

# Energy-Spectral Efficiency Tradeoff of Visible Light Communication Systems

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**Abstract**—In this paper, we propose a new definition of the energy efficiency (EE) for indoor visible light communication (VLC) systems and further investigate the tradeoff between the EE and spectral efficiency (SE). Different from the conventional concept of the EE in wireless communication systems, the newly defined EE in VLC systems utilizes the optical communication power instead of radio frequency power to evaluate the consumed energy. Some key parameters (e.g., horizontal distance and vertical distance) in optical transmission circumstances are included when calculating the SE. The nonlinearity of light-emitting diode (LED) is also considered in the EE-SE tradeoff and such nonlinearity can be eliminated by pre-distortion. In particular, the relation between the electrical power and optical power can be derived based on the LED nonlinearity. Numerical results verify the presented EE-SE tradeoff. It is shown that the geometric parameters and the LED turn-on voltage (TOV) can significantly affect the relationship of the EE and SE.

**Index Terms**—VLC, LED, energy efficiency, spectral efficiency, energy-spectral efficiency tradeoff.

## I. INTRODUCTION

With the increasing number of wireless applications and devices, there will be a concern about the scarcity of spectral resource. To solve this problem, VLC is used as a complementary communication technology especially in indoor scenarios in recent years [1]. In VLC, visible light is used as information transmission medium. The wavelength of visible light ranges from 375 nm to 780 nm. Commercial LEDs can be used as signal transmitters. Nowadays, efficient and cheap LEDs are widely used and it is a trend that LEDs will replace incandescent light bulbs. So LEDs can be used as a kind of communication devices [3].

Compared to radio frequency (RF) communications, there are some different characteristics of VLC. Visible light belongs to non-regulated frequency and its frequency is much higher than that of RF [4]. The VLC has good security performance and the frequency reuse can be realized easily due to the fact that visible light cannot travel through walls. It can also be used in electromagnetic sensitive scenarios because there is no interference between VLC and RF systems. Communications between vehicles and traffic lights can be realized using automotive light. In VLC, the electrical signal is sent to LED through the drive circuit and the LED light is able to carry information. By modulating the signal into light intensity and

detect the signal directly, it has the potential to offer high speed communication [5]. To gain a better performance, a DC bias point is often added on LEDs.

Tremendous wireless communication devices will be used in the near future, leading to huge energy consumption [6]. The improvement of battery is much slower than what we need, which will cause a gap between the required and available battery [7]. EE and SE are two basic metrics for wireless communications [8]. How to provide guaranteed quality of service (QoS) to users with affordable energy is a challenge [9]. EE and SE in a specific system are used to evaluate the performances of energy and spectral usage. In particular, the tradeoff between EE and SE should be delicately considered in the design of high performance wireless communication applications. The EE-SE tradeoff in varying cognitive radio networks (CRNs) and wireless communication systems have been discussed in [10], [11]. The area spectral efficiency (ASE) has been evaluated for a heterogeneous network combining VLC and RF femtocell systems in [12]. The energy efficiency model of small cell backhaul networks has been discussed in [13].

The EE and SE in VLC and the corresponding EE-SE tradeoff should be discussed. In this paper, we mainly investigate the relationship between EE and SE in VLC. In particular, the inherent characteristics of LED are considered to analyze power consumption in VLC. Based on the characteristics of input electrical power and output optical power, the definitions of EE and SE are provided. Moreover, the closed form of EE-SE tradeoff in VLC is derived. The effect of transmission parameters is also analysed.

The remainder of this paper is organised as follows. The system model, LED model and channel model are introduced in Section II. Section III describes the definitions of EE and SE. The EE-SE tradeoff is also analysed in this section. Numerical results with analysis are provided in Section IV and the conclusion is given in Section V.

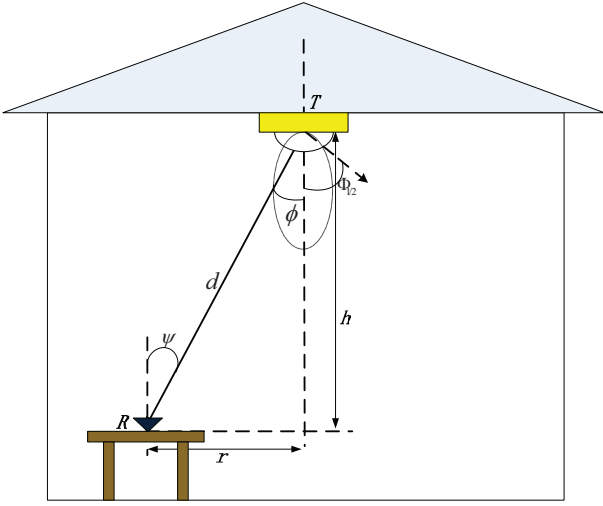


Fig. 1. A geometric scenario of indoor VLC systems.

## II. SYSTEM MODEL

### A. Geometric Scenario

The geometric scenario about the system is an empty room depicted in Fig. 1. We make some assumptions as common settings [14], for example, the transmitter is placed in the center of the ceiling and it is vertical to the ceiling. The receiver is placed vertically up on a desk. The receiver field of view (FOV) is large enough to receive the light signal at any places of the room. The vertical and horizontal distance between the transmitter and receiver are denoted by  $h$  and  $r$ , respectively. The direct distance between the transmitter and the receiver is denoted by  $d$ . The simplest scenario that one user share the whole bandwidth  $B$  is considered. The transmitter LED is denoted by  $T$  and  $R$  denotes the photo detector (PD). The light radiance angle and the light incidence angle are  $\phi$  and  $\psi$ , respectively.  $\Phi_{1/2}$  is the half intensity viewing angle of the LED.

### B. LED Model

The LED is a nonlinear device, in which the relationship between the forward voltage  $U$  and the forward current  $I$  is not completely linear. However the nonlinearity can be compensated by pre-distortion method [15], we can see the relation as a linear function. The most important feature of LED is that it has a TOV  $U_r$ . Only when the forward voltage is higher than  $U_r$ , its forward current is not zero.

To gain a faster reaction speed and a better performance, we add a DC bias on LED. The bias is defined as

$$U_b = \frac{U_{\max} + U_r}{2} \quad (1)$$

where  $U_{\max}$  is the maximum allowable voltage. The input electrical signal  $X(t)$  is a random signal and its power is  $P_s$ . We make an assumption that the signal amplitude is smaller than  $\frac{U_{\max} - U_r}{2}$ . The slope of the line can be defined as

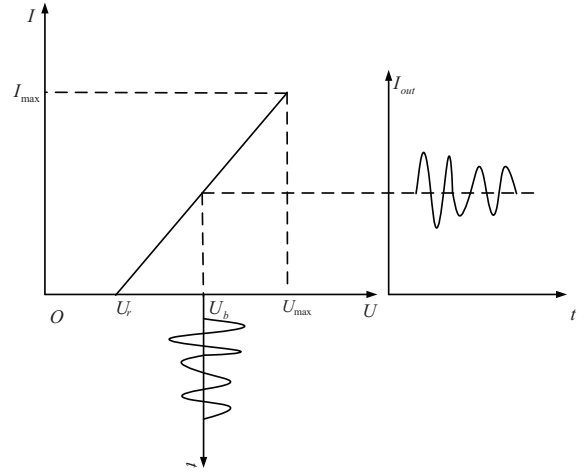


Fig. 2. The relationship between forward voltage and forward current.

$k = \frac{I_{\max}}{U_{\max} - U_r}$ , and  $I_{\max}$  is the maximum allowable current. The details of the input voltage and current are shown in Fig. 2.

### C. Channel Model

The channel of VLC is different from that of RF. The area of PD is much larger than the wavelength of visible light, and little change can be observed when the receiver moves in a distance of few wavelength [16]. Moreover, path loss and inter-symbol interference (ISI) affect channel very much. But in this work we only consider LOS path because the LOS path contains most of the energy [17]. The DC gain from the transmitter to the receiver is given as [17]–[19]

$$g = \frac{(m+1)A_{pd}}{2\pi d^2} \cos^m(\phi) \cos(\psi) T(\psi) G(\psi) \text{rect}\left(\frac{\psi}{FOV}\right) \quad (2)$$

where  $m$  is the Lambertian order, which is defined as  $m = -1/\log_2(\cos\Phi_{1/2})$  [20],  $A_{pd}$  is the area of PD.  $T(\psi)$  is the gain of the optical filter, for simplification it is set to 1.  $G(\psi)$  is the concentrator gain, and it is given as

$$G(\psi) = \begin{cases} \frac{n^2}{\sin(\Psi_c)}, & 0 \leq \psi \leq \Psi_c \\ 0, & \psi > \Psi_c \end{cases} \quad (3)$$

where  $\Psi_c$  models the concentrator FOV (semiangle).  $n$  is the reflective index of receiver optics. The rectangular function in (2) is given as

$$\text{rect}(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1. \end{cases} \quad (4)$$

The noise in VLC can be described as AWGN and the optical wireless communication channel can be described as follows [21]

$$Y(t) = SX(t) \otimes g + N(t) \quad (5)$$

where  $S$  is the PD responsivity,  $X(t)$  is the transmitted optical signal,  $g$  is the channel DC gain,  $\otimes$  represents convolution,  $N(t)$  is the channel AWGN noise,  $Y(t)$  is the received signal.

#### D. Noise Analysis

There are mainly two kinds of noises at optical receiver, shot noise and thermal noise which can be expressed as AWGN noise [22]. A shot noise can be denoted by [16], [23]

$$\sigma_{\text{shot}}^2 = 2qSP_oB + 2qI_{bg}B \quad (6)$$

where the electron charge is denoted by  $q$ ,  $P_o$  is the received optical power at PD. The whole bandwidth is  $B$ , and  $I_{bg}$  is the background current.

The thermal noise is dominated by the noise that generated at the resistor  $R_L$ , and it can be expressed as [24]

$$\sigma_{\text{thermal}}^2 = \frac{4k_BTB}{R_L} \quad (7)$$

where  $k_B$  is the Boltzmann's constant,  $T$  is the absolute temperature of device. The load resistance is denoted by  $R_L$ . The total noise power can be written as

$$P_n = \sigma_{\text{shot}}^2 + \sigma_{\text{thermal}}^2 \quad (8)$$

### III. SPECTRAL EFFICIENCY AND ENERGY EFFICIENCY IN INDOOR VLC

#### A. Energy Efficiency

Energy efficiency is defined as the ratio of the link capacity  $C$  over the average power consumption  $P$  [10]

$$\eta_{EE} = \frac{C}{P} \quad (9)$$

As for the VLC, the power for illumination can be used for communication at the same time. So there is no need to separate the power. For simplification, we do not discuss illumination requirement. The average optical power can be considered as the illumination power  $P_o$ . The random signal passing through LED which will cause a power that can be regarded as communication power  $P_{o,c}$ . In this paper, we use the power consumed by LED  $P_{\text{ele}}$  in our definition of EE, i.e.,

$$\begin{aligned} \eta_{EE} &= \frac{C}{P} \\ &= \frac{B \log_2(1 + \gamma)}{P_{\text{ele}}} \\ &= \frac{B \log_2(1 + \frac{(SgP_{o,c})^2}{P_n})}{P_{\text{ele}}} \end{aligned} \quad (10)$$

where  $P_{\text{ele}}$  can be expressed as

$$\begin{aligned} P_{\text{ele}} &= E\{ui\} \\ &= E\{(U_b + X(t))k(U_b + X(t) - U_r)\} \\ &= kU_b(U_b - U_r) + P_s \end{aligned} \quad (11)$$

where  $u$  and  $i$  are the voltage and current on LED, respectively. The optical power is approximately linear to the forward current  $I$ . So the communication optical power can be defined as  $P_{o,c} = RkX(t)$ , where  $R$  is the responsivity of LED with a unit of W/A. The value of  $R$  is on the basis of some high-power white LEDs [25]. In VLC, the LED must satisfy some illumination conditions, but in this paper we don't consider them for the purpose of simplicity. After converting

optical power into current, the received electrical power can be expressed as

$$\begin{aligned} P_{r,c} &= E\{(SgP_{o,c})^2\} \\ &= (RSg)^2kE\{kX(t)^2\} \\ &= (RSg)^2kP_s. \end{aligned} \quad (12)$$

Based on (11) and (12), the relationship between  $P_{\text{ele}}$  and  $P_{r,c}$  can be expressed as

$$P_{\text{ele}} = kU_b(U_b - U_r) + \frac{P_{r,c}}{(RSg)^2k} \quad (13)$$

From Fig. 1 we can see that

$$\cos(\psi) = \cos(\phi) = \frac{h}{\sqrt{(h^2 + r^2)}} \quad (14)$$

Taking (14) into (2), we can get

$$g = K \cdot \frac{h^{m+1}}{(r^2 + h^2)^{\frac{m+1}{2}}} \quad (15)$$

where  $K = \frac{(m+1)A_{pd}}{2\pi} \cdot G(\psi)$ . In this transformation,  $g$  changes to a function of horizontal distance  $r$  and vertical distance  $h$ . So the EE is a function of  $r$  and  $h$ . The SNR  $\gamma$  can be written as follows

$$\gamma = \frac{P_{r,c}}{P_n} = \frac{(RSK)^2kP_s}{P_n} \cdot \frac{h^{2(m+1)}}{(r^2 + h^2)^{m+3}} \quad (16)$$

#### B. Spectral Efficiency

Spectrum efficiency is defined as the ratio of the link capacity over the link bandwidth [11]

$$\eta_{SE} = \frac{C}{B} = \log_2(1 + \gamma) \quad (17)$$

where  $R$  is the capacity of the system and  $B$  is the bandwidth. Taking (16) into (17), we can get

$$\eta_{SE} = \log_2\left\{1 + \frac{(RSK)^2kP_s}{P_n} \cdot \frac{h^{2(m+1)}}{(r^2 + h^2)^{m+3}}\right\} \quad (18)$$

We can see that  $\eta_{SE}$  is a function of horizontal distance  $r$  and vertical distance  $h$ . The horizontal distance  $r$  and the vertical distance  $h$  can affect the SE in a practical VLC circumstance.

#### C. EE-SE Tradeoff

By solving the simultaneous equations (10), (13) and (17), the tradeoff between EE and SE can be deduced as

$$\begin{aligned} \eta_{EE} &= \frac{B\eta_{SE}}{P_{\text{ele}}} \\ &= \frac{B\eta_{SE}}{kU_b(U_b - U_r) + \frac{P_{r,c}}{(RSg)^2k}} \\ &= \frac{Bk(RSg)^2\eta_{SE}}{(RSgk)^2U_b(U_b - U_r) + (2^{\eta_{SE}} - 1)P_n}. \end{aligned} \quad (19)$$

*Proof:* From (16) we can get

$$P_{r,c} = (2^{\eta_{SE}} - 1)P_n \quad (20)$$

Taking (20) into (13), we can calculate

$$P_{\text{ele}} = kU_b(U_b - U_r) + \frac{(2^{\eta_{SE}} - 1)P_n}{(RSg)^2k}. \quad (21)$$

□

So we can see that  $\eta_{EE}$  is a function of  $\eta_{SE}$  under the parameter  $r$ ,  $h$  and turn-on voltage  $U_r$ . The EE-SE tradeoff can be evaluated with varying circumstance parameters  $r$ ,  $h$  and  $U_r$ .

TABLE I  
SIMULATION PARAMETERS

Parameter	Value
Bandwidth, $B$	20 MHz
Responsivity of LED, $R$	0.40 W/A
Maximum voltage, $U_{\text{max}}$	3.4 V
Maximum current, $I_{\text{max}}$	0.7 A
Half intensity viewing angle, $\Phi_{1/2}$	60°
Lambertian order, $m$	1
Electron charge, $q$	$1.602 \times 10^{-19}$ C
Absolute temperature, $T$	295 K
Boltzmann's constant, $k_B$	$1.380610 \times 10^{-23}$ J/K
Load resistance, $R_L$	10 kΩ
Background current, $I_{bg}$	5.1 mA
Physical area of detector, $A_{pd}$	1 cm <sup>2</sup>
Receiver FOV semiangle, $\Phi_c$	85°
Responsivity of PD, $S$	0.28 A/W
Reflective index, $n$	1.5

#### IV. SIMULATION RESULTS AND ANALYSIS

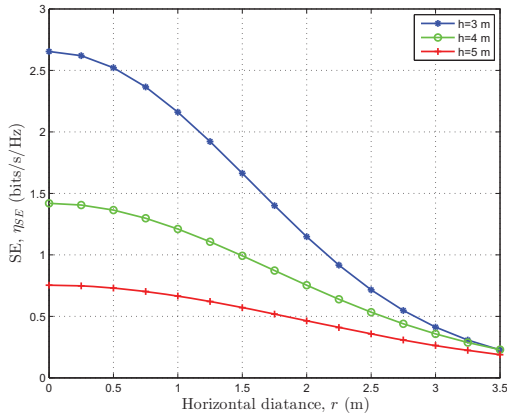


Fig. 3. The SE vs. the horizontal distance  $r$  with varying  $h$ .

In the simulations, the indoor VLC circumstance is exploited to evaluate EE and SE. The simulation parameters are summarized in Table I. The effect of the circumstance parameters on the SE is evaluated in Fig. 3, in which the horizontal distance  $r$  is set to 0 ~ 3.5 m. The vertical distance  $h$  is set to 3, 4 and 5 m and  $U_r$  is 2.75 V. We can observe that SE decreases with the increase of  $r$  and  $h$ . The EE-SE tradeoff with varying  $r$  is shown in Fig. 4, in which the horizontal

distance is set to 0, 1 and 2 m and  $h$  and  $U_r$  are 1 m and 2.75 V respectively. The tradeoff is evaluated by the function of EE and SE. According to the EE function with varying SE, we can see that an optimal EE can be achieved for a given specific SE range. For  $r = 2$  m, in the SE ranges 2 ~ 5 bits/s/Hz, an optimal EE about 3.1 kbits/Joule can be achieved.

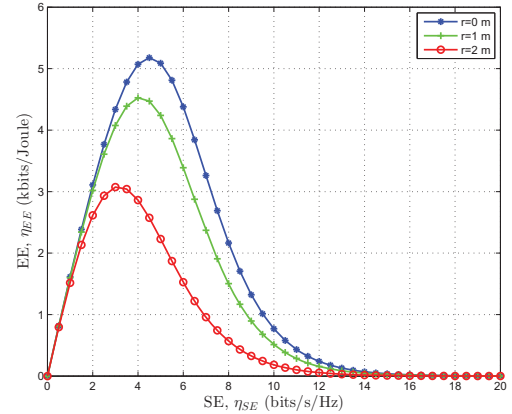


Fig. 4. The EE vs. SE with varying  $r$ .

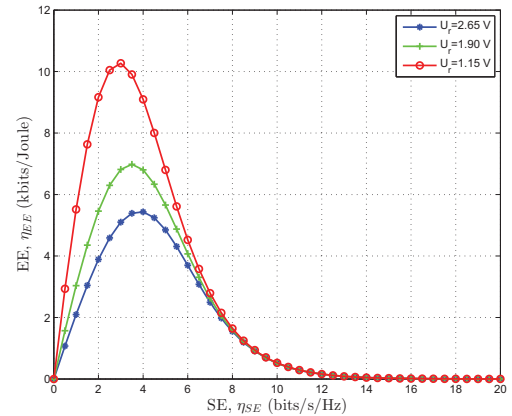


Fig. 5. The EE vs. SE with varying TOV  $U_r$ .

The EE-SE tradeoff with varying TOV  $U_r$  is shown in Fig. 5, in which the optimal EE can be achieved for SE with specific TOVs. We can see that with lower TOV, the superior EE-SE tradeoff can be achieved. Therefore, an efficient LED with small TOV is required. Similarly to Fig. 4, the effect of the vertical distance  $h$  on the EE-SE tradeoff is evaluate in Fig. 6, in which  $h$  is set to 3, 4 and 5 m. According to Fig. 4 and Fig. 6, we can see that the transmission circumstance can significantly affect the EE and SE performance.

From the above simulation results, we can conclude that the spacial transmission circumstance is crucial to VLC, leading

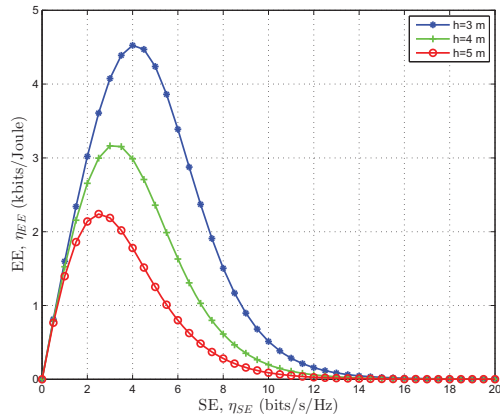


Fig. 6. The EE vs. SE with varying vertical distance  $h$ .

to a relatively small transmission cell is ideal. Optimal EE-SE tradeoff can be achieved by delicately designing the VLC transmission model.

## V. CONCLUSIONS

In this paper, a new definition of the EE in VLC systems has been proposed. Based on the new definition, the EE-SE tradeoff has been further deduced in a closed form. Such tradeoff has been given as an EE function in terms of the SE with key transmission parameters. The simulation results have verified the proposed EE-SE tradeoff in VLC systems. We have found that the optimal EE can be achieved. The transmission circumstance parameters can significantly affect the performance of the EE and SE. A practical and efficient VLC system can be designed according to the theoretical results of this paper.

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