A Side Information Embedded PTS Scheme in the OFDM Communication System

Heung-Gyoon Ryu, Seon-Ae Kim
Department of Electronic Engineering,
Chungbuk National University,
Cheongju, Chungbuk, 361-763, KOREA
ecomm@cb.ac.kr, i-mayo@hanmail.net

Cheng-Xiang Wang
JRI in SIP, School of EPS
Heriot-Watt University
Edinburgh EH14 4AS, UK
cheng-xiang.wang@hw.ac.uk

***Abstract*-- Partial transmit sequence (PTS) scheme has been widely studied to reduce the peak-to-average power ratio (PAPR) of Orthogonal frequency-division multiplexing (OFDM) signal since it is flexible and any spectral regrowth is not made. A necessity of the PTS scheme is to transmit the side information about the phase rotation factors for correct OFDM symbol recovery. In this paper, a new side information embedded PTS scheme using the reference symbols is proposed for the PAPR reduction and the BER performance is analyzed when the erroneous side information is received. In this proposed method, the information about the rotation factors is expressed by the phase of reference symbols. The proposed method maintains the same PAPR reduction performance as the conventional PTS scheme and OFDM symbols are correctly recovered by the side information to meet the required BER level. Also, the proposed method can be easily realized for all kinds of digital modulation formats in the OFDM communication system.**

***Index terms* - OFDM; PTS; PAPR; phase rotation; side information; nonlinearity; bandwidth efficiency**

I. INTRODUCTION

OFDM system is suitable for the wireless high-speed data transmission because of the robustness to the frequency selective fading channel [1]. However, the high PAPR causes the nonlinear distortion as the number of subcarriers is increased. The high PAPR signal is distorted when it passes through the non-linear devices such as high power amplifier (HPA).

Many methods have been proposed for the PAPR reduction in the OFDM communication system. The simplest solution is to clip the OFDM signal before HPA amplification [2], but the clip method results in performance degradation. Another method is block coding [3] in which the desired data sequence is encoded into a larger sequence and only a special subset of the low peak power in all the possible sequences is used for OFDM symbol transmission. 3-dB PAPR can be achieved with a large loss of bandwidth efficiency. Also, the size of look-up tables is exponentially enlarged as the number of subcarriers is increased.

Two kinds of the phase control schemes to reduce the PAPR of the OFDM signal have been proposed: the selective mapping (SLM) [4] and partial transmit sequence (PTS) [5] approaches. In SLM, one signal of the lowest PAPR is selected in a set of several signals which all represent the same information. In PTS, the lowest PAPR signal is made to transmit by optimally combining the signal subblocks. They are very flexible scheme and have an effective performance of the PAPR reduction without any signal degradation. PTS introduce additional complexity and a little bit loss of the spectral efficiency due to the side information insertion. The side information about the phase rotation factors would be necessary to transmit for correct OFDM symbol recovery. A side information transmission method using marking algorithm in the PTS scheme was published [6]. This method can be used only for the MPSK modulation and suffers no spectral loss. However, the method cannot be applied into the M-QAM modulation, which was indicated in ref. [6].

In this paper, we propose a new method on the side information transmission using reference symbols in the PTS approach. It has a slight spectral loss because of the supplementary subcarriers allocation for the reference symbols, but the loss is very small in case of the large subcarriers *N* and the small number of sbublocks *M*. Also, the proposed method can be applied to all kinds of modulation format used in OFDM system.

II. PAPR AND PTS SCHEME

A. Peak-to-Average Power Ratio

In OFDM system, a block of N symbols, X_n, $n = 0,1,\ldots,N-1$, is formed with each symbol modulating one of a set of N subcarriers, f_n, $n = 0,1,\ldots,N-1$. The N subcarriers are chosen to be orthogonal, that is $f_n = n\Delta f$, where $\Delta f = 1/NT$ and T is the original symbol period. The resulting signal can be expressed as

$$x(t) = \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, \qquad 0 \le t \le NT \tag{1}$$

The PAPR of the transmitted signal in Eq. (1) can be defined as

$$PAPR = \frac{\max|x(t)|^2}{E\left[|x(t)|^2\right]} \tag{2}$$

The PAPR of the continuous-time OFDM signal cannot be precisely computed by the use of the Nyquist sampling rate [7]. In this case, peaks are missed and PAPR estimates are

1-4244-1190-4/07/$25.00 ©2007 IEEE.

not precise. So, oversampling is necessary and factor of 4 is sufficient for accuracy.

B. Theoretical CCDF of PAPR

Theoretical CCDF (complementary cumulative distribution function) of PAPR can be derived by the results in [8]. For an OFDM symbol with N subcarriers, the samples of the complex baseband signal are given by Eq.(1). The amplitude of the OFDM signal therefore has the Rayleigh distribution. The CDF of the peak power per OFDM symbol can be found based on the assumption of the uncorrelated samples. The probability that the PAPR is below the threshold level $PAPR_o$ can be written as

$$\Pr\left(PAPR \le PAPR_o\right) = \left(1 - \exp\left(-PAPR_o\right)\right)^N . \quad (3)$$

As Eq.(3) does not hold for the oversampling case, an approximation is presented in [8]. Adding a certain number of extra independent samples approximates the effect of oversampling. The distribution of the PAPR is the given by

$$\Pr\left(PAPR \le PAPR_o\right) = \left(1 - \exp\left(-PAPR_o\right)\right)^{\alpha N} . \quad (4)$$

$$\Pr\left(PAPR > PAPR_o\right) = 1 - \left(1 - \exp\left(-PAPR_o\right)\right)^{\alpha N} . \quad (5)$$

When $\alpha = 2.4$, the accurate results can be shown for QPSK modulation with N=128.

C. Partial Transmit Sequence (PTS)

In the PTS approach, the input data block is partitioned into disjoint clusters or subblocks that are combined to minimize the PAPR. The data block, $X_n,\ n = 0,1,\ldots,N-1$ is defined as a vector, $\boldsymbol{X} = [X_0\ X_1\ \ldots\ X_{N-1}]^T$. Then, partition $\boldsymbol{X}$ into M disjoint sets, represented by the vectors $X^{(m)},\ m = 1,2,\ldots,M$

$$\boldsymbol{X} = \sum_{m=1}^{M} X^{(m)} . \quad (6)$$

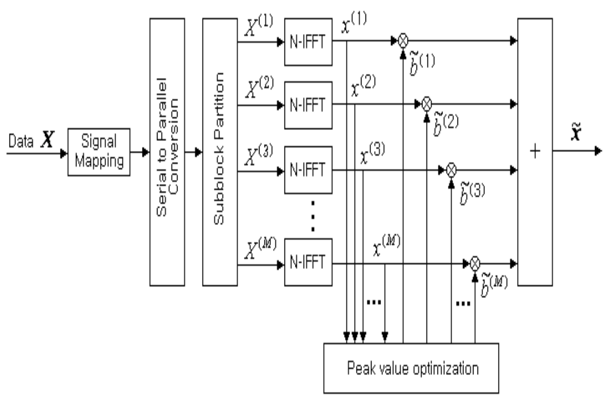

Figure 1. OFDM transmitter of the PTS scheme.

The objective of the PTS approach is to form a weighted combination of the M clusters

$$\hat{\boldsymbol{X}} = \sum_{m=1}^{M} b^{(m)} X^{(m)} \quad (7)$$

where $b^{(m)},\ m = 1,2,\ldots,M$ are weighting factors or phase factors and are assumed to be pure rotations. (i.e., $b^{(m)} = e^{j\Phi m}$). After transforming to the time domain, Eq. (7) becomes

$$\hat{\boldsymbol{x}} = \sum_{m=1}^{M} b^{(m)} x^{(m)} . \quad (8)$$

The vector $x^{(m)}$, called *partial transmit sequence*, is the IFFT of $X^{(m)}$. The weighting factors are chosen to minimize the PAPR by searching for the appropriate combination of each cluster and by corresponding weighting factors.

$$\left\{\tilde{b}^{(1)}, \tilde{b}^{(2)}, \ldots, \tilde{b}^{(M)}\right\} = \underset{\left\{b^{(1)}, b^{(2)}, \ldots, b^{(M)}\right\}}{\arg\min} \left(\max_{0 \le n \le N-1} \left| \sum_{m=1}^{M} b^{(m)} x_n^{(m)} \right| \right) . \quad (9)$$

Optimized transmit sequence is

$$\tilde{\boldsymbol{x}} = \sum_{m=1}^{M} \tilde{b}^{(m)} x^{(m)} . \quad (10)$$

III. Side Information Transmission Using Reference Symbol

To recover the data, the receiver must know what rotation factors are used to reduce the PAPR. In this paper, we present a new approach that transmits the side information about the rotation factors by the reference symbols. The basic strategy is to insert the reference symbols onto the transmitted data that can be used to uniquely identify the rotation factors at the receiver.

A Reference symbol insertion

Data $\boldsymbol{D}$ is partitioned into multiple disjoint subblocks. Then, reference symbol $\boldsymbol{R}$ is inserted in each cluster.

$$\boldsymbol{D} = \sum_{m=1}^{M} D^{(m)}, \quad D = MPSK \ \ or\ MQAM$$

$$\boldsymbol{R} = \sum_{m=1}^{M} R^{(m)}, \quad R = e^{j0^\circ} = 1 \quad (11)$$

where $\boldsymbol{D}$ is data symbol and $\boldsymbol{R}$ is reference symbol. A new signal vector can be shown by

$$\boldsymbol{X} = \boldsymbol{D} + \boldsymbol{R} = \sum_{m=1}^{M} \left(D^{(m)} + R^{(m)}\right) = \sum_{m=1}^{M} X^{(m)} . \quad (12)$$

$\boldsymbol{X}$ is similarly processed through the Eq. (7) ~ (10).

As an example, adjacent, interleaved and pseudo-random subblock partitioning schemes are represented in the Fig. 2, 3 and 4. In each cluster, both data and reference symbols are not overlapped. In the Fig.2, 3 and 4, reference symbols are located in $X_0^{(1)}$, $X_4^{(2)}$, $X_8^{(3)}$, $X_{12}^{(4)}$.

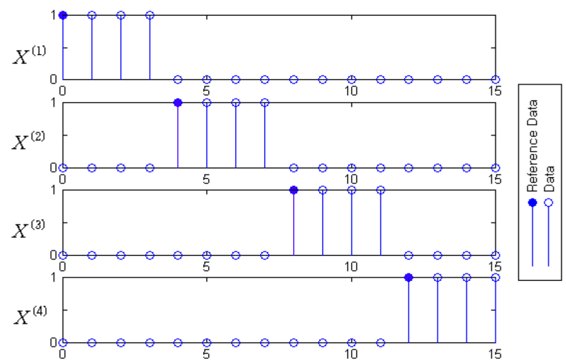

Figure 2. Adjacent subblok partitioning scheme.

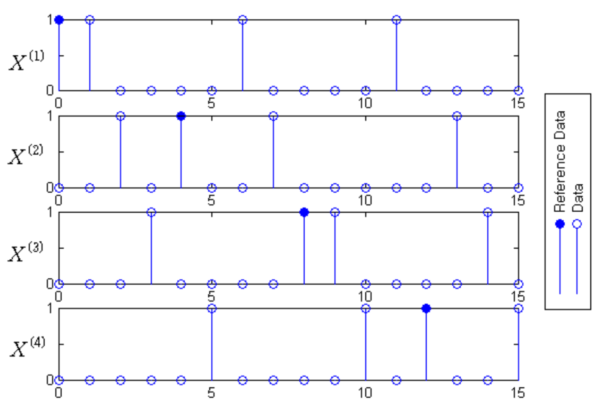

Figure 3. Interleaved subblok partitioning scheme.

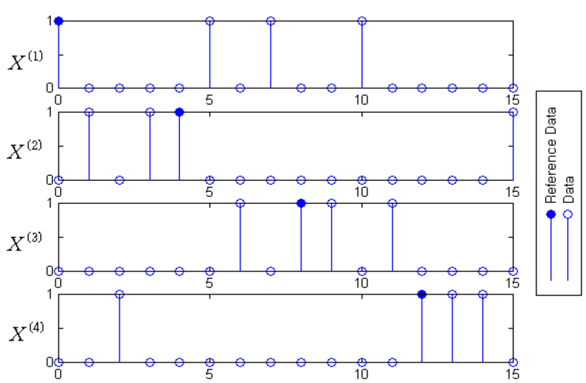

Figure 4. Pseudo-random subblok partitioning scheme.

B. Bit error performance degradation due to the false side information

When the signal is received, receiver first recovers the side information from the received signal to detect correctly. The side information plays an important role in the overall system performance [9].

For simplicity, we only consider in the AWGN channel, which may serve as a lower bound performance for a fading channel may be significantly changed.

P_S, the error probability of the side information, may control the overall bit error probability, $\boldsymbol{P}$, which is written as

$$\boldsymbol{P} = P_b \cdot (1 - P_S) + P_{b|\text{False}} \cdot P_S . \quad (19)$$

P_b is the bit error probability of QPSK in the AWGN channel, which is given by

$$P_b = Q\left(\sqrt{\frac{\sigma_S^2}{\sigma_N^2}}\right) = Q\left(\sqrt{\frac{2E_b}{N_o}}\right) . \quad (20)$$

$P_{b|False}$ is the conditional bit error probability given that the side information is false, and it can be shown that

$$P_{b|\text{False}} = Q\left(\sqrt{\frac{\sigma_S^2}{\sigma_N^2 + \sigma_{\text{False}}^2}}\right) = Q\left(\sqrt{\frac{2E_b}{N_o + (N-M)\cdot 2E_b}}\right) \quad (21)$$

where $\sigma_S^2 = 2E_b$ and $\sigma_{\text{False}}^2 = (N-M)\cdot 2E_b$ are signal variance and the variance of false side information, respectively.

IV. Simulation Results and Discussion

To evaluate the performance of the proposed scheme, we assume that the number of subcarrier is 128 (i.e., N=128) and the subcarriers are divided into 4 or 8 clusters (i.e., M=4, 8). QPSK modulation format is used for the simulation, but all kinds of digital modulation format considered in OFDM communication can be easily extended. Three kinds of subblock partitioning methods are used in the proposed side information embedded PTS scheme: adjacent, interleaved and pseudo-random subblock partitioning method. Here, we consider binary (i.e., $b^{(m)} = \pm 1$) weighting factors and optimal combining algorithm is selected for the combining technique so that there is a combination number of 2^M.

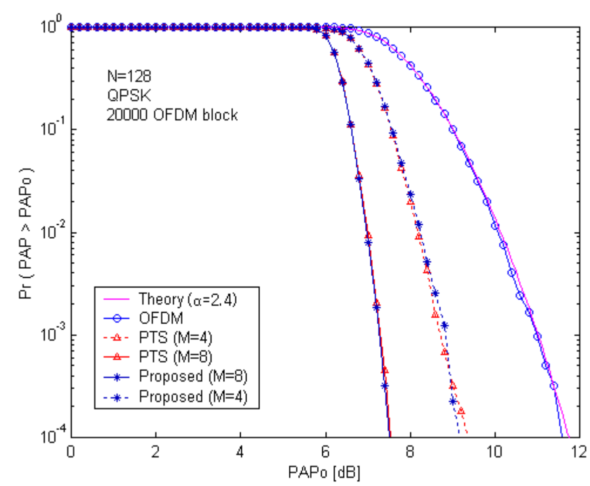

Figure 5. Comparison of the conventional and the proposed PTS scheme.

In Fig. 5, the conventional PTS means that the OFDM receiver, in any ways, is assumed to know the side information about the rotation factors shown in Fig.1. The outer two curves are the theoretical and simulation results. Two curves in the middle express the conventional PTS

(M=4) and the proposed method (M=4). Two inner curves describe the conventional PTS(M=8) and the proposed method(M=8F). Fig. 6 shows the CCDF performance of the adjacent, interleaved and pseudo-random subblock partitioning schemes. Three curves in the middle are the case of M=4 in which the curves of the adjacent, interleaved and pseudo-random subblock partitioning appear in order from the right. The inner three curves are the case of M=8. The pseudo-random partitioning works better than the adjacent and interleaved subblock partition schemes like the case of M=4.

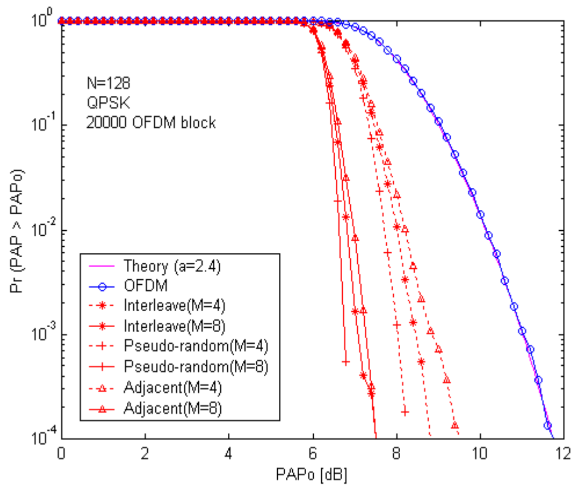

Figure 6. Performance comparison of the three subblock partitioning schemes.

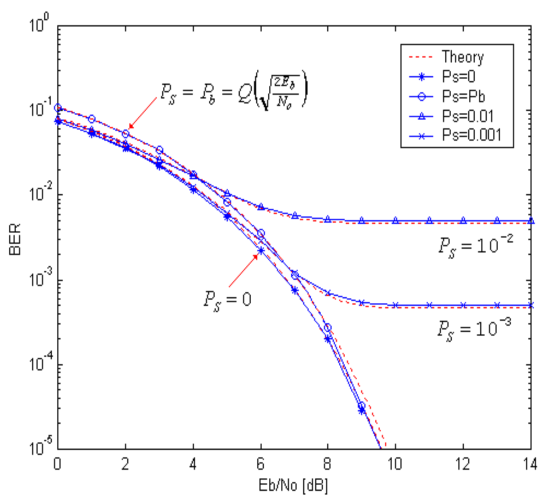

Figure 7. BER performance for the error of the side information, P_S.

Fig. 7 illustrates the overall bit error probability described in Eq. (19) for the several error probabilities of the side information, P_S. In Fig. 7, it can be seen that if there is a high error level in P_S (i.e. 10^{-2}, 10^{-3}), the serious error floor is made. However, the performance degradation due to the false side information is not a problem when $P_S = P_b$ in the most usual cases. If P_S is zero, $\boldsymbol{P}$ is only the bit error probability of QPSK in AWGN channel. We can see that there is a very small difference between the cases of $P_S = P_b$ and $P_S = 0$.

This method has a little spectral loss due to the side information insertion. However, the fraction of the overhead is very small for large *N*. For example, in case of BPSK with *N*=128 and *M*=8, the spectral loss is 8/128=0.0625. This fraction of overhead is very small for large N so that the spectral loss is not significant.

V. Conclusion

In this paper, a side information insertion in the PTS scheme is studied for the PAPR reduction in the OFDM system. The proposed scheme has the same PAPR reduction performance as the conventional PTS scheme. Whereas the conventional PTS without the side information does not meet the required BER performance, the proposed scheme can correctly recover the OFDM data and satisfy the required BER. Also, It can be easily extended to higher-order PSK and QAM modulation formats.

References

[1] L. J. Cimini Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. on Commun.*, vol. COM-33, pp.665-675, July 1985.

[2] R. O'Neill and L. N. Lopes, "Envelope variations and spectral splatter in clipped multicarrier Signals," *Proc. PIMRC '95*, pp. 71-75, 1995.

[3] A. E. Jones, T. A. Wilkinson, and S. K. Barton, "Block coding scheme for reduction of peak to mean envelope power ratio of multicarrier transmission scheme," *IEE Electron. Letters*, Vol. 30, no. 25, pp. 2098-2099, Dec. 1994.

[4] R. W. Bauml, R. F. H. Fischer, and J. B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulation by selective mapping," *IEE Electron. Letters*, vol. 32, no. 22, pp.2056-2057, Oct. 1996.

[5] S. H. Muller and J. B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *IEE Electron. Letters*, vol. 33, no.5, pp. 368-369, Feb. 1997.

[6] L. J. Cimini, N. R. Sollenberger, "Peak-to-Average Power Ratio Reduction of an OFDM Signal Using Partial Transmit Sequences With Embedded Side Information," *GLOBECOM '00. IEEE*, vol. 2, pp. 746-750, 2000.

[7] C. Tellambura, "Phase optimization criterion for reducing peak-to-average power ratio of an OFDM signal," *IEE Electron. Letters*, vol. 34, pp. 169-170, Jan. 1998.

[8] R. van Nee, *OFDM wireless multimedia communication.* Boston, London: Artech House, 2000.

[9] H. Ochiai and H. Imai, "OFDM-CDMA with peak power reduction based on the spreading sequences," *International Conference on IEEE*, vol. 3, pp. 1299-1303, 1998.