Tensor-computing-based Spectrum Usage Framework for 6G

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Abstract*—In this paper, we coin a new concept of tensor-computing, which is based on tensor theory and designed for future sixth generation (6G) wireless communication systems. Two different types of tensors, namely spectrum-tensor and system-tensor, are defined and analysed to develop a new spectrum usage framework for 6G. The spectrum-tensor encapsulates high dimensional spectrum big data into the format of a compact tensor. The system-tensor summarizes key system performance, including data rate, bandwidth, delay, spectral efficiency, and energy efficiency, into a multi-dimension tensor. The concepts of spectrum-tensor and system-tensor enable unique tensor-based computing and analysis with the help of high efficiency tensor-computing tools, such as tensor completion and tensor decomposition. In the new spectrum usage framework, a value-based spectrum fusion scheme is designed. The maximum system value is achieved under the constraint that the individual value of single user should be guaranteed. The proposed tensor-computing framework builds a bridge between 6G wireless functions with real-world high dimension data processing tools, such as TensorFlow and Tensor Processing Unit (TPU). The authors hope this paper will shine a beam of tensor theory in and open a new research field of tensor-computing for future 6G wireless communications.

***Index Terms*—Tensor, tensor-computing, spectrum-tensor, system-tensor, spectrum usage framework, 6G.**

I. INTRODUCTION

Tensor theory has been widely used in many research fields, such as physics, mathematics, chemometrics, signal processing and so on [1]–[3]. Different from tensor field in physics and mathematics, the notation of ”tensor” here refers to multidimensional array, which encapsulates high dimensional big data information in a compact format. Tensor provides an efficient way to represent high dimensional data, which is prevalent in future wireless communication systems, such as the sixth generation (6G) wireless communication systems.

The vision, concept, and development of 6G wireless communication systems have attracted growing interests in the research community [4]–[9]. Compared to 5G systems, it is expected that 6G systems will have three distinctive new features: artificial intelligence (AI) enabled computing, very large scale and high dimensional data, and extremely aggressive spectrum reuse and sharing. It is expected that the development of 6G systems will face several unique limiting factors, including limited computing resources, limited data storage, and limited spectrum. This necessitates the development of new system architecture and spectrum usage frameworks that are tailored for the unique challenges faced by the developed of 6G systems.

We propose to apply tensor theory to develop new system architecture and spectrum usage framework for 6G systems. Tensor theory can provide efficient solutions to the new feature and challenges faced by 6G systems. For example, tensor has been widely used in the AI community as an elementary data structure, and the application of tensor is a natural selection of AI-enabled computing. Regarding limited storage, tensor can be used to efficiently encapsulate and condense the large scale data into a compact format. Tensor-based spectrum usage framework can provide efficient and scalable solutions to spectrum sharing, spectrum aggregation, and spectrum fusion [10]. Therefore, the unique features and challenges of 6G systems can benefit tremendously from the application of tensor theories in the design and analysis of 6G systems.

Tensor theory provides an efficient way to handle multidimensional array based on the following three facts [11]–[13]:

- efficient data storage based on compact tensor representation;
- AI-enabled computing with tensor-computing tools such as tensor completion tensor decomposition;
- leveraging connections between applications and processing hardware.

Tensor can be considered as the extension of random matrix with multiple dimensions. Random matrix theory has found lots of applications in wireless communications [14], [15]. Based on matrix completion, tensor completion has found some useful applications in signal processing and wireless communications [13], [16]. Tensor decomposition provides an efficient way to analyze big data by decomposing a tensor into multiple low-rank tensors or matrices. A whole tensor can also be analyzed from a specific dimension [1], [3]. Like random matrix theory, tensor rank and tensor eigenvalue are still useful tools to analyze tensors [17], [18]. However, till now the rank of a specific tensor is tough to be determined and tensor rank has completely different meaning with matrix rank [1], [19]. The determinations of tensor rank and tensor eigenvalue are still open problems for researchers.

The main contributions of this paper can be summarized as follows:

1) a new concept of tensor-computing has been proposed with tensor theory. The tensor-computing provides a potential connection between the practical applications and the hardware in the physical layers;
2) a new spectrum usage framework for 6G systems has been designed. The spectrum-tensor and system-tensor have been defined to deal with spectrum big data and system big data;
3) a spectrum fusion scheme has been proposed for spectrum usage framework. The proposed spectrum fusion scheme tries to achieve the maximum system value under the constraint that each user value is guaranteed. The tensor-computing provides an efficient way to analyze spectrum-tensor with a specific dimension like value.

II. Math Preliminaries and System Models

A. Math Preliminaries

1) Spectrum-tensor and System-tensor Definitions: Let $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times I_3 \times I_4 \times I_5}$ denote a fifth-order [1] spectrum-tensor of sizes $I_1 \times I_2 \times I_3 \times I_4 \times I_5$, where $I_1 = U, I_2 = V, I_3 = T, I_4 = S$, and $I_5 = F$ denote the numbers of users, value, time, space, and frequency, respectively.

A vivid schematic of $\mathcal{X}$ is indicated in Fig.1, where the sizes are $U = 3, V = 4, T = 2, S = 2, F = 2$ and there are in total 12 third-order sub-spectrum-tensors. When $u = 3$ and $v = 3$, the third-order sub-spectrum-tensor is $\mathcal{M} = \mathcal{X}(3, 3, :, :, :)$, which is shown in Cartesian coordinates with t, s, and f. In the sub-spectrum-tensor $\mathcal{M}$, a $T \times S$ spectrum matrix $\mathbf{N}_{t\times s} = \mathcal{X}(3, 3, :, :, 1)$ can be generated and the spectrum entry is defined as $x_{i_1 i_2 i_3 i_4 i_5} = p_\alpha$, where p_α is the power of the received signal α. In Fig. 1, $x_{3,3,2,1,2} = p_\alpha$ when $i_1 = 3, i_2 = 3, i_3 = 2, i_4 = 1, i_5 = 2$ for a specific spectrum entry in a fifth-order spectrum-tensor. With the spectrum-tensor $\mathcal{X}$, the whole information of the received signal α can be determined from five dimensions. In particular, $\mathcal{X}$ can be regarded as an elementary computing unit for 6G systems.

A fifth-order system-tensor is defined as $\mathcal{Y} \in \mathbb{R}^{J_1 \times J_2 \times J_3 \times J_4 \times J_5}$ with the sizes of $J_1 \times J_2 \times J_3 \times J_4 \times J_5$, where $J_1 = D, J_2 = B, J_3 = L, J_4 = \Theta_f$, and $J_5 = \Theta_e$ denote data rate, bandwidth, time-delay, spectrum-efficiency, and energy efficiency, respectively. The system entry is in the form of a discrete value and defined as $y_{j_1 j_2 j_3 j_4 j_5} \in \{0, 1\}$, where 1 and 0 denote whether there exists a specific system performance at a given position in a system-tensor $\mathcal{Y}$. For simplicity, the schematic of the system-tensor is not presented here.

[1] also refer to the dimensions of a tensor as modes (ways) and the order is the number of modes, i.e., a third-order tensor refers equally to a three-mode tensor.

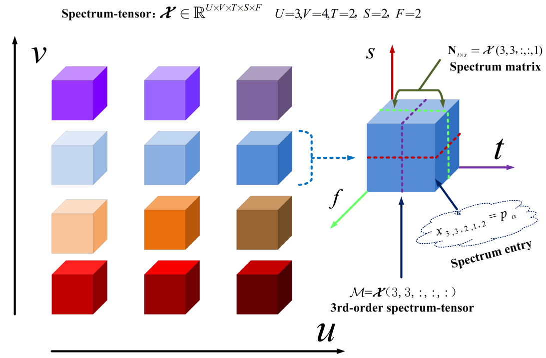

Fig. 1: Schematic of a 5th-order spectrum-tensor.

2) Tensor-computing: In this paper, we coin a novel concept of tensor-computing. The key point of the tensor-computing is that both the spectrum-tensor and the system-tensor are processed as single units instead of multiple distributed entries in the future 6G systems, which will essentially enhance system computing efficiency. To fulfill the core concept, the spectrum-tensor and system-tensor should be constructed from spectrum big data and system big data from multiple dimensions. In this process, the completion operation will be carried to fill spectrum big data and system big data by the process of detection and estimation. Both spectrum-tensor and system-tensor should be sparse even after data completion. Therefore, some key tensor characteristics such as eigenvalues or eigenvectors of spectrum-tensor and system-tensor are required to evaluate the target tensors.

For spectrum-tensor and system-tensor with high-order, say more than three, the system feature in a specific dimension can be obtained by tensor decomposition. The operation of tensor decomposition can transfer a tensor into multiple low-order sub-tensors, matrices, slices, and fibers. Consequently the existing analysis techniques for matrix and vectors can be utilized to analyse the whole tensors. As shown in Fig. 1, the 5th-order spectrum-tensor $\mathcal{X}$ can decomposed into 12 3rd-order $2 \times 2 \times 2$ sub-tensors. For the sub-spectrum-tensor $\mathcal{M}$, a 2×2 spectrum matrix $\mathbf{N}$ can be generated with three fixed modes $u = 3, v = 3, f = 1$. Consequently, the system performance can be obtained by a specific spectrum matrix or a spectrum vector with fixed dimensions. Furthermore, the spectrum-tensor can also be decomposed into other formats with matrices and vectors. In the same way, the system-tensor $\mathcal{Y}$ can be completed from system big data and some key characteristics can be obtained by tensor decomposition.

B. System Models

A novel spectrum usage framework for 6G systems is shown in Fig. 2, where a spectrum fusion engine (SFE) is designed to sense, record, regulate and utilize all available spectrum including licensed bands, unlicensed bands, and potential bands. The spectrum information in the form of spectrum-tensor and the system information in the form of system-tensor are processed in the SFE. The key functions of tensor-computing

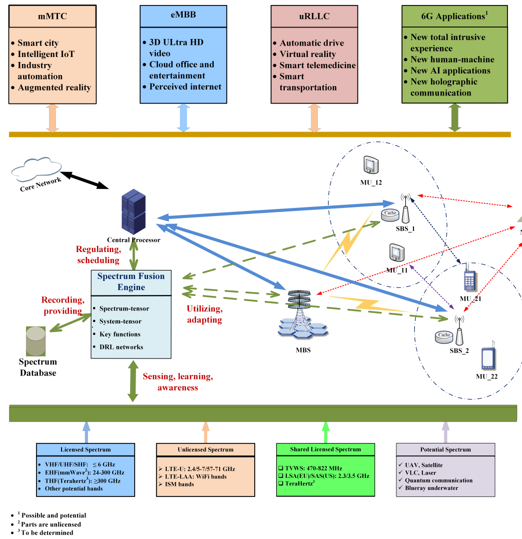

Fig. 2: Spectrum usage framework for 6G systems.

in the SFE are used to analyze spectrum-tensor and system-tensor to obtain the characteristics of spectrum and system. Some sophisticated techniques such as deep reinforcement learning can be potentially used to determine the optimal spectrum usage police.

The SFE utilizes spectrum-tensor and system-tensor to determine the available spectrum that is stored in a spectrum database. The macro-base station (MBS) and the small base station (SBS) with their mobile users (MUs) and non-cell MUs (N-MUs) are authorized to access the spectrum by a central processor. Some new and more unforeseeable applications will be fulfilled in 6G operation scenarios, which should be extended from existing 5G applications including massive Machine Type Communications (mMTC), enhanced Mobile Broadband (eMBB), and Ultra Reliable Low Latency Communications (uRLLC). Currently, people have imagined some potential 6G applications, such as new total intrusive experience, new human-machine, new AI applications, and new holographic communications [7]–[9], all of which will require large amount of spectrum and new spectrum usage model.

III. Tensor-computing for 6G

A. Tensor Completion and Decomposition

The completion and decomposition operations are both hot topics in the research field of tensor theory. From the view of big data, completion and decomposition seem much important due to the intrinsic essence of deficiency and sparsity for original big data.

For a Mth-order spectrum-tensor $\mathcal{X}$ with some missing or incomplete entries, the completion can be formulated by solving the following optimization [12]

$$\begin{aligned} \min_{\mathcal{X}} \quad & \left[\sum_{m=1}^{M} \gamma_m \|\mathbf{X}_{(m)}\|_* \right] \\ \text{s.t.} \quad & \mathcal{X}_{\mathbf{\Phi}} = \mathcal{T}_{\mathbf{\Phi}} \end{aligned} \tag{1}$$

where the constant $\gamma_m \geq 0$ and $\sum_{m=1}^{M} \gamma_m = 1$, $\mathbf{X}_{(m)} \in \mathbb{R}^{I_m \times I_1 I_2 \cdots I_{m-1} I_{m+1} \cdots I_M}$ denotes the mode-m matricization of $\mathcal{X}$, $\|\mathbf{X}_{(m)}\|_* := \text{trace}\sqrt{\mathbf{X}_{(m)} \mathbf{X}_{(m)}^{\mathrm{T}}} = \sum_{i=1}^{\min\{I_m, I_1 I_2 \cdots I_{m-1} I_{m+1}\}} \left[\sigma_i\left(\mathbf{X}_{(m)}\right)\right]$ is the trace norm (also known as nuclear norm) and σ_i is the i-th singular value of $\mathbf{X}_{(m)}$, $\mathcal{T}_{\mathbf{\Phi}}$ is also a Mth-order spectrum-tensor with the identical size of each mode with $\mathcal{X}$ and some of its entries are filled by a known spectrum data set $\mathbf{\Phi}$ and other entries are unknown or missing. Note that (1) provides a general tensor completion scheme based on tensor matricization with total M modes and single value decomposition on each matrix. A convex combination of M unfolded matrices of $\mathcal{X}$ is fulfilled with (1), which essentially aims to calculate the minimum rank of $\text{rank}\,(\mathcal{X})$. The determination of the rank of a high-order (say, $M > 3$) tensor goes to a NP-hard problem [19]. This is the reason why the matricization or decomposition schemes are used to tackle tensor completion under the given constraints [12]. The known spectrum data in $\mathbf{\Phi}$ are indicated at the same position in $\mathcal{X}_{\mathbf{\Phi}}$ and $\mathcal{T}_{\mathbf{\Phi}}$. With the existing decomposition schemes like CANDECOMP/PARAFAC decomposition (CPD), Tucker decompostion (TD), and Tensor train decomposition (TTD), the spectrum-tensor $\mathcal{X}_{\mathbf{\Phi}}$ can be decomposed into low rank tensors (matrices) and more simple completion schemes can be designed.

The spectrum-tensor can be decomposed into low-rank tensors or their combinations with decomposition schemes like CPD, TD, TTD, etc. The M-order spectrum-tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_M}$ can be decomposed into R rank-one tensors with the CPD scheme [1], [2]

$$\begin{aligned} \mathcal{X} &\approx \sum_{r=1}^{R} \mathbf{a}_r^{(I_1)} \circ \mathbf{a}_r^{(I_2)} \circ \cdots \circ \mathbf{a}_r^{(I_M)} \\ &= [\![\mathbf{A}^{(I_1)}, \mathbf{A}^{(I_2)}, \cdots, \mathbf{A}^{(I_M)}]\!] \end{aligned} \tag{2}$$

where R is the rank of $\mathcal{X}$, $\mathbf{a}_r^{(I_m)} \in \mathbb{R}^{I_m}$ is the r-th vector with the length of I_m, $\mathbf{A}^{(I_m)} \in \mathbb{R}^{I_m \times R}$, the symbol "$\circ$" denotes the vector outer product. The TD can be formulated as

$$\begin{aligned} \mathcal{X} &\approx \mathcal{G} \times_1 \mathbf{A}^{(I_1)} \times_2 \mathbf{A}^{(I_2)} \cdots \times_{I_M} \mathbf{A}^{(I_M)} \\ &= [\![\mathcal{G}; \mathbf{A}^{(I_1)}, \cdots, \mathbf{A}^{(I_M)}]\!] \end{aligned} \tag{3}$$

where $\mathcal{G} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_M}$ denotes the core tensor. When $\mathcal{G}$ is superdiagonal and $I_1 = I_2 = \cdots = I_M$, the CPD can be considered as a special case of the TD [1].

B. Tensor Rank and Tensor Eigenvalues

The rank of a tensor $\mathcal{X}$ is denoted by $\text{rank}\,(\mathcal{X})$. The properties of tensor and matrix ranks are different [1]. For a specific tensor, the rank determination is NP-hard problem [19] and only some upper bounds for high-order tensor can be determined. Moreover, the rank of a practical tensor may be different over real number field $\mathbb{R}$ and complex number field $\mathbb{C}$. Till now, it is still an open problem to determine an efficient algorithm for calculating the rank of a specific tensor.

For a square matrix $\mathbf{A} \in \mathcal{R}^{M\times M}$, the eigenvalues can be determined by

$$\mathbf{A}\xi = \lambda\xi \tag{4}$$

where λ denotes the eigenvalue and ξ denotes an $M\times 1$ eigenvector. For a full-rank matrix $M = \text{rank}(\mathbf{A})$, the M ordered eigenvalues can be written as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. In random matrix theory, the eigenvalues are widely used in many fields such as estimation and detections. Like tensor rank, tensor eigenvalues are more difficult to be determined due to obscure knowledge of tensor rank. For an M-th order supersymmetric tensor $\mathcal{X} \in \mathbb{R}^{\overbrace{N\times N\times\cdots\times N}^{M}}$ with unchanged entries under permuting indices, the tensor eigenvalue λ can be determined [17], [18]

$$\mathcal{X}\times_1 \mathbf{v}\times_2 \mathbf{v}\times_3\cdots\times_M \mathbf{v} = \lambda\mathbf{v} \tag{5}$$

where $\mathbf{v}\in\mathbb{R}^N$ is the corresponding eigenvector and λ is an eigenvalue. Note that "supersymmetric" is a very strict constraint for a practical tensor with big data.

C. Tensor Processing Unit and TensorFlow

The big data in the form of tensors can be tackled with TPU in hardware and TensorFlow in software both developed by Google. The TPU is a tensor-based AI accelerator application-specific integrated circuit (ASIC) chip, which is designed for TensorFlow framework and specially used for neural network machine learning [20]. TensorFlow is a free and open-source software library for big data analysis and processing. Both TPU and TensorFlow provide tensor-based spectrum big data analysis from the view of hardware and software. In this paper, the system-tensor and spectrum-tensor can be designed to be processed with TPU and TensorFlow.

IV. Spectrum Usage Framework and Tensor-based Spectrum Fusion

A. New Spectrum Usage Framework

The 6G systems will require large amount of spectrum including licensed, unlicensed, shared licensed, and potential spectrum. Based on the spectrum framework for 6G systems shown in Fig. 2, the SFE is the core part and its functions mainly include sensing (learning), recording (providing), regulating (scheduling), and utilizing (adapting) and so on. New spectrum usage mode should be considered to fulfill huge spectrum requirements for 6G systems. For the new applications in 6G systems like new holographic communications, multiple kinds of spectrum should be jointly used to fulfill their rigid system requirements in terms of time latency and data rate.

For the SFE, the spectrum-tensor $\mathcal{X}\in\mathbb{R}^{U\times V\times T\times S\times F}$ can be updated by sensing its each entry

$$\mathcal{X}_0 \xrightarrow[i_1=1:I_1,i_2=1:I_2,i_3=1:I_3,i_4=1:I_4,i_5=1:I_5]{x_{i_1i_2i_3i_4i_5}=p_a} \mathcal{X}_1 \tag{6}$$

where p_a denotes the power of the received signal a at a specific entry. We use the energy of the received signal to indicate the state of the required entry. Following this point of view, a spectrum-tensor can be evaluated with received signal energy and a vivid M-dimension energy tensor can be achieved.

Based on spectrum-tensor, the spectrum state can evaluated by two schemes. First, the total energy of a M-th order spectrum-tensor $\mathcal{X}$ can be calculated

$$E_{\mathcal{X}} = \sum_{i_1=1}^{I_1}\cdots\sum_{i_M=1}^{I_M}|x_{i_1\cdots i_M}|^2 = \|\mathcal{X}\|^2 \tag{7}$$

where $\|\cdot\|$ denotes the norm and the energy of $I_1\times I_2\times\cdots\times I_M$ entries are summarized. Secondly, for a specific dimension or mode i_k, the energy of the "slice" $\mathbf{X}^{(i_k)}$ can be calculated

$$\begin{aligned} E_{\mathbf{X}^{(i_k)}} &= \|\mathbf{X}^{(i_k)}\|^2 \qquad (8)\\ &= \sum_{i_1=1}^{I_1}\sum_{i_2=1}^{I_2}\cdots\sum_{i_{k-1}=1}^{I_{k-1}}\sum_{i_{k+1}=1}^{I_{k+1}}\cdots\sum_{i_M=1}^{I_M}|x_{i_1\cdots i_k\cdots i_M}|^2. \end{aligned}$$

The above formulation can also be expressed as

$$E_{\mathbf{X}^{(i_k)}} = \sum_{\forall(i_1,i_2,\cdots i_{k-1},i_{k+1},\cdots,i_M)\cap(i_k=\beta)}|x_{i_1\cdots i_k\cdots i_M}|^2 \tag{9}$$

where i_k is fixed value and $\beta\in[1:I_k]$. For a spectrum-tensor $\mathcal{X}\in\mathbb{R}^{U\times V\times T\times S\times F}$ shown in Fig. 1, the corresponding energy calculated by (7) indicates the total power distribution for all five modes. If we want to check the status of a specific dimension (say $v=1$), the power can be calculated by

$$E_{\mathbf{X}^{(v=1)}} = \sum_{u_1=1}^{U}\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{f=1}^{F}|x_{u1tsf}|^2. \tag{10}$$

When $U=3, V=4, T=2, S=2, F=2$ in Fig.1, $E_{\mathbf{X}^{(v=1)}}$ denotes the power of three cubes located in the bottom line. With this scheme, the power of a specific mode (or dimension) can be evaluated. We can calculate the signal power of a specific user (fixed u) or a specific spot (fixed s). For other functions shown in Fig. 2 such as recording, utilizing, and regulating, the corresponding new functions can also be defined with the spectrum-tensor, leading to a simple and compact expression.

B. Tensor-based Spectrum Fusion Scheme

Based on spectrum sharing and spectrum aggregation, a new spectrum usage framework has been discussed in [10], which jointly use spectrum sharing and spectrum aggregation for an enhanced cognitive radio networks. In this paper, a value-based spectrum fusion scheme is proposed for 6G systems. A spectrum-tensor with five modes has been indicated in Fig. 1, in which the spectrum information can be evaluated with a specific dimension. For example, we can obtain the specific spectrum information with a fixed mode, say $v=1$.

In the new spectrum fusion scheme based on spectrum-tensor, we can jointly analyze the spectrum with varying modes, such as time, space, user, and value. The proposed spectrum fusion scheme can be formulated as

$$\max_{\mathcal{X}} \quad \left[\sum_{u=1}^{U} V_u(\mathcal{X})\right] \quad \text{s.t.} \quad V_u(\mathcal{X}) \geq V_{th} \tag{11}$$

where the value function $V_u(\mathcal{X})$ calculates the value of the user u with $\mathcal{X}$ and V_{th} denotes a value threshold for a user. For 6G systems, the spectrum fusion scheme tries to achieve the maximum value for all users in a tensor space spanned by five modes. Different from spectrum sharing and spectrum aggregating, the proposed spectrum fusion scheme aims to maximize the spectrum value in a fixed system under the constraint that each user's spectrum value should be larger than a given value threshold.

We use the tensor decomposition scheme to fulfill the value function $V_u(\mathcal{X})$. Based the CPD scheme shown in (2), a spectrum-tensor $\mathcal{X} \in \mathbb{R}^{U\times V\times T\times S\times F}$ can be decomposed into five matrices as

$$\begin{aligned}\mathcal{X} &\approx \sum_{r=1}^{R} \mathbf{a}_r^{(U)} \circ \mathbf{a}_r^{(V)} \circ \mathbf{a}_r^{(T)} \circ \mathbf{a}_r^{(S)} \circ \mathbf{a}_r^{(F)} \\ &= [\![\mathbf{A}^{(U)}, \mathbf{A}^{(V)}, \mathbf{A}^{(T)}, \mathbf{A}^{(S)}, \mathbf{A}^{(F)}]\!]\end{aligned} \tag{12}$$

where the rank R can be determined numerically [1] and the dimensions of these matrices are also determined. The tensor-spectrum can be further evaluated from five separated modes. Let's take the value matrix $\mathbf{A}^{(V)} \in \mathbb{R}^{V\times R}$ as an example to construct the value function $V_u(\mathcal{X})$,

$$V_u(\mathcal{X}) = \sum_{v=1}^{V} \beta_v \|\mathbf{A}^{(V)}(v,:)\|^2 \tag{13}$$

where $\beta_v \in [0,1]$ denotes the coefficient for the vth value and $\mathbf{A}^{(V)}(v,:) \in \mathbb{R}^{V\times 1}$ denotes the vth vector of $\mathbf{A}^{(V)}$. The value factor β_v can be jointly determined by the spectrum value in terms of data rate or spectrum efficiency.

A more sophisticated spectrum-tensor analysis scheme utilizes tensor eigenvalues or matrix eigenvalues, which indicate the essential characteristics of spectrum-tensors or matrices decomposed by CPD or TD. Based on (5), we can potentially calculate M eigenvalues for a M-th order tensor $\mathcal{X}$. However, to the best of our knowledge, only the eigenvalues of low order (say 3) supersymmetric tensor can be theoretically determined with different results in terms of real and complex [17]. The determination of eigenvalues for high-order tensors are still an open problem to us.

However, the eigenvalues of matrix have been well studied and the distributions of eigenvalue have been widely used in wireless communications and signal processing [21]. For a matrix $\mathbf{A}^{(V)} \in \mathbb{R}^{V\times R}$ decomposed from a spectrum tensor $\mathcal{X}$, a Wishart matrix $\mathbf{W} \in \mathbb{R}^{K\times K}$ $(K = \min\{V, R\})$ can be generated by

$$\mathbf{W} = \mathbf{A}^{(V)} \times \left(\mathbf{A}^{(V)}\right)^{\mathrm{T}} \tag{14}$$

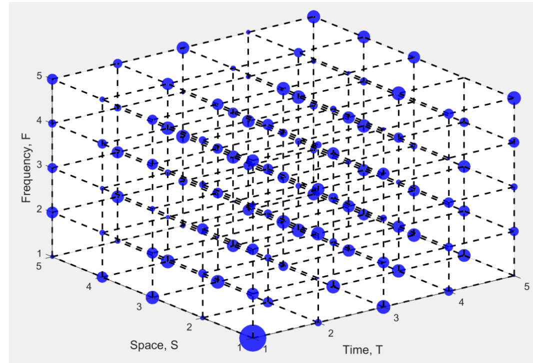

Fig. 3: A 3-order spectrum-tensor of received signal (5dB).

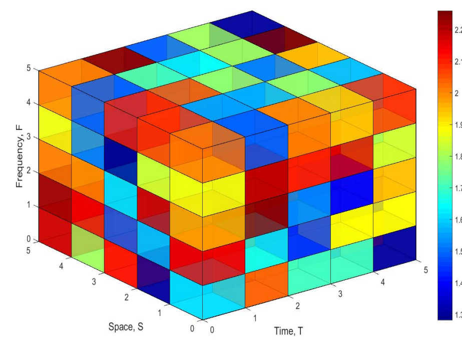

Fig. 4: A 3-order spectrum-tensor of received signal (2dB).

where T denotes transpose and suppose $V \leq R$. For $\mathbf{W}$, K ordered eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K$ can be determined. Till now, lots of existing results including eigenvalue, condition number in random matrix theory can be applied for spectrum analysis.

V. Numerical results and Evaluations

In this section, we provide some vivid illustrations of high order tensors and their decomposed matrices. Moreover, a spectrum sensing scheme based on spectrum-tensor is illustrated.

A 3-order spectrum-tensor $\mathcal{X} \in \mathbb{R}^{T\times S\times F}$ is shown in Fig. 3, in which $T = S = F = 5$ and the area of blue circles denotes the power of received signal and noise, the SNR of the signal is set to 5dB. The $T \times S \times F = 125$ entries of the spectrum-tensor $\mathcal{X}$ indicate the power of signal and noise at specific dimension (t, s, f). Let $y_{t,s,f} = h * z_{t,s,f} + n_{t,s,f}$ denote the received signal z with the flat channel h and the noise n. The area of the blue circle at t, s, f denote the power of signal and noise $|y_{t,s,f}|^2$. We can see that Fig.3 provides a 3-dimension model for the spectrum-tensor $\mathcal{X}$.

A more vivid spectrum-tensor is shown in Fig. 4, in which the power of the received signal (2 dB) and noise is indicated with varying color. With this figure, we can see directly the power distribution of the received signal in a whole spectrum-tensor. Note that a cube from $(0, 0, 0)$ to $(1, 1, 1)$ is used to

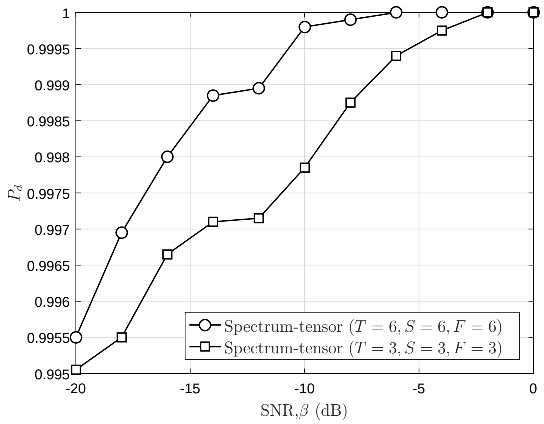

Fig. 5: A spectrum sensing scheme based on spectrum-tensor with 2dB received signal.

illustrate a specific entry of the spectrum-tensor $\mathcal{X} \in \mathbb{R}^{T\times S\times F}$ and its color is used to denote its corresponding signal power.

As for the application of spectrum-tensor, we illustrate a spectrum sensing scheme based on the received signal power in Fig. 5. A hypothesis test is still used in this scheme and the power (tensor norm) of received signal is used to construct the detector. We compare the detectors based on a 6-order spectrum-tensor and a 3-order spectrum-tensor. Both detectors can achieve high sensing performance and the 6-order spectrum-tensor based scheme outperforms that of the 3-order spectrum-tensor. With this scheme, the spectrum sensing performance for a specific dimension (say time or space) can be evaluated. Furthermore, the energy performance of the slice or matrix of the spectrum-tensor can be achieved by tensor decomposition.

VI. Conclusions

In this paper, a new concept of tensor-computing has been proposed and discussed for the future 6G systems. The new spectrum-tensor and system-tensor have been designed. A new spectrum usage framework based on the proposed tensor-computing has been designed. A spectrum fusion scheme based on total system value and single user value has also be proposed to achieve higher spectrum efficiency. Some key tensor operations such as completion, decomposition, eigen-value calculation have been discussed for spectrum-tensor and system-tensor. The authors hope that the newly coined tensor-computing can provide a sharp tool for the coming 6G wireless communication systems.

ACKNOWLEDGMENT

This work was supported by the National Key R&D Program of China (2018YFB1801101), the National Natural Science Foundation of China (NSFC) (61960206006), the Fundamental Research Funds of Shandong University (2017JC029), the China Scholarship Council (CSC) (201806225029), the High Level Innovation and Entrepreneurial Talent Introduction Program in Jiangsu, the Research Fund of National Mobile Communications Research Laboratory, Southeast University,(2020B01), the Fundamental Research Funds for the Central Universities (2242019R30001), the EU H2020 RISE TESTBED2 project (872172), Huawei cooperation project, and Taishan Scholar Program of Shandong Province. The work of J. Wu was supported in part by the U.S. National Science Foundation (NSF) (ECCS-1405403 and ECCS-1711087).

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