive RRs [10]. The advantage of heterogeneous CR networks is particularly promising with the adoption of cooperative communication schemes, which creates heterogeneous wireless channels in the system. Significant performance gains can be achieved when the heterogeneous RRs are carefully assigned to the heterogeneous channels [10]. For example, the licensed RR is better used for long range communications, while the cognitive RR is better used for short-range communications to facilitate local cooperation.

Previous works on heterogeneous cooperative CR networks have focused on system-level studies [10]–[14]. To our best knowledge, the link-level performance of heterogeneous cooperative CR networks has never been studied from a rigorous information-theoretic perspective. This paper makes the first attempt to study a novel type of relay channel called heterogeneous cognitive Gaussian relay channel (HCGRC). In HCGRC, the source broadcast to a relay and a destination using the licensed RR, while the relay forward information to the destination using the cognitive RR in a duplex fashion. HCGRC is fundamentally different from the conventional Gaussian relay channel in two aspects: First, the source and relay are subject to separate resource constraints rather than a total resource constraint. The corresponding research question, therefore, is not about how to properly divide the radio resource between the source and relay, but how much cognitive RR is required at the relay to match the licensed RR and achieve the optimal performance. Second, the cognitive RR is opportunistic in nature, hence it should be characterized by not only power and bandwidth, but also reliability. The corresponding research question is to understand the impact of spectrum reliability on the power-bandwidth allocation of cognitive RRs.

The main contributions of this paper are as follows:

- Taking an information-theoretic perspective, the fundamental performance bounds of HCGRC is analyzed thoroughly in terms of capacity, SE, and EE.
- With regard to each performance metric, we derive closed-form formulas to characterize the optimal power-bandwidth allocation scheme for the cognitive relay.
- The impacts of CR spectrum reliability and relay location on the performance of HCGRC are analyzed and discussed.

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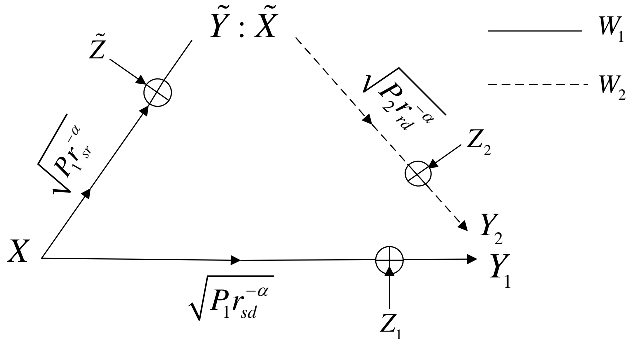

Fig. 1. The Heterogeneous Cognitive Gaussian Relay Channel (HCGRC) model.

II. System Model

The proposed HCGRC channel model is motivated by a practical scenario where CR is applied in cellular networks as the wireless backhaul of relay stations. By upgrading the infrastructure (i.e., base stations and relay stations) with CR-enabled air interface, this type of cognitive relay system can harness more RRs to improve the network capacity, while imposing no change on user terminals.

From the operators' perspective, we are interested in the capacity, SE, and EE of the relay channel. Apart from the obvious capacity metric, the SE metric indicates how effectively the cognitive spectrum is used. Because cognitive spectrum may come with a cost (e.g., via spectrum leasing from other incumbents), the SE metric is particularly useful for operators to decide how much cognitive spectrum is needed. On the other hand, the EE metric indicates the energy efficiency of the relay system. It is particularly relevant to the emerging 'green radio' paradigm to reduce power consumption and operational costs.

In this paper, we consider a simple three-node HCGRC with a source, a destination, and a relay. The source broadcasts to the relay and destination using a licensed RR, while the relay forward information to the destination using a cognitive RR. Because the licensed and cognitive RRs use different frequency bands, we assume that the relay node can receive and transmit at the same time, i.e., work in a full-duplex fashion. As shown in Fig. 1 , the solid and dashed lines indicate transmissions in the licensed and cognitive bands, respectively.

The HCGRC model has two new features compared with the classic Gaussian relay channels [15] or the recently studied parallel relay channels [16]. First, the licensed and cognitive RRs have different average power constraints denoted by P_1 and P_2, respectively. Similarly, the bandwidth of the licensed and cognitive RRs are denoted by W_1 and W_2, respectively. To clearly indicate the relationship between licensed and cognitive RRs, we define bandwidth ratio θ and power ratio φ as $\theta = W_2/W_1, \varphi = P_2/P_1$, respectively. Second, due to the instability of cognitive RR, the relay channel is only active with a certain probability $\bar{\varepsilon}$.

In Fig. 1 the relationships between input and output symbols of HCGRC can be written as

$$\begin{aligned} Y_1 &= \sqrt{P_1 r_{sd}^{-\alpha}} X + Z_1 \\ Y_2 &= \varepsilon \left(\sqrt{P_2 r_{rd}^{-\alpha}} \tilde{X} + Z_2 \right) \\ \tilde{Y} &= \sqrt{P_1 r_{sr}^{-\alpha}} X + \tilde{Z} \end{aligned} \quad (1)$$

where X and $\tilde{X}$ are inputs of the licensed and cognitive channels, respectively, r_{sr}, r_{rd} and r_{sd} are distances from source-to-relay, relay-to-destination and source-to-destination, respectively, $\alpha = 4$ is the path loss exponent, and $Z_1, Z_2, \tilde{Z}$ are independent white Gaussian noises. The transmit signal-to-noise ratios (SNRs) of the source-to-destination link and source-to-relay link can be written as $\rho_1 = (P_1/N_0 W_1)$ and $\rho_3 = (P_1/\tilde{N}_0 W_1)$, respectively, where N_0 and $\tilde{N}_0$ are noise spectral densities at the destination and relay, respectively. The transmit SNR of the relay-to-destination link can be expressed as $\rho_2 = \rho_1 \varphi/\theta$. In (1), ε is a binary random variable defined on probability space [0,1] to represent the opportunistic nature of the cognitive channel, with $\varepsilon = 0$ indicating that the cognitive channel is unavailable. The mean of ε is $\bar{\varepsilon}$, which indicates the proportion of available time and can be understood as a 'reliability' measure of the cognitive channel. We assume that both the CR transmitter and receiver have perfect information of the channel availability. So when $\varepsilon = 0$, both the transmitter and receiver stop working and do not consume any extra power.

III. Performance Analysis of HCGRC

A. Capacity

In this subsection, we study the capacity bounds of HCGRC and further derive the optimal power and bandwidth allocation schemes that achieve the capacity bounds.

The HCGRC model is similar to the Gaussian orthogonal relay model studied in [16]. Both types of channel have the same capacity cut set bounds given by [16]

$$C_{lower} = \sup_{p(x)p(\tilde{x})} \min \left\{ I(X;Y_1) + I\left(\tilde{X};Y_2\right), I\left(X;\tilde{Y}\right) \right\} \quad (2)$$

$$C_{upper} = \sup_{p(x)p(\tilde{x})} \min \left\{ I(X;Y_1) + I\left(\tilde{X};Y_2\right), I\left(X;\tilde{Y},Y_1\right) \right\}. \quad (3)$$

Unlike [16],the source node and relay node in HCGRC are not subject to a total bandwidth constraint. Therefore we have a different capacity theorem as follows.

Theorem 1: Given θ and φ, the lower bound and upper bound of the HCGRC capacity are given by

$$C_{lower}(\theta,\varphi) = \min\left\{C_{1,low}(\theta,\varphi), C_{2,low}\right\} \quad (4)$$

$$C_{upper}(\theta,\varphi) = \min\left\{C_{1,up}(\theta,\varphi), C_{2,up}\right\} \quad (5)$$

respectively, where

$$C_{1,low} = W_1 \log\left(1+\rho_1 r_{sd}^{-4}\right) + W_1 \theta \bar{\varepsilon} \log\left(1+\frac{\rho_1 \varphi r_{rd}^{-4}}{\theta}\right) \quad (6)$$

$$C_{2,low} = W_1 \log\left(1+\rho_3 r_{sr}^{-4}\right) \quad (7)$$

$$C_{1,up} = C_{1,low} \quad (8)$$

$$C_{2,up} = W_1 \log\left(1+\rho_3 r_{sr}^{-4} + \rho_1 r_{sd}^{-4}\right). \quad (9)$$

Proof: The derivation of Theorem 1 can take reference from the relay channel with orthogonal receiver components [17]. According to the maximum entropy theory in [18], the lower bound can be obtained when input symbols follow Gaussian distribution. The upper bound can be directly derived from the RFD (receiver frequency-division) Gaussian relay channel following the proof in [17].

According to (4) and (5), the lower bound $C_{lower}(\theta,\varphi)$ and upper bound $C_{upper}(\theta,\varphi)$ are functions of resource allocation parameters θ and φ. We use (θ_c^*,φ_c^*) and (θ_c^o,φ_c^o) to denote the optimal solutions that maximize the lower bound and upper bound of the capacity with the least RRs, respectively. Specifically, we define $\theta_c^* = \min\arg\sup_{\theta}(C_{lower}(\theta,\varphi_c^*))$. The following theorem can be obtained. ■

Theorem 2: With given ρ_1,ρ_3 and availability probability $\bar{\varepsilon}$, the optimal resource allocation solution (θ_c^*,φ_c^*) that maximize the capacity lower bound is

$$\varphi_c^* = \Psi(\theta_c^*) = \frac{\theta_c^*}{\rho_1 r_{rd}^{-4}}\left[\left(\frac{1+\rho_3 r_{sr}^{-4}}{1+\rho_1 r_{sd}^{-4}}\right)^{\frac{1}{\theta_c^*\bar{\varepsilon}}}-1\right] \quad (10)$$

and the optimal resource allocation solution (θ_c^o,φ_c^o) that maximize the capacity upper bound is

$$\varphi_c^o = \Phi(\theta_c^o) = \frac{\theta_c^o}{\rho_1 r_{rd}^{-4}}\left[\left(\frac{1+\rho_3 r_{sr}^{-4}+\rho_1 r_{sd}^{-4}}{1+\rho_1 r_{sd}^{-4}}\right)^{\frac{1}{\theta_c^o\bar{\varepsilon}}}-1\right]. \quad (11)$$

Proof: The difference between the lower and upper bounds lies on the second item $C_{2,low}$ and $C_{2,up}$, both of which have non-zero values and are unrelated to θ and φ. On the other hand, $C_{1,low}$ and $C_{1,up}$ are monotonically increasing functions of both θ and φ, which means they vary from zero to infinite as θ and φ increase. Based on (4) and (5), the capacity bounds start with the first items and are clipped by the second items. Therefore, (10) and (11) can be derived as the solutions of $C_{1,low}(\theta,\varphi) = C_{2,low}$ and $C_{1,up}(\theta,\varphi) = C_{2,up}$, respectively. ■

B. Spectral Efficiency

In this subsection, we address another performance metric of the HCGRC: spectral efficiency (SE). From (4)-(9), we can obtain the corresponding lower and upper bounds of SE as

$$S_{lower}(\theta,\varphi) = \min\{S_{1,low}(\theta,\varphi), S_{2,low}\} \quad (12)$$
$$S_{upper}(\theta,\varphi) = \min\{S_{1,up}(\theta,\varphi), S_{2,up}\} \quad (13)$$

respectively, where

$$S_{1,low} = \frac{\log\left(1+\rho_1 r_{sd}^{-4}\right)+\theta\bar{\varepsilon}\log\left(1+\varphi\rho_1 r_{rd}^{-4}/\theta\right)}{(\theta\bar{\varepsilon}+1)} \quad (14)$$
$$S_{2,low} = S_{2,low} = \left(\log\left(1+\rho_3 r_{sr}^{-4}\right)\right)/(\theta\bar{\varepsilon}+1) \quad (15)$$
$$S_{1,up} = S_{1,low} \quad (16)$$
$$S_{2,up} = S_{2,up} = \left(\log\left(1+\rho_3 r_{sr}^{-4}+\rho_1 r_{sd}^{-4}\right)\right)/(\theta\bar{\varepsilon}+1). \quad (17)$$

Due to the similarity between lower and upper bounds, our subsequent discussions will only focus on the lower bound. We note that the analysis can be easily extended to upper bounds.

Theorem 3: The SE lower bound is a monotonically increasing function of φ. There exists a threshold φ_{th}, for $\varphi < \varphi_{th}$, the optimal resource allocation pair (θ_s^*,φ_s^*) that achieves the SE lower bound is

$$\theta_{s(1)}^* = \frac{\varphi_s^*\rho_1 r_{rd}^{-4} W_0(k_1)}{\varphi_s^*\rho_1 r_{rd}^{-4}\bar{\varepsilon}-1-W_0(k_1)} \quad (18)$$

where $k_1 = \left(\varphi_s^*\rho_1 r_{rd}^{-4}\bar{\varepsilon}-1\right)e^{-1}/\left(1+\rho_1 r_{sd}^{-4}\right)$, and $W_0(x)$ is the Lambert W function which satisfies $W(x)e^{W(x)} = x$ for any complex number x [19]. For $\varphi > \varphi_{th}$,the optimal resource allocation pair $(\theta_{s(2)}^*,\varphi_s^*)$ that achieves the SE lower bound is

$$\theta_{s(2)}^* = \Psi^{-1}(\varphi_s^*) \quad (19)$$

where $\Psi(\cdot)$ is defined in (10) and $\Psi^{-1}(\cdot)$ denotes its inverse function. The threshold φ_{th} is the unique solution of $\theta_{s(1)}^*(\varphi) = \theta_{s(2)}^*(\varphi)$.

Proof: Analyzing the derivatives of (14) and (15) with respect to θ and φ, we can show that $S_{1,low}$ is a monotonically increasing function of φ and a convex function of θ. In addition, $S_{2,low}$ is a monotonically decreasing function of θ but unrelated to φ. Further observing (14) and (15), we can get $S_{1,low}(0,\varphi) < S_{2,low}(0,\varphi)$. Moreover, both $S_{1,low}$ and $S_{2,low}$ approach zero as θ goes to infinity. Consequently, for any given φ, there exists a unique $\theta_{s(1)}^*$ that maximizes $S_{1,low}$ and one solution $\theta_{s(2)}^*$ that satisfies $S_{1,low}(\theta_{s(2)}^*) = S_{2,low}(\theta_{s(2)}^*)$. The optimal θ_s^* that achieves the highest SE is either $\theta_{s(1)}^*$ or $\theta_{s(2)}^*$. It can be further drawn that there is a unique threshold point φ_{th} that gives $\theta_{s(1)}^* = \theta_{s(2)}^*$.

For a given power ratio φ, if $\varphi < \varphi_{th}$, the optimal resource allocation is $\theta_s^* = \theta_{s(1)}^*$, and the corresponding spectral efficiency is $S_{lower} = S_{1,low}(\theta_{s(1)}^*)$; otherwise, the optimal resource allocation is $\theta_s^* = \theta_{s(2)}^*$, and the corresponding spectral efficiency is $S_{lower} = S_{1,low}(\theta_{s(2)}^*) = S_{2,low}(\theta_{s(2)}^*)$.

Recall that $\theta_{s(1)}^*$ is the maximum point of $S_{1,low}$. Since $S_{1,low}$ is a continuous and differentiable function, we have $\frac{\partial S_{1,low}}{\partial\theta}(\theta_{s(1)}^*) = 0$, i.e.,

$$\log\left(1+\frac{\varphi\rho_1 r_{rd}^{-4}}{\theta_{s(1)}^*}\right)-\log\left(1+\rho_1 r_{sd}^{-4}\right)-\frac{\varphi\rho_1 r_{rd}^{-4}\left(\theta_{s(1)}^*\bar{\varepsilon}+1\right)}{\ln 2\left(\theta_{s(1)}^*+\varphi\rho_1 r_{rd}^{-4}\right)} = 0 \quad (20)$$

Let $a = \varphi\rho_1 r_{rd}^{-4}, b = 1+\rho_1 r_{sd}^{-4}$, $x = (\theta+a)/(\theta b)$, (20) can be rewritten as

$$\ln(x) = 1+\frac{a\bar{\varepsilon}-1}{bx} \quad (21)$$

Let $\beta = (a\bar{\varepsilon}-1)/b$, (21) can be rearranged into the common form of the Lambert W function as $\frac{\beta}{e} = \frac{\beta}{x}e^{\frac{\beta}{x}}$. Consequently, the solution of (21) is $x = \beta/W_0(\beta/e)$. It follows that $\theta_{s(1)}^*$ can be calculated by (18).

Recall that $\theta_{s(2)}^*$ is defined as the intersection of two curves, i.e., $S_{1,low}(\theta_{s(2)}^*) = S_{2,low}(\theta_{s(2)}^*)$. It follows that $\theta_{s(2)}^*$ should lay on the curve of $S_{2,low}$ and it is easy to show that $\theta_{s(2)}^* = \Psi^{-1}(\varphi_s^*)$. ■

Theorem 4: For $\varphi < \varphi_{th}$, the optimal resource allocation solution $\left(\theta_{s(1)}^*,\varphi_s^*\right)$ derived in (18) also achieves the upper bound of SE. For $\varphi > \varphi_{th}$, the optimal resource allocation pair $\left(\theta^*_{s(2)}, \varphi^*_s\right)$ that achieves the SE upper bound is

$$\theta^*_{s(2)} = \Phi^{-1}(\varphi^*_s) \tag{22}$$

where $\Phi(\cdot)$ is defined in (11).

Proof: It has been shown that $\theta^*_{s(1)}$ maximizes $S_{1,low}$. Moreover, $S_{1,low}(\theta) < S_{2,low}(\theta)$ when $\theta < \theta^*_{s(1)}$. Because $S_{1,up}(\theta) = S_{1,low}(\theta)$, it's clear that $\theta^*_{s(1)}$ also maximizes $S_{1,up}$. Consequently, when $\theta < \theta^*_{s(1)}$, we have

$$S_{1,low}(\theta) = S_{1,up}(\theta) < S_{2,low}(\theta) < S_{2,up}(\theta) \tag{23}$$

and $\theta^*_{s(1)}$ maximizes both the lower bound and upper bound of SE.

In Theorems 3 and 4, the threshold φ_{th} is defined as the solution of $\theta^*_{s(1)}(\varphi) = \theta^*_{s(2)}(\varphi)$. Although this equation cannot be solved analytically, φ_{th} can be easily calculated via numerical methods as both $\theta^*_{s(1)}$ and $\theta^*_{s(2)}$ are tractable. For example, with $\bar{\varepsilon} = 1$ and $(r_{sd}, r_{rd}, r_{sr}) = (1, 1/2, 1/2)$, we have $\varphi_{th} = 0.56$. ■

C. Energy Efficiency

The EE metric evaluates the average number of bits per Joule spent. In this paper we consider the total energy consumption of the source and relay. When the cognitive spectrum is unavailable (i.e., $\varepsilon = 0$), the relay consumes no power. According to (4)-(9), the corresponding lower and upper bounds of EE is given by

$$E_{lower}(\theta, \varphi) = \min\{E_{1,low}(\theta, \varphi), E_{2,low}(\theta, \varphi)\} \tag{24}$$

$$E_{upper}(\theta, \varphi) = \min\{E_{1,up}(\theta, \varphi), E_{2,up}(\theta, \varphi)\} \tag{25}$$

respectively, where

$$E_{1,low} = \frac{W_1\left(\log(1+\rho_1 r_{sd}^{-4}) + \theta\bar{\varepsilon}\log\left(1+\varphi\rho_1 r_{rd}^{-4}/\theta\right)\right)}{(P_1 + P_1\varphi\bar{\varepsilon})} \tag{26}$$

$$E_{2,low} = \left(W_1 \log\left(1+\rho_3 r_{sr}^{-4}\right)\right) / (P_1 + P_1\varphi\bar{\varepsilon}) \tag{27}$$

$$E_{1,up} = E_{1,low} \tag{28}$$

$$E_{2,up} = \left(W_1 \log\left(1+\rho_3 r_{sr}^{-4} + \rho_1 r_{sd}^{-4}\right)\right) / (P_1 + P_1\varphi\bar{\varepsilon}). \tag{29}$$

Theorem 5: The EE lower bound is a monotonically increasing function of θ. There exists a threshold θ_{th}, for $\theta < \theta_{th}$, the optimal resource allocation pair $(\theta^*_e, \varphi^*_e)$ that achieves the EE lower bound is

$$\varphi^*_{e(1)} = -\frac{-\rho_1 r_{rd}^{-4} + \theta^*_e\bar{\varepsilon} + \theta^*_e\bar{\varepsilon}W_0(k_2)}{\rho_1 r_{rd}^{-4}\bar{\varepsilon}W_0(k_2)} \tag{30}$$

where $k_2 = \left(\rho_1 r_{rd}^{-4} - \theta^*_e\bar{\varepsilon}\right) e^{\frac{-\theta^*_e\bar{\varepsilon} + \ln\left(1+\rho_1 r_{sd}^{-4}\right)}{\theta^*_e\bar{\varepsilon}}} / \theta^*_e\bar{\varepsilon}$, For $\theta > \theta_{th}$, the optimal allocation pair $(\theta^*_e, \varphi^*_e)$ that achieves the EE lower bound is

$$\varphi^*_{e(2)} = \Psi(\theta^*_e) \tag{31}$$

Where $\Psi(\cdot)$ is defined in (10). The threshold θ_{th} is the unique solution of $\varphi^*_{e(1)}(\theta) = \varphi^*_{e(2)}(\theta)$.

Proof: The analysis of EE is very similar to that of SE and there exists a strong duality between Theorems 3 and 5. The derivatives of $E_{1,low}$ and $E_{2,low}$ indicates that $E_{1,low}$ is a monotonically increasing function of θ and a convex function of φ. In addition, $E_{2,low}$ is a monotonically decreasing function of φ but unrelated to θ. Define $\varphi^*_{e(1)}$ as the maximum point of $E_{1,low}$. The calculation of $\varphi^*_{e(1)}$ can follow the same procedure as the derivation of $\theta^*_{s(1)}$ in Theorem 3. Define $\varphi^*_{e(2)}$ as the intersection of $E_{1,low}(\varphi)$ and $E_{2,low}(\varphi)$, it can be calculated as the inverse function of (10). Furthermore, the threshold θ_{th} is the solution of $\varphi^*_{e(1)}(\theta) = \varphi^*_{e(2)}(\theta)$. It also can be concluded that the lower bound and upper bound of EE overlaps when $\theta < \theta_{th}$. ■

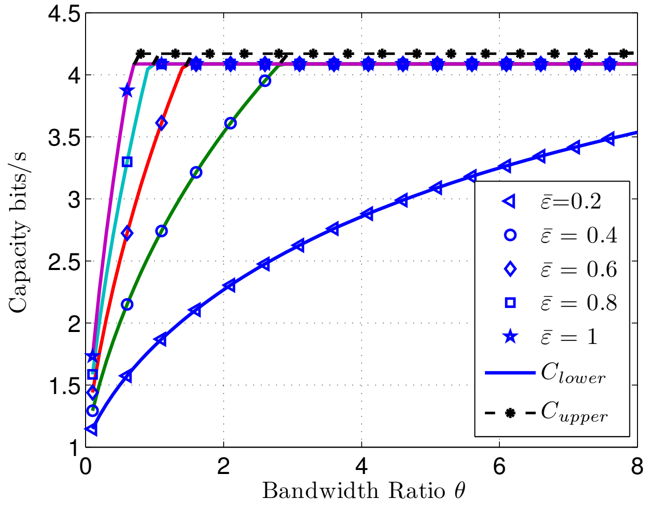

Fig. 2. Upper and lower bounds of the capacity as a function of θ with varying $\bar{\varepsilon}$ ($\varphi = 1$).

IV. Numerical Results and Discussions

This section presents numerical results based on our previous analysis. The licensed RR is assumed to be given and our discussions will focus on the cognitive RR. Without loss of generality, the licensed bandwidth W_1 and transmission power P_1 are normalized to 1. Except otherwise mentioned, we assume that the relay lies in the middle of the source and destination. The normalized distances are set to be $(r_{sd}, r_{rd}, r_{sr}) = (1, 1/2, 1/2)$.

Fig. 2 illustrates the capacity bounds based on (4) and (5), where capacity bounds are shown as functions of the cognitive RR characterized by three parameters, θ, φ, and $\bar{\varepsilon}$. The following observations are made. First, the difference between upper and lower bounds of the capacity is insignificant. In fact, a close look into the equations reveal that such difference will remain small as long as the SNR of the source-to-relay channel is significantly larger than the SNR of the source-to-destination channel. Second, capacity increases with increasing θ, φ or $\bar{\varepsilon}$ until it reaches the maximum values. Such maximum values, as defined by (7) and (9), are entirely determined by the licensed RR and relay location. This observation implies that the allocation of cognitive RR should 'match' the licensed RR to avoid waste of resources. Third, because the cognitive RR is typically unreliable but has a wide bandwidth, we are interested in the trade-off between reliability $\bar{\varepsilon}$ and bandwidth θ. Fig. 2 shows that with relatively high reliability ($\bar{\varepsilon} > 0.4$), decreasing

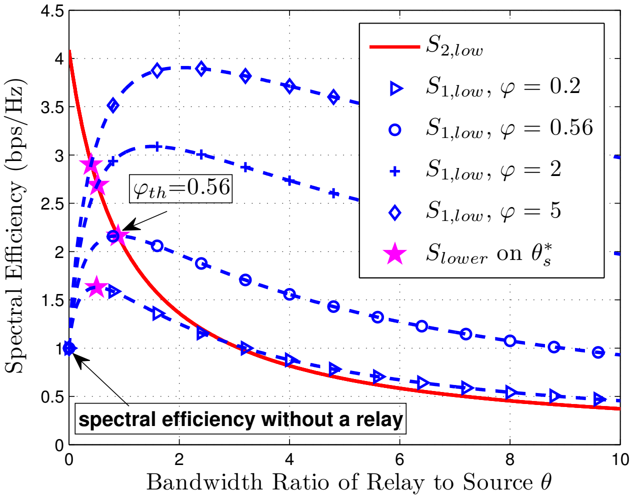

Fig. 3. SE lower bound of HCGRC as a function of θ with varying φ ($\bar{\varepsilon} = 1$).

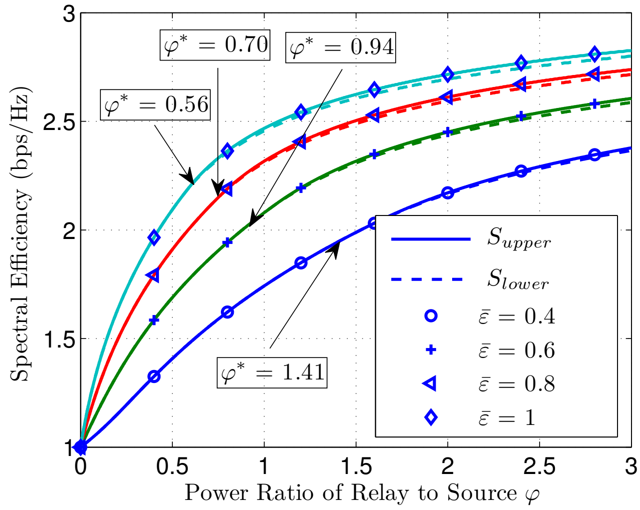

Fig. 4. Upper and lower bounds of the optimal SE as a function φ with varying $\bar{\varepsilon}$.

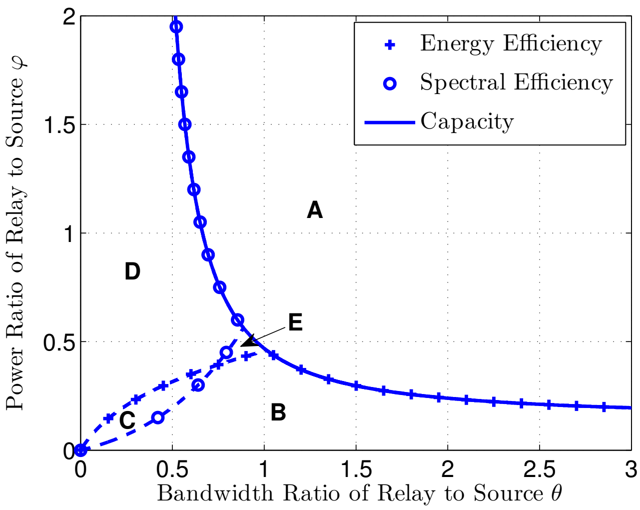

Fig. 5. Optimal bandwidth-power ($\theta - \varphi$) allocation curves with respect to capacity, SE, and EE ($\bar{\varepsilon} = 1$).

reliability can be compensated by increasing bandwidth with the same proportion. In other words, roughly the same capacity is achieved when the product of $\bar{\varepsilon}$ and θ is a constant. This is because the relay channel currently operates in the power-limited regime and the capacity scales linearly with bandwidth. However, when the reliability is relatively low ($\bar{\varepsilon} < 0.4$), much more bandwidth is needed to compensate for a slight decrease of reliability. This is because the relay channel now operates in the bandwidth-limited regime, therefore increasing bandwidth become less effective in improving the capacity.

Fig. 3 shows the SE as a function of θ with varying φ. Because the upper and lower bounds are very close, only the lower bound is illustrated based on (12). The 'star' signs represent where the optimal SE is achieved. It is shown that with given φ, there exists a unique bandwidth ratio θ that maximizes the SE. With increasing φ, the optimal SE is initially given by the maximum of $S_{1,low}$ and latter by the intersection of $S_{1,low}$ and $S_{2,low}$. In our case, this transition takes place when $\varphi_{th} = 0.56$ ($\bar{\varepsilon} = 1$). Moreover, either short-supply or over-provision of cognitive bandwidth θ has negative impacts on SE. Therefore, the key of SE optimization lies on bandwidth allocation. Finally, it is shown that the optimal SE of the cognitive relay system is always greater than the SE of the direct link (i.e., without a relay).

Fig. 4 shows the upper and lower bounds of the optimal SE (i.e., maximum SE over possible values of θ) as a function of φ and $\bar{\varepsilon}$. It is observed that for $\varphi < \varphi_{th}$, the lower bound overlaps with the upper bound, implying that the exact SE values are obtained. Even for $\varphi > \varphi_{th}$, the gaps between the two bounds are observed to be small. Moreover, increasing the cognitive power φ is always beneficial to the SE but become less effective when $\varphi > \varphi_{th}$. Finally, it is shown that SE degrades with decreasing reliability.

Based on Theorems 2, 3, and 5, Fig. 5 shows the optimal power-bandwidth allocation curve with respect to the lower bounds of the capacity, SE, and EE, respectively. The three curves divide the power-bandwidth plane into five areas. The implication of each area is explained as follows: (A) Resource excess area: Power and bandwidth are over-provisioned and have caused negative impacts on SE and EE; (B) Power hungry area: increasing power will improve all three metrics, while increasing bandwidth will improve capacity and EE but degrade SE. (C) Power and bandwidth hungry area: increasing either power or bandwidth will improve all three metrics. (D) Bandwidth hungry area: increasing bandwidth will improve all three metrics, while increasing power will improve capacity and SE but degrade EE. (E) Trade-off area: increasing power will improve capacity and SE but degrade EE, while increasing bandwidth will improve capacity and EE but degrade SE. For an arbitrary resource allocation pair (θ, φ), it must locate in one of the five areas. The above insights can provide useful guidelines for the cognitive relay to harness proper amount of cognitive RRs.

Finally, we briefly discuss the impact of relay location on the system performance. The problem of optimal relay location

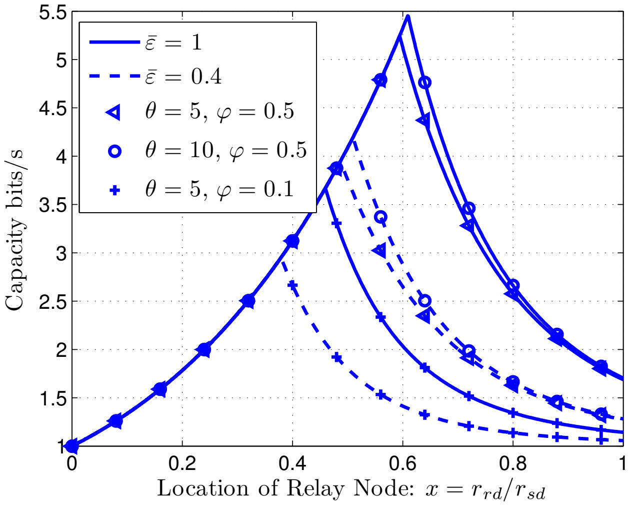

Fig. 6. Capacity of HCGRC as a function of relay location with varying θ, φ, and $\bar{\varepsilon}$.

in conventional relay channels has been studied in [20]–[22]. Their conclusions, however, cannot be directly applied to HCGRC because HCGRC have distinct features in terms of radio resource constraints. In this paper, we are particularly interested in understanding how the optimal relay location changes with varying cognitive RRs. In Fig. 6, we vary parameters θ, φ and $\bar{\varepsilon}$ and calculate the capacity as a function of the relay position. The relay is assumed to move from the source to the destination in a straight line. It is observed that given the cognitive RR, a single optimal location exists to achieve the maximum capacity. Interestingly, the same location is also optimal in terms of SE and EE. This is because all the three metrics have the same trade-off structure (see (4),(12) and (24)) with regard to location variations. As a general trend, improving the quality of the cognitive RR will move the optimal relay location towards the source.

V. Conclusion

In this paper, we have studied a new type of relay channels called HCGRC. The fundamental properties of HCGRC have been analyzed thoroughly in terms of capacity, SE, and EE. For each metric, the upper and lower bounds have been derived analytically. Moreover, the optimal resource allocation strategies with respect to each metric have been proposed to provide guidelines for the proper usage of cognitive RRs. It has been shown that improving the EE and SE of HCGRC are not necessarily conflicting objectives. With proper bandwidth and power allocation, HCGRC based relay can deliver significant performance gains compared with direct transmissions. Moreover, the performance gains of HCGRC can be further improved if the relay location is optimized according to the available cognitive RRs.

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