# A NEW DETERMINISTIC PROCESS BASED GENERATIVE MODEL FOR CHARACTERIZING BURSTY ERROR SEQUENCES

**Cheng-Xiang Wang and Matthias Pätzold**

Faculty of Engineering and Science, Agder University College, Grooseveien 36, NO-4876 Grimstad, Norway
E-mail: cheng.wang@hia.no; matthias.paetzold@hia.no

**Abstract -** Efficient and accurate generative models are of great importance for the design and performance evaluation of wireless communication protocols as well as error control schemes. In this paper, deterministic processes are utilized to derive a new generative model for the simulation of bursty error sequences encountered in digital mobile fading channels. The proposed deterministic process based generative model (DPBGM) is simply a properly parameterized and sampled deterministic process followed by a threshold detector and two parallel mappers. The target error sequence is generated by a computer simulation of a frequency hopping (FH) convolutionally coded Gaussian minimum shift keying (GMSK) transmission system with Rayleigh fading. Simulation results show that this generative model enables us to match very closely any given gap distribution (GD), error-free run distribution (EFRD), error cluster distribution (ECD), error burst distribution (EBD), error-free burst distribution (EFBD), block error probability distribution (BEPD), and bit error correlation function (BECF) of the underlying descriptive model.

**Keywords -** Generative models, deterministic processes, burst error statistics, digital frequency hopping systems.

## I. INTRODUCTION

A digital wireless propagation channel is characterized by a variety of impairments resulting in the fact that errors tend to occur in clusters or bursts separated by fairly long error-free gaps. Many research studies have shown that the performance of high layer protocols as well as error control systems is very sensitive to the statistical properties of the underlying error sequences [1], [2]. Therefore, it is of great significance to develop accurate and efficient error models for characterizing bursty error sequences. Error models can be classified as descriptive models [3] and generative models [4]. A descriptive model often obtains target error sequences from a real digital channel or a computer simulation of the overall communication link. A generative model specifies an underlying mechanism that generates error sequences with desired statistics. Compared with a descriptive model, the main advantage of a generative model is that it greatly reduces the computational effort for generating long error sequences and therefore speeds up simulations.

For modeling of error sequences, various generative models have been presented based on finite [2], [4], [5] or infinite [4] state Markov chains or hidden Markov chains [6], [7]. In particular, much attention was devoted to simplified Frichman's models (SFMs) with only one error state [5], [8]. Recently, it was shown that alternative error generation mechanisms, different from Markov chains, can be used to produce error sequences. For instance, generative models based on chaotic equations [9] and context-free grammars [10] were proposed to simulate bursty error sequences. Interestingly, deterministic processes [11], which originally go back to Rice's sum-of-sinusoids [12], [13], were successfully applied to the development of generative models with good burst error statistics [14-16]. The DPBGM in [14] enables us to match closely any given GD, EFRD, EBD, and EFBD of the descriptive model. The GD, EFRD, and ECD of the DPBGMs in [15], [16] can be fitted very well to those of the descriptive model. However, all these DPBGMs in [14-16] failed to approximate the BEPD and the BECF of the descriptive model with high precision. The BEPD is an important quantity for the proper choice of error control strategies, while the BECF is useful for the design of bit interleavers [4]. Both statistics have great impact on the throughput and delay performance of communication protocols. The aim of this paper is to develop an improved DPBGM in such a way that it approximates very well not only any given GD, EFRD, ECD, EBD, EFBD, but also the BEPD and BACF of the underlying descriptive model.

The rest of the paper is organized as follows. Section II briefly reviews some terms and the relevant burst error statistics. A novel DPBGM is proposed in Section III. Section IV presents the underlying descriptive model, while Section V compares the burst error statistics of the adopted descriptive model, the proposed generative model, and a SFM. Finally, the conclusions are drawn in Section VI.

## II. BURST ERROR STATISTICS

For the sake of clarity, let us first introduce some terms used to describe the relevant burst error statistics. An error sequence is represented by a binary sequence of ones and zeros, where "1" and "0" denote error bits and correct bits, respectively. A gap is defined as a string of consecutive zeros between two ones, having a length equal to the number of zeros. An error cluster is a region where the errors occur consecutively and has a length equal to the number of ones. An error-free burst is defined as an all-zero sequence with a length of at least $\eta$ bits, where $\eta$ is a positive integer. Compared to a gap, an error-free burst has the minimum length of $\eta$ and is not necessarily located between two errors. An error burst is a sequence of zeros and ones starting and ending with a one, and separated from neighboring error bursts by error-free bursts. It should be observed that the number of consecutive error-free bits within an error burst is less than $\eta$. Hence, the local error density inside an error burst is greater than $\Delta = 1/\eta$.

With the above terms in mind, the following burst error statistics will be investigated:

1) $G(m_g)$: the GD, which is defined as the cumulative distribution of gap lengths $m_g$.
2) $P(0^{m_0}/1)$: the EFRD, which is the probability that an error is followed by at least $m_0$ error-free bits. The EFRD can be calculated from the GD. Note that $P(0^{m_0}/1)$ is a monotonically decreasing function of $m_0$ such that $P(0^0/1) = 1$ and $P(0^{m_0}/1) \to 0$ as $m_0 \to \infty$.
3) $P(1^{m_c}/0)$: the ECD, which is the probability that a correct bit is followed by $m_c$ or more error bits.
4) $P_{EB}(m_e)$: the EBD, which is the cumulative distribution of error burst lengths $m_e$.
5) $P_{EFB}(m_{\bar{e}})$: the EFBD, which is the cumulative distribution of error-free burst lengths $m_{\bar{e}}$.
6) $P(m, n)$: the BEPD, which is defined as the probability that a block of $n$ bits will contain at least $m$ errors. This quantity is important for determining the performance of error-correcting schemes.
7) $\rho(\Delta k)$: the BECF, which is defined as the probability of two error bits occurring at a distance of $\Delta k$ bits apart.

Note that an error sequence is the combination of consecutive error bursts and error-free bursts, while error bursts can further be subdivided into gaps and error clusters. To avoid a bit-by-bit processing of an error sequence, it is sensible to compress the error data by listing the successive error burst lengths and error-free burst lengths. Consequently, an error burst recorder $\mathbf{EB}_{rec}$ and an error-free burst recorder $\mathbf{EFB}_{rec}$ are obtained. Here, $\mathbf{EB}_{rec}$ is a vector which keeps a record of successive error burst lengths, while $\mathbf{EFB}_{rec}$ records successive error-free burst lengths. Let us denote the minimum value as $m_{B1}$ and the maximum value as $m_{B2}$ in $\mathbf{EB}_{rec}$. By analogy, the minimum value and the maximum value in $\mathbf{EFB}_{rec}$ are denoted as $m_{\bar{B}1}$ and $m_{\bar{B}2}$, respectively. For the derivation of the generative model in Section III, it is convenient to further define the following quantities:

1) $N_t$: the total length of the target error sequence.
2) $\mathcal{N}_{EB}$: the total number of error bursts, which equals the number of entries in $\mathbf{EB}_{rec}$.
3) $\mathcal{N}_{EFB}$: the total number of error-free bursts, which equals the number of entries in $\mathbf{EFB}_{rec}$.
4) $N_{EB}(m_e)$: the number of error bursts of length $m_e$ in $\mathbf{EB}_{rec}$. Apparently, $\sum_{m_e=m_{B1}}^{m_{B2}} N_{EB}(m_e) = \mathcal{N}_{EB}$ holds.
5) $N_{EFB}(m_{\bar{e}})$: the number of error-free bursts of length $m_{\bar{e}}$ in $\mathbf{EFB}_{rec}$. Similarly, $\sum_{m_{\bar{e}}=m_{\bar{B}1}}^{m_{\bar{B}2}} N_{EFB}(m_{\bar{e}}) = \mathcal{N}_{EFB}$ holds.
6) $\mathcal{R}_B$: the ratio of the mean value $M_{EB}$ of error bursts to the mean value $M_{EFB}$ of error-free bursts, i.e., $\mathcal{R}_B = M_{EB}/M_{EFB}$.
7) $\mathbf{ECG}_i$: a vector which lists successive error cluster lengths and gap lengths corresponding to each entry of $\mathbf{EB}_{rec}$. Clearly, $i = 1, 2, \ldots, \mathcal{N}_{EB}$. Note that $\mathbf{ECG}_i$ has an odd number of entries, with error cluster lengths as odd entries and gap lengths as even entries.

## III. The Generative Model

It is commonly accepted that the second order statistics of fading envelope processes are closely related to the statistics of burst errors. This indicates the potential of developing generative models by using fading processes. It is well known that deterministic processes [11], basing on the principle of Rice's sum-of-sinusoids [12], [13], are advantageous to be employed as channel simulators due to the easy determination of the model parameters, the efficient implementation on a computer, and their excellent statistical properties. Inspired by these promising advantages, we will show in the following how to utilize deterministic processes as a proper mechanism to generate bursty error sequences with desired statistics.

It is intuitive to relate error bursts and error-free bursts to fading intervals and inter-fade intervals, respectively, of a deterministic process. The idea of the proposed generative model is to derive directly from a deterministic process an error burst length generator and an error-free burst length generator. However, the employed deterministic process $\tilde{\zeta}(t)$ has to be properly parameterized and sampled with a certain sampling interval $T_A$. Here, $T_A$ can simply be equated with the symbol duration $T_s$ of the reference transmission system. The sampled deterministic process $\tilde{\zeta}(kT_A)$, where $k$ is a nonnegative integer, is then followed by a threshold detector. Error-free bursts are produced at the model's output if the level of $\tilde{\zeta}(kT_A)$ is above a given threshold $r_{th}$. The lengths of the generated error-free bursts equal the numbers of samples in the corresponding inter-fade intervals of $\tilde{\zeta}(kT_A)$. On the other hand, when the level of $\tilde{\zeta}(kT_A)$ falls below $r_{th}$, then this implies the occurrence of error bursts. The error burst lengths equal the numbers of samples in the corresponding fading intervals of $\tilde{\zeta}(kT_A)$. Consequently, an error burst length generator $\widetilde{\mathbf{EB}}_{rec}$ and an error-free burst length generator $\widetilde{\mathbf{EFB}}_{rec}$ are obtained. For the generative model, we use similar notations to those introduced in Section II by simply putting the tilde sign on all affected symbols, i.e., we write $\tilde{m}_{B1}$, $\tilde{\mathcal{N}}_{EFB}$, $\tilde{N}_{EB}(m_e)$, etc.

### A. The parametrization of the deterministic process

The parameters of the deterministic process are determined as follows. The level-crossing rate (LCR) $\tilde{N}_{\zeta}(r_{th})$ at the chosen threshold $r_{th}$ is fitted to the desired occurrence rate $R_{EB} = \mathcal{N}_{EB}/T_t$ of error bursts. Here, $T_t$ denotes the total transmission time of the reference transmission system, from which the target error sequence of length $N_t$ is obtained. Also, the ratio $\tilde{\mathcal{R}}_B$ of the average duration of fades (ADF) $\tilde{T}_{\zeta_-}(r_{th})$ at $r_{th}$ to the average duration of inter-fades (ADIF) $\tilde{T}_{\zeta_+}(r_{th})$ at $r_{th}$ is adapted to the ratio $\mathcal{R}_B = M_{EB}/M_{EFB}$. Let us consider the following continuous-time deterministic process [11]

$$\tilde{\zeta}(t) = |\tilde{\mu}_1(t) + j\tilde{\mu}_2(t)| \tag{1}$$

where

$$\tilde{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}) \ , \quad i = 1, 2 \ . \tag{2}$$

In (2), $N_i$ defines the number of sinusoids, $c_{i,n}$, $f_{i,n}$, and $\theta_{i,n}$ are called the gains, the discrete frequencies, and the phases, respectively. By using the method of exact Doppler spread (MEDS) [11], the phases $\theta_{i,n}$ are considered as the realizations of a random generator uniformly distributed over $(0, 2\pi]$, while $c_{i,n}$ and $f_{i,n}$ are given by

$$c_{i,n} = \sigma_0 \sqrt{\frac{2}{N_i}} \tag{3}$$

$$f_{i,n} \quad = \quad f_{max} \sin\left[\frac{\pi}{2N_i}(n - \frac{1}{2})\right] \tag{4}$$

respectively. Here, $\sigma_0$ is the square root of the mean power of $\tilde{\mu}_i(t)$ and $f_{max}$ represents the maximum Doppler frequency.

When using the MEDS with $N_i \geq 7$, it has been shown in [11] that the LCR $\tilde{N}_\zeta(r)$ of $\tilde{\zeta}(t)$ is very close to the LCR $N_\zeta(r)$ of a Rayleigh process, which is given by

$$N_\zeta(r) = \sqrt{\frac{\beta}{2\pi}} p_\zeta(r) \ , \quad r \geq 0 \tag{5}$$

where

$$\beta = 2(\pi\sigma_0 f_{max})^2 \tag{6}$$

and

$$p_\zeta(r) = \frac{r}{\sigma_0^2} \exp(-\frac{r^2}{2\sigma_0^2}) \ , \quad r \geq 0 \tag{7}$$

denotes the Rayleigh distribution. It can also be shown that the ADF $\tilde{T}_{\zeta_-}(r)$ and the ADIF $\tilde{T}_{\zeta_+}(r)$ of $\tilde{\zeta}(t)$ approximate very well the desired quantities $T_{\zeta_-}(r)$ and $T_{\zeta_+}(r)$, respectively, of a Rayleigh process. They can be expressed as

$$T_{\zeta_-}(r) = \sqrt{\frac{2\pi}{\beta}} \frac{\sigma_0^2}{r} \left[\exp(\frac{r^2}{2\sigma_0^2}) - 1\right] \ , \quad r \geq 0 \tag{8}$$

$$T_{\zeta_+}(r) = \sqrt{\frac{2\pi}{\beta}} \frac{\sigma_0^2}{r} \ , \quad r \geq 0 \ . \tag{9}$$

Consequently, the ratio $\tilde{\mathcal{R}}_B$ can be determined as follows

$$\tilde{\mathcal{R}}_B = \frac{\tilde{T}_{\zeta_-}(r_{th})}{\tilde{T}_{\zeta_+}(r_{th})} \approx \frac{T_{\zeta_-}(r_{th})}{T_{\zeta_+}(r_{th})} = \exp(\frac{r_{th}^2}{2\sigma_0^2}) - 1 \ . \tag{10}$$

Now, the task at hand is to find a proper parameter vector $\mathbf{\Psi} = (N_1, N_2, r_{th}, \sigma_0, f_{max})$ in order to fulfill the following conditions: $R_{EB} = N_\zeta(r_{th})$ and $\mathcal{R}_B = \frac{T_{\zeta_-}(r_{th})}{T_{\zeta_+}(r_{th})}$. To solve this problem, we first fix $N_1$, $N_2$, and $r_{th}$ by choosing reasonable values, e.g., $N_1 = 9$, $N_2 = 10$, and $r_{th} = 0.09$. Then, performing $\mathcal{R}_B = \frac{T_{\zeta_-}(r_{th})}{T_{\zeta_+}(r_{th})}$, $\sigma_0$ can be calculated according to the following expression

$$\sigma_0 = \frac{r_{th}}{\sqrt{2\ln(1 + \mathcal{R}_B)}} \ . \tag{11}$$

With the help of the relation $R_{EB} = N_\zeta(r_{th})$, $f_{max}$ is given by

$$f_{max} = \frac{\mathcal{N}_{EB}}{\sqrt{\pi}\sigma_0 T_t p_\zeta(r_{th})} \ . \tag{12}$$

Using (7), (12) can finally be expressed as

$$f_{max} = \frac{\mathcal{N}_{EB}(1 + \mathcal{R}_B)}{T_t\sqrt{2\pi\ln(1 + \mathcal{R}_B)}} \ . \tag{13}$$

By using the obtained parameter vector $\mathbf{\Psi}$, a sampled deterministic process $\tilde{\zeta}(kT_A)$ is generated within the necessary time interval $[0, \tilde{T}_t]$, i.e., $kT_A \leq \tilde{T}_t$. Here, $\tilde{T}_t = T_t\tilde{N}_t/N_t$ with $\tilde{N}_t$ denoting the required length of the generated error sequence. The total numbers of the generated error busts $\tilde{\mathcal{N}}_{EB}$ and error-free bursts $\tilde{\mathcal{N}}_{EFB}$ can be estimated from $\tilde{\mathcal{N}}_{EB} = \lfloor \frac{\tilde{N}_t}{N_t}\mathcal{N}_{EB} \rfloor$ and $\tilde{\mathcal{N}}_{EFB} = \lfloor \frac{\tilde{N}_t}{N_t}\mathcal{N}_{EFB} \rfloor$, respectively. Here, $\lfloor x \rfloor$ stands for the nearest integer to $x$ towards minus infinity. In this manner, an error burst length recorder $\widetilde{\mathbf{EB}}_{rec}$ with $\tilde{\mathcal{N}}_{EB}$ entries and an error-free burst length recorder $\widetilde{\mathbf{EFB}}_{rec}$ with $\tilde{\mathcal{N}}_{EFB}$ entries are derived.

### *B. The mappers*

We have found that the obtained recorders $\widetilde{\mathbf{EB}}_{rec}$ and $\widetilde{\mathbf{EFB}}_{rec}$ are not suitable to directly generate an acceptable EBD and EFBD, respectively. A proper procedure is required to adapt the EBD and EFBD of the developed generative model to those of the descriptive model. Two mappers are therefore introduced, which map the lengths of the generated error bursts and error-free bursts to the corresponding desired lengths. The idea of the mappers is to modify $\widetilde{\mathbf{EB}}_{rec}$ and $\widetilde{\mathbf{EFB}}_{rec}$ in such a way that $\tilde{N}_{EB}(m_e) = N'_{EB}(m_e)$ and $\tilde{N}_{EFB}(m_{\bar{e}}) = N'_{EFB}(m_{\bar{e}})$ hold, respectively. Here, $N'_{EB}(m_e)$ equals $\lfloor \frac{\tilde{N}_t}{N_t} N_{EB}(m_e) \rfloor$ or $\lfloor \frac{\tilde{N}_t}{N_t} N_{EB}(m_e) \rfloor + 1$ for different error burst lengths $m_e$ in order to fulfill $\sum_{m_e=m_{B1}}^{m_{B2}} N'_{EB}(m_e) = \tilde{\mathcal{N}}_{EB}$. Similarly, $N'_{EFB}(m_{\bar{e}})$ equals $\lfloor \frac{\tilde{N}_t}{N_t} N_{EFB}(m_{\bar{e}}) \rfloor$ or $\lfloor \frac{\tilde{N}_t}{N_t} N_{EFB}(m_{\bar{e}}) \rfloor + 1$ for different error-free burst lengths $m_{\bar{e}}$ to satisfy $\sum_{m_{\bar{e}}=m_{\bar{B}1}}^{m_{\bar{B}2}} N'_{EFB}(m_{\bar{e}}) = \tilde{\mathcal{N}}_{EFB}$. Note that the resulting EBD $\tilde{P}_{EB}(m_e)$ will be close to the desired EBD $P_{EB}(m_e)$, since $\tilde{N}_{EB}(m_e)$ is almost proportional to $N_{EB}(m_e)$. Also, the resulting EFBD $\tilde{P}_{EFB}(m_{\bar{e}})$ will match well the desired one $P_{EFB}(m_{\bar{e}})$.

Next, we will only concentrate on the procedure of properly modifying $\widetilde{\mathbf{EB}}_{rec}$. The same procedure applies also to $\widetilde{\mathbf{EFB}}_{rec}$. For each error burst length value $m_e$ ($m_{B1} \leq m_e \leq m_{B2}$), we first find the corresponding values $\ell^1_{m_e}$ and $\ell^2_{m_e}$ ($\tilde{m}_{B1} \leq \ell^1_{m_e}$, $\ell^2_{m_e} \leq \tilde{m}_{B2}$) in $\widetilde{\mathbf{EB}}_{rec}$ to satisfy the following conditions

$$\sum_{l=\ell^1_{m_e}}^{\ell^2_{m_e}-1} \tilde{N}_{EB}(l) \; < \; N'_{EB}(m_e) \tag{14}$$

$$\sum_{l=\ell^1_{m_e}}^{\ell^2_{m_e}} \tilde{N}_{EB}(l) \; \geq \; N'_{EB}(m_e)\,. \tag{15}$$

Let us define

$$N_{\ell^2_{m_e}} = N'_{EB}(m_e) - \sum_{l=\ell^1_{m_e}}^{\ell^2_{m_e}-1} \tilde{N}_{EB}(l)\,. \tag{16}$$

Clearly, $\sum_{l=\ell^1_{m_e}}^{\ell^2_{m_e}-1} \tilde{N}_{EB}(l) + N_{\ell^2_{m_e}} = N'_{EB}(m_e)$ holds. This indicates that if we map all error burst lengths between $\ell^1_{m_e}$ and $\ell^2_{m_e} - 1$, while only $N_{\ell^2_{m_e}}$ error burst lengths of $\ell^2_{m_e}$ in $\widetilde{\mathbf{EB}}_{rec}$ to $m_e$, then $\tilde{N}_{EB}(m_e) = N'_{EB}(m_e)$ will be satisfied. Note that $\ell^1_{m_{B1}} = \tilde{m}_{B1}$ and $\ell^2_{m_{B2}} = \tilde{m}_{B2}$ hold. In summary, the mapper for the error burst length generator works as follows: if $l$ ($\ell^1_{m_e} \leq l < \ell^2_{m_e} - 1$) samples of the deterministic process are observed in a fading interval, then a mapping $l \to m_e$ is first performed and afterwards an error burst with length $m_e$ is generated.

### C. The generation of error sequences

In this subsection, from the modified recorders $\widetilde{\mathbf{EB}}_{rec}$ and $\widetilde{\mathbf{EFB}}_{rec}$, an approach is described to enable the generation of error sequences. For generating error-free bursts, each entry of $\widetilde{\mathbf{EFB}}_{rec}$ is interpreted as the number of consecutive zeros. For generating error bursts, it is convenient to first construct parameter vectors $\widetilde{\mathbf{ECG}}_j$ ($j = 1, 2, \ldots, \widetilde{\mathcal{N}}_{EB}$), which indicate the infrastructure of each error burst in $\widetilde{\mathbf{EB}}_{rec}$ by listing the corresponding consecutive cluster lengths and gap lengths. To reach this aim, we first have to find all vectors $\mathbf{ECG}_i$ corresponding to error bursts with length $m_e$ in $\mathbf{EB}_{rec}$. Then, for all error bursts with the same length $m_e$ in $\widetilde{\mathbf{EB}}_{rec}$, we assign randomly $\widetilde{\mathbf{ECG}}_j$ from all possible vectors $\mathbf{ECG}_i$. With such a vector $\widetilde{\mathbf{ECG}}_j$, an error burst is generated by combining consecutive error clusters (ones) and gaps (zeros). The resulting error sequence is simply the combination of consecutively generated error bursts and error-free bursts. The block diagram of the obtained generative model is depicted in Fig. 1. We stress that, although the simulation set-up phase (determining the parameters and designing the mappers) of the DPBGM requires relatively long time, the simulation run phase (generation of error sequences) is fast, since it determines directly error burst and error-free burst lengths instead of bit sequences.

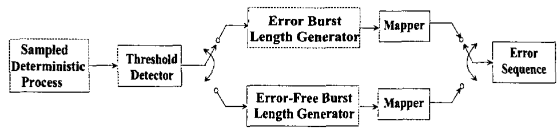


Fig. 1. The block diagram of the proposed DPBGM.

## IV. The Descriptive Model

In this paper, a FH convolutionally coded GMSK transmission system in the presence of Rayleigh fading channels was adopted to generate target error sequences. The transmitter part consists of a convolutional encoder, a block interleaver, a GMSK modulator, and a frequency hopper, while the receiver includes correspondingly a frequency dehopper, a GMSK demodulator, a block deinterleaver, and a convolutional decoder. In order to take into account the frequency correlations of different FH channels with insufficient frequency separations in practice, we have employed here a realistic FH Rayleigh fading channel simulator [17] to model FH channels. The convolutional encoder has the same structure as specified for the GSM system. The convolutional decoder is based on the Viterbi algorithm. The block interleaver has an interleaving size of $60 \times 10$. The frame length equals 60 symbols for transmission via one of the FH channels. A cyclic FH pattern was selected with 5 hopping frequencies separated by 1 MHz. The mobile speed was set to be $v = 30$ km/h. Fig. 2 depicts the resulting average bit error probability (BEP) of the FH coded GMSK transmission system, which was obtained by transmitting $N_t = 10 \times 10^6$ bits with a transmission rate of $F_s = 1/T_s = 270.8$ kb/s. The total transmission time is therefore $T_t \approx 36.93$ s. For reasons of comparison, the simulated BEPs for the uncoded GMSK system (without interleaving and FH) and the non-FH coded system (with interleaving) have also been shown in Fig. 2. A target error sequence of length $10 \times 10^6$ was extracted from the simulated FH transmission system with a signal-to-noise ratio of 15 dB. The corresponding BEP is $2.8955 \times 10^{-3}$. The relevant burst error statistics were computed from the resulting error sequence. By setting $\eta = 800$, altogether $\mathcal{N}_{EB} = 1818$ error bursts and $\mathcal{N}_{EFB} = 1818$ error-free bursts were obtained. The ratio $\mathcal{R}_B$ equals 0.057.

## V. Simulation Results and Discussions

The procedure described in Section III is applied here for obtaining the DPBGM. The chosen parameter vector for the sampled deterministic process $\tilde{\zeta}(kT_A)$ was $\mathbf{\Psi} = (9, 10, 0.09, 0.2703, 87.9873 \text{ Hz})$. For a generated error sequence with the desired length $\tilde{N}_t = 12 \times 10^6$, the necessary simulation time of $\tilde{\zeta}(kT_A)$ was $\tilde{T}_t = 44.3131$ s. The GD, the EFRD, the ECD, the EBD, the EFBD, the BEPD with blocks of 60 bits ($n$=60), and the BECF calculated from the generated error sequence will be compared to those of the target error sequence. Also, the relevant results of a SFM will

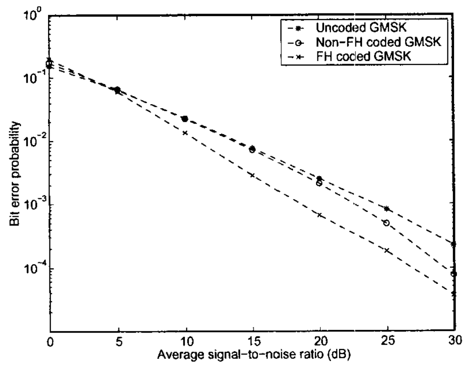


Fig. 2. Comparison of BEPs for uncoded GMSK (without interleaving and FH), non-FH coded GMSK (with interleaving), and FH coded GMSK systems.

be presented for comparison purposes. The parameters of a SFM with $K$ states are obtained by fitting the weighted sum of $K-1$ exponentials to the EFRD $P(0^{m_0}/1)$ [5]. In this paper, a SFM with 6 states was employed. Our experiments have shown that no better fitting can be obtained from SFMs with more than 6 states.

Figs. 3–8 show the GDs, the EFRDs, the ECDs, the EBDs, the BEPDs with blocks of 60 bits, and the BECFs of both generative models and the descriptive model, respectively. The results for the EFBDs of the three models are not shown here since they are very close to each other. As expected, all these curves for the DPBGM have very excellent agreements with the target ones. Especially, compared to the DPBGMs in [14–16], this improved DPBGM can also capture the main features of the BEPD and the BECF of the descriptive model with high accuracy. The SFM enables also a good approximation to the GD and the EFRD of the descriptive model, since this model is based on the fitting of the EFRD. However, relatively large deviations were found for the fittings to the desired ECD, EBD, BEPD, and BECF by using the SFM. This demonstrates that the SFM fails to model the correlation properties of the target error sequence. Both generative models require relatively long time in the simulation set-up phase, but the simulation run phase of the DPBGM is about 6 times faster than that of the SFM.

## VI. Conclusions

This paper has illustrated a general procedure of developing a generative model by using a properly parameterized and sampled deterministic process followed by a threshold detector and two parallel mappers. An approach has also been presented to enable the fast generation of error sequences from the modified error burst length generator and error-free burst length generator of the DPBGM. The reliability of the suggested generative model is confirmed by the excellent match of all interested burst error statistics to those of the underlying descriptive model. Furthermore, this

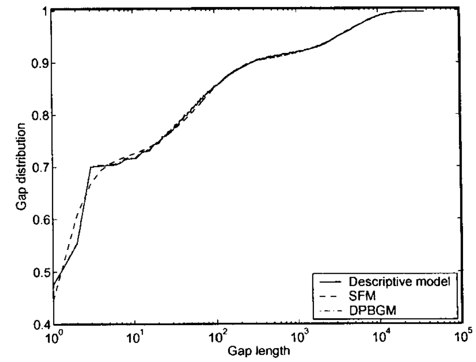


Fig. 3. The gap distributions of the generative models and the descriptive model.

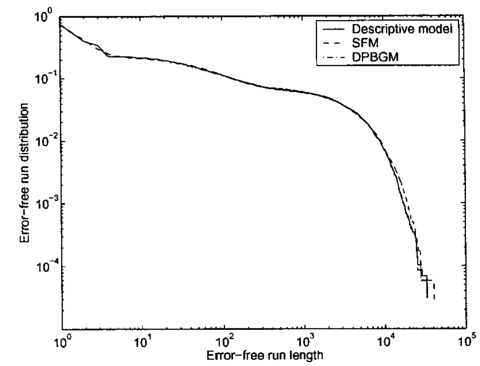


Fig. 4. The error-free run distributions of the generative models and the descriptive model.

improved DPBGM outperforms the often used SFM and the DPBGMs in [14-16] by accurately modeling the correlation properties of error sequences.

## Acknowledgements

The authors would like to thank Mr. Muslim Muhammed Abid for the simulation results shown in Fig. 2.

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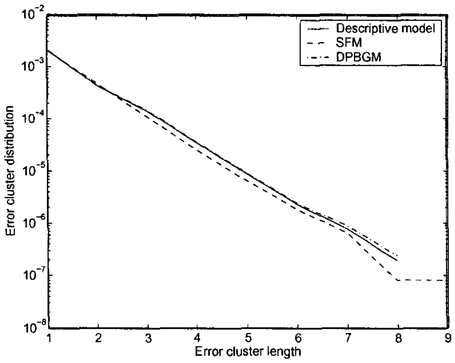


Fig. 5. The error cluster distributions of the generative models and the descriptive model.

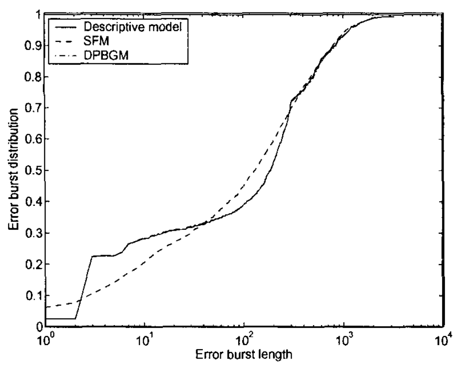


Fig. 6. The error burst distributions of the generative models and the descriptive model.

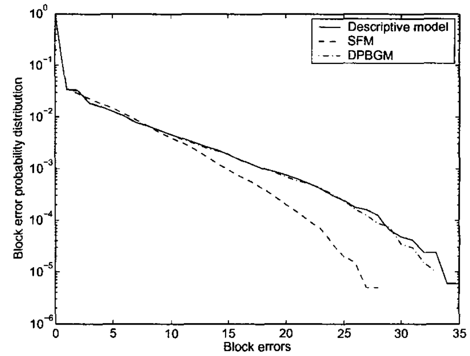


Fig. 7. The block error probability distributions of the generative models and the descriptive model.

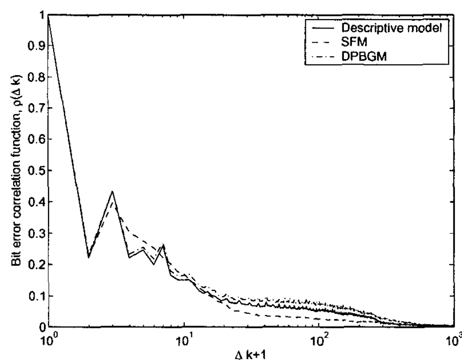


Fig. 8. The bit error correlation functions of the generative models and the descriptive model


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