Different Decoding Methods for Multilevel Coded Modulation over Rayleigh Fading Channels*

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Abstract: Based on "capacity rule", the performance of multilevel coding (MLC) schemes with different decoding methods, which are multistage decoding(MSD) and parallel decoding on level(PDL), is investigated. The work is done for 8ASK modulation and three mapping strategies over Rayleigh fading channels are used. In each scheme BCH codes with different code lengths are used as component codes. Numerical results indicate that MSD is a sub-optimal decoding method of MLC, while PDL is most robust if block partitioning (BP) is used. For Ungerboeck partitioning (UP) and Mixed partitoning (MP) strategy, MSD is strongly recommended to use for MLC system, while for BP strategy, PDL is suggested to use as a simple decoding method compared with MSD.

I. Introduction

A sub-optimal decoding technique called MSD was introduced in [1] for the decoding of multilevel codes. This decoding procedure is done stage by stage and is accomplished by decoding the component codes one at a time. The reliability of MLC system can be improved greatly by using MSD method which is to decode each component code individually starting from the lowest level and using decisions of previous decoding stages. Because of the advantages of MSD, many publications have concentrated on it[2-4].

Another decoding method for MLC proposed by P. Schramm in 1997 is PDL or Independent decoding on levels (IDL)[5]. The complexity and time delay of this decoding method is lower than MSD and it has robustness to different channels[6].

In this paper, we are focusing on the comparison of these two decoding methods for MLC. Based on the calculation for capacities of equivalent channels[6-8], the performance of MLC/MSD and MLC/PDL schemes with three set partitioning strategies in Rayleigh fading channels is investigated, in which BCH codes with different code lengths are chosen as component codes, and 8ASK signal constellation is used.

II. Equivalent Channel and Capacity Rule

For a MLC system, since the mapping M is bijective and hence lossless in the sense of information theory, the mutual information I(Y;A) between the transmitted signal point $a \in \mathbf{A}$ and the received signal $y \in Y$ equals the mutual information $I(Y;X^0,X^1,\ldots X^{l-1})$ between the address vector $X \in \{0,1\}^l$ and the received signal point:

$$I(Y;A) = I(Y;X^0,X^1,\ldots X^{l-1}) \tag{1}$$

Applying the chain rule to the mutual information yields[9]

$$I(Y;X^0,X^1,\ldots X^{l-1}) = I(Y;X^0) + I(Y;X^1 \mid X^0) + \ldots + I(Y;X^{l-1} \mid X^0 X^1 \ldots X^{l-2}) \tag{2}$$

This equation may be interpreted in the following way: the transmission of vectors with binary digits $X^i, i = 0,1,\ldots l-1$,over the physical channel can be virtually separated into the parallel transmission of the digits x^i over ℓ equivalent channels. The equivalent channel i consists of the equivalent mapper i, provided that the digits $x^0 \ldots x^{i-1}$ and the noisy channel are known. The binary symbol x^i is multiply represented in the signal set of the equivalent mapper i for $i < \ell - 2$.

The capacities of the equivalent channels for MLC/MSD scheme are proposed and derived by [10,11] which directly lead to the capacity rule or the rate rule

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design. Given a 2-ary digital modulation scheme, choose the rate R^i at the individual coding level i of a MLC scheme to equal the capacity C^i of the equivalent channel

$$i: \quad R^i = C^i \qquad i = 0,1,\ldots,l-1 \tag{3}$$

The basis of the capacity rule is to characterize the transmission properties of the equivalent channels by its capacities. Operating at the capacity limit of MLC scheme, the capacity rule provides the maximum individual rates to be transmitted with arbitrarily low error probability. Thus, the design of MLC system with an optimum trade-off between power and bandwidth efficiency has to be based on the capacity rule.

The capacity C^i for given a-priori probabilities of signal points yields:

$$C^i = I(Y;X^i/X^0\ldots X^{i-1}) = I(Y;X^i\ldots X^{l-1}/X^0\ldots X^{i-1}) - I(Y;X^{i+1}\ldots X^{l-1}/X^0\ldots X^i) \tag{4}$$

Thus, C^i is given by:

$$\begin{cases} C^i = E_{x^0..x^{i-1}}\{C(A(x^0\ldots x^{i-1}))\} - E_{x^0..x^i}\{C(A(x^0..x^i))\}, & i=1,\ldots,l-1 \\ C^0 = C(A) - E_{x^0}\{C(A(x^0))\} & i=0 \end{cases} \tag{5}$$

III. Comparison of Decoding Methods for MLC Schemes

A. Multistage Decoding for Multilevel Codes

The right side of the chain rule (2) suggests the rule for a low-complex staged decoding procedure that is well-known as multistage decoding(MSD) proposed by Imai in his original work[1]. The component codes C^i are successively decoded by the corresponding decoders D_i. At stage i, decoder D_i processes depending on not only the block $y=(y[1],\ldots,y[N])$, $y[\mu]\in Y$, of received signal points, but also decisions $\hat{x}^j$, j=0, ...,i-1, of previous decoding stages j. The use of previous decoding decisions accomplishes the selection of the current subsets of the equivalent mapper i for the different time instants $\mu=1,\ldots,N$.

Actually, the staged decoding according to the chain rule in (2) would require the transmit symbol x^j instead of the estimate $\hat{x}^j$. But if we assume error-free decisions $\hat{x}^j=x^j$ of decoder D_j, MSD can be interpreted as an implementation of the chain rule. Clearly, in practice, erroneous decisions occur and errors propagate from low levels to higher ones. But it is shown later that error propagation in MSD does not significantly influence the performance of the total scheme.

B. Parallel Decoding for Multilevel Codes

An alternative decoding strategy for multilevel coded transmission is parallel decoding of the individual levels(PDL)[5]. In contrast to the MSD approach, decoder D_i makes no use of decisions of other levels $i\neq j$. All decoders D_i, i=0,1, ...,l-1, are working in parallel. For MLC/PDL the transmission of each address symbol x^i, i=0,1,...,l-1, over the equivalent channel i is based on the entire signal constellation, since there is no preselection of signal points at higher levels due to decoding decisions of other levels. Of course, information is lost by not using estimates from lower levels. Thus, the sum of the capacities C_{sum} of all levels is less than(or equal to) the total capacity of the signal set, i.e.

$$C_{sum} = \sum_i C_i \le C_{set} \tag{6}$$

To be more accurate, the concept of the equivalent channel and its characterizing pdf has to be adopted appropriately for an MLC/PDL scheme. While in the case of MLC/MSD the signal set of the equivalent mapper i is time variant for i>0 depending on the binary digits x^j of lower levels j, j=0, ...,i-1, the equivalent mapper i for the MLC/PDL scheme is time invariant for all i=0, ...,l-1. Since the decoding at one level is done independently of other levels, the equivalent mappers for MLC/PDL comprise the entire signal set **A** in every case. In the signal set of equivalent mapper i the binary symbol b^i is multiply represented by all signal points with address digit $x^i=b^i$, $b^i\in\{0,1\}$.

An advantage of the PDL decoding approach is certainly that error propagation from low to higher levels can be avoided since the levels are decoded independently. Additionally, PDL is favorable in terms of decoding delay since the individual decoders are working in parallel instead of serial in the staged decoding approach MSD.

From the results of capacities for MLC/MSD scheme with three mapping strategies (Ungerboeck Partitioning--UP, Block Partitioning--BP, Mixed

Partitioning--MP)and 8ASK modulation, the rate design values of MLC/MSD over Rayleigh fading channels are obtained. Table1 lists the results over Rayleigh fading channels when R is 2bits/symbol.

IV. Results and Discussions

According to the discussion of channel capacity in the previous section and the code rates in Table 1, the performance comparison of MLC/MSD and MLC/PDL over Rayleigh fading channels was performed by means of simulation. The presented results are bit error rates(BERs or P_b) as a function of E_b/N_0, where E_b denotes average energy per information bit. The modulation scheme is 8ASK, BCH codes with different code lengths are chosen as component codes on three levels.

Fig.1 shows the results for BCH codes with code lengths of 127, while Fig.2 is the results for BCH codes with code lengths of 255. The total rates of all schemes are R=2bits/symbol. For comparison, the performance of uncoded 4ASK modulation scheme is also given. From the simulation results, we can see:

(a) For any set partitioning strategy, MSD is superior to PDL for MLC scheme over Rayleigh fading channels. Therefore, the MLC/MSD scheme is proved to be an asymptotically optimum approach to coded modulation for Rayleigh fading channels. The condition for this optimality is that the rates of the component codes are chosen to be equal to the capacities of the equivalent channels.

(b) Comparing Fig.1 with Fig.2 we can see that for each of the corresponding scheme, the performance of MLC/MSD and MLC/PDL schemes will be better using component codes with longer code lengths when code rates are designed according to "capacity rule". Compared with uncoded 4ASK scheme, they all have high coding gains.

(c) As shown in Fig.1 and Fig.2, for UP and MP strategies, the power efficiency of MLC/MSD and MLC/PDL is nearly the same at lower SNR when E_b/N_0 is between 4dB and 16dB. At high SNR, the performance difference will be larger. When $Pb=10^{-3}$, MLC/MSD scheme is superior to MLC/PDL by 3~5dB coding gain for UP strategy using BCH codes with code length of 127, as shown in Fig.1, while by 2~4dB coding gain for MP strategy using BCH codes with code lengths of 255, as shown in Fig.2.

(d) As Fig.1 and Fig.2 show that PDL and MSD lead to approximately the same performance for MLC scheme with BP mapping strategy. Therefore, PDL, the simple and pragmatic decoding method, can be used instead of MSD, the complex and iterative decoding method with long time delay, for MLC system over Rayleigh fading channels when BP strategy is employed. This conclusion has great importance and significance for multilevel coding schemes with more levels, e.g. MLC/MSD scheme with 64QAM, because the complexity and time delay of MSD will be decreased greatly.

V. Conclusions

From simulation results and discussions, some conclusions can be got:

(a) For any set partitioning strategy, MLC/MSD scheme is superior to MLC/PDL over Rayleigh fading channels. In each scheme the code rates of component codes with different code lengths are all designed based on "capacity rule". Therefore, MSD is the sub-optimal decoding method for multilevel coding system.

(b)As long as BP strategy is used, the performance of MLC/PDL is nearly the same with that of MLC/MSD scheme for Rayleith fading channels. Therefore, PDL can be used as a more attractive and simple decoding method instead of MSD for MLC system. This conclusion has great significance for designing the MLC system with higher bandwidth efficiency, e.g. there is more than three levels in MLC system.

(c)The performance of MLC scheme with different decoding methods is related to set partitioning strategies. For UP and MP strategy, MSD method is strongly recommended to use because the performance of MLC/MSD scheme is much better than that of MLC/PDL. For BP strategy, PDL is suggested to use as a simple decoding method because the performance of MLC scheme with two decoding methods is nearly the same.

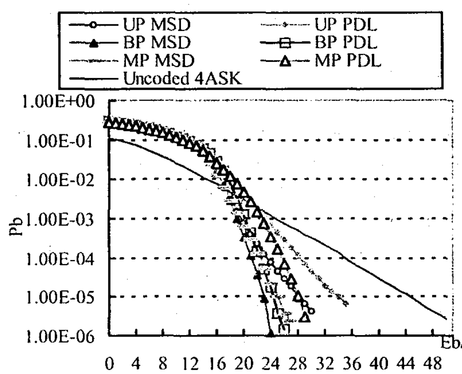
Reference

1. H. Imai and S. Hirakawa, "A new multilevel coding method using error correcting codes", IEEE Trans. Inform. Theory, Vol. 23, No.5, pp371~377, 1977

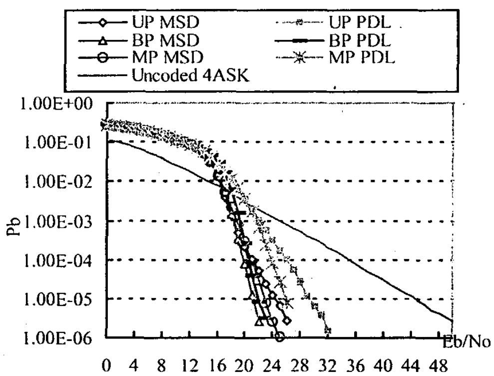
2. N. Seshadri, C-E.W. Sundberg, "Multilevel Trellis Coded Modulations for the Rayleigh Fading Channel", IEEE Trans. on Com., Vol.41, pp1300-1310, Sept. 1993
3. L. Zhang and B. Vucetic, "Multilevel block codes for Rayleigh fading channels", IEEE Trans. on Com., Vol. 43, No.1, pp.24-31, Jan. 1995
4. E. J. Leonardo, L. Zhang and B. Vucetic, "Sub-optimum Multistage decoding of Multilevel Block Codes", ICC'96, pp984-988, Dallas, Texas, U.S.A., June 1996
5. P. Schramm, "Multilevel Coding with Independent Decoding on Levels for Efficient Communication on Static and Interleaved Fading Channels", PIMRC'97, pp1196-1200, Helsinki, Finland, Sept. 1997
6. D. F. Yuan, Z. G. Cao, D. W. Schill and J. B. Huber, "Robust signal constellation design for AWGN and Rayleigh fading channels for softly degrading communication scheme using multilevel codes", To appear in Acta Electronica Sinica, English edition, 2000
7. D. W. Shill, D. F. Yuan and J. B. Huber, "Efficient Hierarchical Broadcasting Using Multi-Level Codes", IEEE ITW'99, Greece, June 1999
8. D. F. Yuan, D. W. Shill and J.B. Huber, "Robust Hierarchical Broadcasting for AWGN and Flat Rayleigh Fading Channels Using Multi-Level Codes", ISCTA'99, U.K., July 1999
9. R. G. Gallager, *Information Theory and Reliable Communication*. John Wiley Sons, Inc., New York, 1968
10. J. B. Huber and Udo Wachsmann, "Capacity of equivalent channels in multilevel coding schemes", Electron. Lett., pp557~558, 1994, 30(7)
11. U. Wachsmann, R. F. H. Fischer and J. B. Huber, "Multilevel Codes: Theoretical Concepts and Practical Design Rules", IEEE Trans. Inform. Theory, pp1361~1391, 1999, 45(5)

Table I
Different Rates of MLC/MSD with 8ASKModulation Based on Capacity Rule (R=2bits/symbol)

	UP	BP	MP
Rayleigh Channels	$C_0=R_0=0.3125$ $C_1=R_1=0.75$ $C_2=R_2=0.9375$	$C_0=R_0=0.8125$ $C_1=R_1=0.6875$ $C_2=R_2=0.5$	$C_0=R_0=0.8475$ $C_1=R_1=0.35$ $C_2=R_2=0.8025$



UP: C0(127,36,15);C1(127,99,4); C2(127,120,1)
BP: C0(127,99,4); C1(127,92,5); C2(127,64,10)
MP: C0(127,106,3);C1(127,43,14);C2(127,106,3)
Fig.1 Comparison of MLC/MSD and MLC/PDL with 8ASK over Rayleigh channel



UP: C0(255,79,27);C1(255,191,8);C2(255,239,2)
BP: C0(255,207,6);C1(255,171,11);C2(255,131,18)
MP: C0(255,215,5);C1(255,87,26);C2(255,207,6)
Fig.2 Comparison of MLC/MSD and MLC/PDL with 8ASK over Rayleigh channel