Design of a Stochastic Frequency Hopping Rayleigh Fading Channel Simulator with Given Correlation Properties

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ABSTRACT

In this paper, a stochastic Rayleigh fading channel simulator with excellent frequency hopping (FH) capabilities is described. The proposed simulator is based on a finite double-sum of properly weighted harmonic functions. Closed-form expressions can be derived for all parameters of the channel simulator. It is shown that the occurrence of a frequency hop in the physical channel model corresponds to phase hops in the simulation model, whereas the other simulation model parameters remain unchanged. Furthermore, the performance of our simulation model is evaluated for typical GSM FH mobile radio data transmission scenarios. The statistical properties of the simulation model are in excellent conformity with those of the underlying reference model.

I. INTRODUCTION

In modern wireless communication systems, FH is an effective technique for combating fast fading, reducing interleaving depth, and enabling efficient frequency reuse by taking advantage of frequency diversity. It is typically assumed in FH systems that the consecutive hopping channels with different carrier frequencies are uncorrelated, which implies that the channel separation is larger than the coherence bandwidth. However, such an assumption is not always justified since channel correlation within coherence bandwidth usually occurs in practical FH systems [1]. This is the motivation for us to investigate the correlation properties of the received signals at different carrier frequencies in order to study the influence of frequency correlation in realistic FH systems.

Channel simulators are essential for the design, optimization and test of FH wireless communication systems. A channel simulator that models accurately the physical channel statistics determined by the frequency separation between two carriers in FH systems is called a FH channel simulator. A deterministic FH simulation model for Rayleigh fading channels has been presented in a previous paper [2]. Although this simulator can easily be designed with low complexity, some of its statistical properties are only in rough agreement with those of the underlying reference model. In this paper, we propose a novel stochastic channel simulator, which can emulate all of the correlation properties of FH Rayleigh fading channels with high precision.

II. THE REFERENCE MODEL AND THE SIMULATION MODEL

For reasons of simplicity, we restrict our investigations to frequency-nonselective fading channels, and assume that no line-of-sight path exists between the base station and the mobile station antennas. Then, a suitable reference model for the envelopes of the received signals at two different carrier frequencies, denoted by F_1 and F_2, is given by the following Rayleigh processes:

$$\zeta(t) = |\mu_1(t) + j\mu_2(t)| \ , \ \text{at } F_1 \ , \tag{1a}$$
$$\zeta'(t) = |\mu_1'(t) + j\mu_2'(t)| \ , \ \text{at } F_2 \ . \tag{1b}$$

Here, both $\mu_i(t)$ and $\mu_i'(t)$ $(i = 1, 2)$ are real Gaussian noise processes, each with zero mean and variance σ_0^2. The existence of different time delays over the propagation paths causes the statistical properties of two signals with different frequencies to become essentially uncorrelated if the absolute value of the frequency separation $\chi = F_2 - F_1$ is sufficiently large. Here, χ is a measure for a frequency hop from F_1 to F_2. But in general, the received signals $\zeta(t)$ and $\zeta'(t)$ are statistically correlated due to the limited bandwidth available for the GSM system. The correlation properties of two signals received at F_1 and F_2 are completely determined by those of the underlying Gaussian random processes $\mu_i(t)$ and $\mu_j'(t)$ $(i, j = 1, 2)$. Therefore, we will restrict our investigations to the following autocorrelation and cross-correlation functions, which have been derived by Jakes [3]:

$$r_{\mu_i\mu_i}(\tau) = r_{\mu_i'\mu_i'}(\tau) = \sigma_0^2 J_0(2\pi f_{max}\tau) \ , \tag{2a}$$
$$r_{\mu_1\mu_2}(\tau) = r_{\mu_1'\mu_2'}(\tau) = 0 \ , \tag{2b}$$
$$r_{\mu_i\mu_i'}(\tau, \chi) = \frac{\sigma_0^2 J_0(2\pi f_{max}\tau)}{1 + (2\pi\alpha\chi)^2} \ , \tag{2c}$$
$$r_{\mu_1\mu_2'}(\tau, \chi) = -r_{\mu_2\mu_1'}(\tau, \chi) = -2\pi\alpha\chi r_{\mu_i\mu_i'}(\tau, \chi) \ . \tag{2d}$$

In (2), τ denotes the time separation, f_{max} is the maximum Doppler frequency, and α is a real constant, which is related to the delay spread of the channel.

Concerning the simulation model, we approximate the statistics of the underlying complex Gaussian random processes by the following finite double-sum of properly weighted exponential functions:

$$\hat{\mu}(t) = \hat{\mu}_1(t) + j\hat{\mu}_2(t) = \sum_{n=-N+1}^{N} \sum_{m=1}^{M} c_{n,m} e^{j(2\pi f_n t - \theta_m - \hat{\theta}_m)} \ , \ \text{at } F_1 \ , \tag{3a}$$

$$\hat{\mu}'(t) = \hat{\mu}_1'(t) + j\hat{\mu}_2'(t) = \sum_{n=-N+1}^{N} \sum_{m=1}^{M} c_{n,m} e^{j(2\pi f_n t - \theta_m' - \hat{\theta}_m)} \ , \ \text{at } F_2 \ , \tag{3b}$$

with

$$\theta_m = 2\pi F_1 \varphi_m \ , \quad \theta_m' = 2\pi(F_1 + \chi)\varphi_m \ , \tag{4a,b}$$

where 2N-by-M defines the number of exponential functions determining the realization expenditure and the performance of the resulting channel simulator. The phases $\hat{\theta}_m$ are random variables uniformly distributed over $[0, 2\pi)$. The so-called Doppler coefficients $c_{n,m}$, discrete Doppler frequencies f_n, and φ_m are simulation model parameters, which are determined in such a way that the statistical properties of $\hat{\mu}_i(t)$ and $\hat{\mu}_j'(t)$ are close to those of the stochastic processes $\mu_i(t)$ and $\mu_j'(t)$ $(i, j = 1, 2)$, respectively. The expressions (3) and (4) indicate that a frequency hop $F_1 \rightarrow F_2$ of size $\chi = F_2 - F_1$ in the physical channel model corresponds to phase hopes $\theta_m \rightarrow \theta_m'$ of sizes $2\pi\chi\varphi_m = \theta_m' - \theta_m$ $(m = 1, 2, \cdots, M)$ in our simulation model, while maintaining all other parameters unchanged. For the simulation model, the following correlation functions can be derived:

$$\hat{r}_{\mu_i\mu_i}(\tau) = \hat{r}_{\mu_i'\mu_i'}(\tau) = \sum_{n=-N+1}^{N} \sum_{m=1}^{M} \frac{c_{n,m}^2}{2} \cos(2\pi f_n \tau) , \tag{5a}$$

$$\hat{r}_{\mu_1\mu_2}(\tau) = \hat{r}_{\mu_1'\mu_2'}(\tau) = \sum_{n=-N+1}^{N} \sum_{m=1}^{M} \frac{c_{n,m}^2}{2} \sin(2\pi f_n \tau) , \tag{5b}$$

$$\hat{r}_{\mu_i\mu_i'}(\tau, \chi) = \sum_{n=-N+1}^{N} \sum_{m=1}^{M} \frac{c_{n,m}^2}{2} \cos(2\pi f_n \tau - 2\pi \varphi_m \chi) , \tag{5c}$$

$$\hat{r}_{\mu_1\mu_2'}(\tau, \chi) = -\hat{r}_{\mu_2\mu_1'}(\tau, \chi) = \sum_{n=-N+1}^{N} \sum_{m=1}^{M} \frac{c_{n,m}^2}{2} \sin(2\pi f_n \tau - 2\pi \varphi_m \chi) . \tag{5d}$$

III. NUMERICAL RESULTS AND CONCLUSION

A novel stochastic FH Rayleigh fading channel simulator has been established and all parameters can be determined in terms of closed-form expressions. It can also be proved that all correlation functions of our simulation model converge to those of the underlying reference model if the number of exponential functions tends to infinity. However, even a limited number of parameters will also give excellent approximation results, as shown in Figs. 1–5. The presented results are valid for the rural area (RA) profile specified by COST 207 [4] for the GSM system. Due to the fact that the proposed stochastic channel simulator has excellent FH capabilities, it is expected to play a significant role in future realistic FH mobile communication systems.

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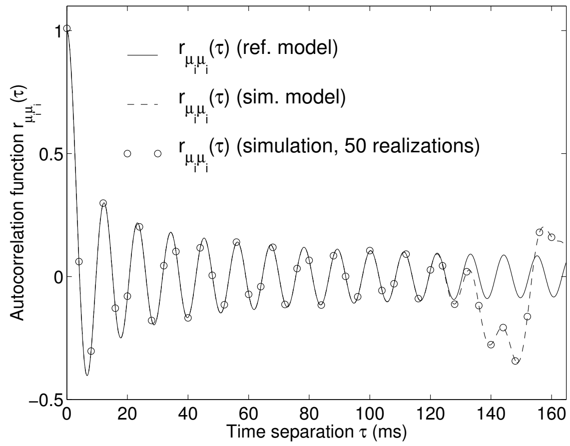

Fig. 1: Autocorrelation function $r_{\mu_i\mu_i}(\tau)$ (ref. model) in comparison with $\hat{r}_{\mu_i\mu_i}(\tau)$ (sim. model, $N = 20$, $M = 20$) for $f_{max} = 91$ Hz and $\sigma_0^2 = 1$.

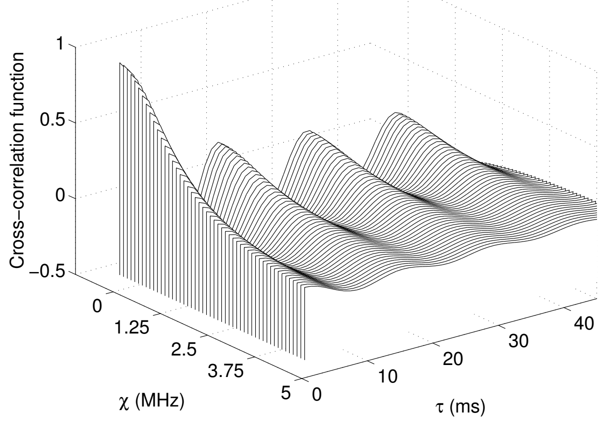

Fig. 2: Cross-correlation function $r_{\mu_i\mu_i'}(\tau, \chi)$ (ref. model) for the COST 207 RA profile ($\alpha = 0.1086$ μs, $f_{max} = 91$ Hz, $\sigma_0^2 = 1$).

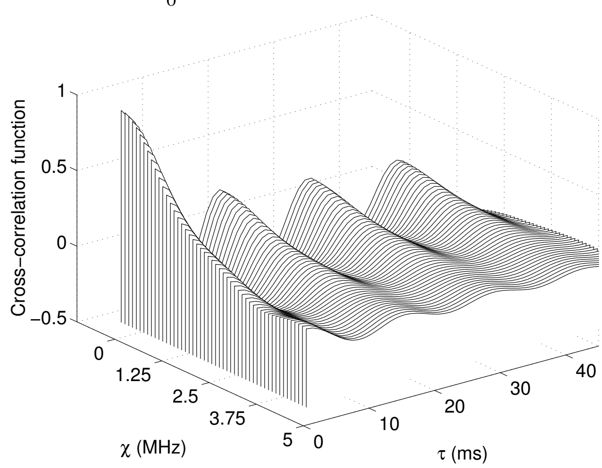

Fig. 3: Cross-correlation function $\hat{r}_{\mu_i\mu_i'}(\tau, \chi)$ (sim. model, $N = 20$, $M = 20$) for the COST 207 RA profile ($\alpha = 0.1086$ μs, $f_{max} = 91$ Hz, $\sigma_0^2 = 1$).

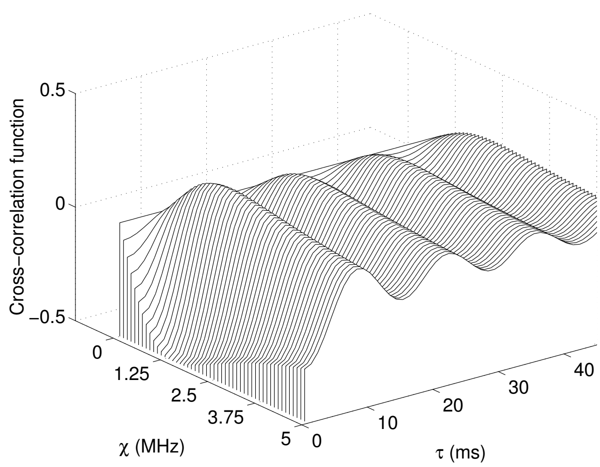

Fig. 4: Cross-correlation function $r_{\mu_1\mu_2'}(\tau, \chi)$ (ref. model) for the COST 207 RA profile ($\alpha = 0.1086$ μs, $f_{max} = 91$ Hz, $\sigma_0^2 = 1$).

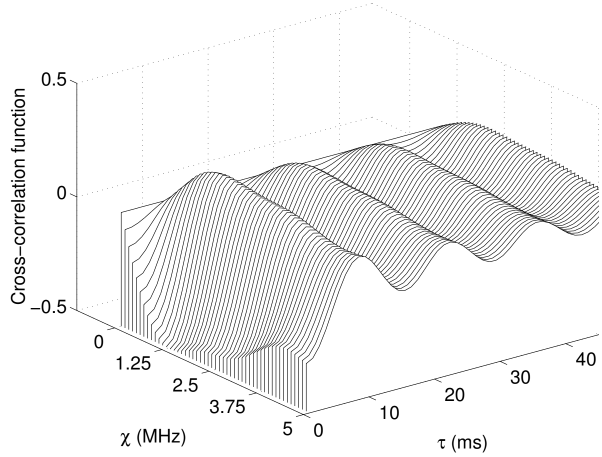

Fig. 5: Cross-correlation function $\hat{r}_{\mu_1\mu_2'}(\tau, \chi)$ (sim. model, $N = 20$, $M = 20$) for the COST 207 RA profile ($\alpha = 0.1086$ μs, $f_{max} = 91$ Hz, $\sigma_0^2 = 1$).