AMC-HARQ System Baded On RC-LDPC Codes in MIMO Channels

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***Abstract*—In this paper, we propose a cross-layer design framework combining adaptive modulation and coding (AMC) with hybrid automatic repeat request (HARQ) based on rate-compatible low-density parity-check codes (RC-LDPC) in multiple-input multiple-output (MIMO) fading channels with estimation errors. We derive the expressions for the throughput of the system and investigated the effect of channel estimation errors on the system throughput. Numerical results show that the joint design of AMC and ARQ based on RC-LDPC codes can achieve considerable spectral efficiency gain.**

***Keywords*—AMC, HARQ, LDPC, STBC.**

I. INTRODUCTION

The explosive growth of wireless packet data applications such as wireless Internet, interactive mobile multimedia applications, and interactive gaming has driven an unprecedented revolution in wireless networks. Various applications in wireless networks require different quality of service (QoS). In order to achieve efficient utilization of scarce radio resources with different QoS requirements, the cross-layer design approach has drawn significant research attention allowing information sharing between different layers of wireless networks. In the literature, various cross-layer design schemes have been proposed. In [1], adaptive modulation and coding (AMC) at the physical layer and automatic repeat request (ARQ) at the data link layer were jointly designed in order to maximize network capacity under constrained QoS requirements. However, perfect channel state information (CSI) was assumed and only single input single output (SISO) scenario was considered in [1]. This observation motivates us to extend the cross-layer design in SISO channels to multiple-input multiple-output (MIMO) channels using space-time block codes (STBCs) with channel estimation errors.

The rest of this paper is organized as follows. In Section II, we present the system model of the cross-layer design combining the AMC and HARQ in MIMO fading channels using STBCs. How to calculate the effective signal-to-noise ratio (SNR) and channel estimations are explained in Section III, while the principle of the cross-layer design is illustrated in Section IV. In Section V, we apply RC-LDPC codes to AMC-ARQ systems over MIMO fading channels with STBC and get numerical results through simulations. Finally, some conclusions are drawn in Section VI.

II. SYSTEM MODEL

The system model of an AMC-ARQ system based on RC-LDPC codes in MIMO channels using STBCs is shown in Fig.1. Assuming that there are N_T transmit antennas and N_R receive antennas, then the diversity order is defined as $K \triangleq N_T N_R$. The MIMO fading channel can be expressed as a matrix $\mathbf{H} = [h_{ij}]_{i,j=1}^{N_R,N_T}$, where h_{ij} is the channel coefficient between the jth transmit antenna and the ith receive antenna. Under the assumption of independent Rayleigh fading, the channel coefficients h_{ij} are modeled as independent and identically distributed (i.i.d.) complex circular Gaussian random variables with zero mean and unit variance. The received signal can be expressed as

$$\mathbf{Y} = \mathbf{HX} + \mathbf{V}, \tag{1}$$

where $\mathbf{Y}$ is a $N_R \times \mathbf{T}$ matrix of received symbols with $\mathbf{T}$ representing the number of symbols per antenna, $\mathbf{X}$ is a $N_T \times \mathbf{T}$ matrix of transmitted symbols, and $\mathbf{V}$ is a $N_R \times \mathbf{T}$ noise matrix with elements modeled as i.i.d. complex circular Gaussian random variables having zero mean and unit variance.

At the physical layer, there are multiple modulation and coding schemes (MCSs) available. The CSI is estimated at the receiver and then sent back through a feedback channel to the AMC controller, which chooses the appropriate MCS in the next transmission accordingly.

At the data link layer, the selective repeat ARQ protocol is adopted to control packet retransmissions. When an error is detected in a packet, a retransmission request is generated and sent back to the transmitter via a feed back channel. For simplicity, we adopt hybrid type-I ARQ scheme and assume that the feed back channel is error free and has zero delay.

III. EFFECTIVE SNR AND CHINA ESTIMATION

In our system model shown in Fig. 1, the STBC

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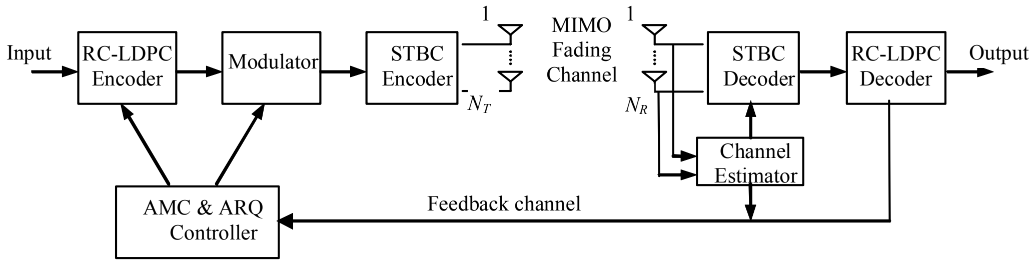

Fig. 1. System model.

encoder maps $R \le T$ complex modulated symbols into N_T orthogonal complex symbol sequences of length T and then transmits them by N_T transmit antennas simultaneously. The coding rate of a STBC is therefore $R_c = R/T$. Let us define the average transmit power per stream/antenna as P_s. According to the effective SISO channel model for STBCs described in [2], the received symbol y before the maximum likelihood (ML) detection can be expressed as

$$y = \|\mathbf{H}\|_F^2 s + v, \tag{2}$$

where s is the real or imaginary part of the transmitted complex symbol, v is the noise symbol with mean power σ^2 after STBC decoding, $\|\cdot\|_F^2$ denotes the squared matrix Frobenius norm, and $\|\mathbf{H}\|_F^2 = \sum_{i,j} h_{ij}^2$. At the receiver, the SNR is given by [3]

$$\gamma = \frac{P_s}{\sigma^2}\|\mathbf{H}\|_F^2 = \frac{P_T}{\sigma^2 N_T R_c}\|\mathbf{H}\|_F^2 = \frac{\bar{\gamma}}{N_T R_c}\|\mathbf{H}\|_F^2, \tag{3}$$

where P_T is the total transmit power transmitted on N_T antennas per symbol duration and $\bar{\gamma} = P_T/\sigma^2$ is defined to be the average pseudo SNR. Since $\|\mathbf{H}\|_F^2$ is the sum of $2K$ i.i.d. χ^2 random variables, we can get the probability density function (PDF) of γ as follows [4]

$$p_\gamma(\gamma) = \frac{\gamma^{K-1}}{\Gamma(K)}\left(\frac{N_T R_c}{\bar{\gamma}}\right)^K \exp\left(-\frac{N_T R_c}{\bar{\gamma}}\gamma\right), \quad \gamma \ge 0$$
(4)

where $\Gamma(\cdot)$ is the Gamma function.

Assuming that the receiver performs the minimum mean square error (MMSE) estimation of the channel, then $\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}$ holds, where $\hat{\mathbf{H}}$ is the estimated channel matrix and $\mathbf{E}$ is the estimation error. We further assume that $\hat{\mathbf{H}}$ and $\mathbf{E}$ are uncorrelated. The entries of $\mathbf{E}$ are also i.i.d. zero-mean circularly symmetric complex Gaussian distributed random variables with variance $\sigma_e^2 = E(h_{ij}^2) - E(\hat{h}_{ij}^2)$. The estimated SNR $\hat{\gamma}$ has the following relationship with the instantaneous SNR γ [5]

$$\hat{\gamma} = \frac{1-\sigma_e^2}{1+\sigma_e^2 P_T}\gamma. \tag{5}$$

Consequently, we can derive the PDF of the estimated SNR $\hat{\gamma}$ [4]

$$p_{\hat{\gamma}}(\hat{\gamma}) = \frac{\lambda^K}{\Gamma(K)}\hat{\gamma}^{K-1} e^{-\lambda\hat{\gamma}}, \quad \gamma \ge 0, \tag{6}$$

where $\lambda = \dfrac{N_T R_c (1+\sigma_e^2 P_T)}{(1-\sigma_e^2)\bar{\gamma}}$.

The correlation between h_{ij} and its estimation $\hat{h}_{ij}$ is [4]

$$u = \frac{E(h_{ij}\hat{h}_{ij})}{\sqrt{\mathrm{E}(h_{ij}^2)\mathrm{E}(\hat{h}_{ij}^2)}} = \frac{1}{\sqrt{1+\sigma_e^2}}, \tag{7}$$

which indicates the quality of the channel estimation. From (5) and (7), it is clear that $\hat{\gamma} = \gamma$ and $u = 1$ hold, respectively, if $\sigma_e^2 = 0$. This is actually corresponding to the perfect channel estimation. The expression (7) further tells us that the correlation u between h_{ij} and $\hat{h}_{ij}$ is getting smaller with the increase of σ_e^2, which means that the channel estimation is becoming more inaccurate and will cause severe degradation of the system performance.

IV. Cross-Layer Design in MIMO Channels

The cross-layer design considered in this paper involves two layers, i.e., the physical layer and the data link layer. At the data link layer, the Nr truncated ARQ protocol is adopted. Packets received incorrectly after Nr retransmissions will be dropped, thus inducing packet loss. In order to meet the system delay constraint, for a given packet loss probability PER_{link} at the data link layer, the packet error rate (PER) P_{target} at the physical layer should be [1]

$$P_{\text{target}} = PER_{\text{link}}^{\;1/(N_r+1)}. \tag{8}$$

Since the AMC is implemented at the physical layer according to the target PER, it is clear that P_{target} is the cross-layer information.

Suppose that there are N MCSs at the physical layer with increasing rates R_n (n=1, 2, …, N) in terms of information bits per symbol. We will consider the modulation method with the MQAM signal constellation, where M denotes the number of points in each signal constellation. If the coding rate of a MCS is R_L, we have $R_n = R_L \cdot (\log_2 M)$. As in [1], we assume constant power transmission and adopt the equivalent SISO channel model to describe the estimated instantaneous channel

SNR $\hat{\gamma}$. The whole SNR range is divided into N+1 intervals based on N thresholds γ_n, n = 1, 2, …, N. When $\gamma_n \le \hat{\gamma} < \gamma_{n+1}$, MCS n with the rate R_n will be chosen for the next transmission. Our first task is to determine the thresholds γ_n.

For LDPC codes, the relationship between the PER and $\hat{\gamma}$ is given by [6]:

$$\mathrm{PER}_n(\hat{\gamma}) = \begin{cases} 1, & if \quad 0 < \hat{\gamma} < \gamma_{cf} \\ \left(\dfrac{1}{1+\exp\{c_n(\hat{\gamma}-b_n)\}}\right)^{a_n}, & if \ \hat{\gamma} \ge \gamma_{cf} \end{cases} , \tag{9}$$

where a_n, b_n, c_n and γ_{cf} are parameters obtained by fitting (9) to the simulation results. Considering the LDPC-coded modulation schemes listed in Table I, Fig. 2 impressively shows the excellent accordance between the theoretical approximation (9) and the exact PER.

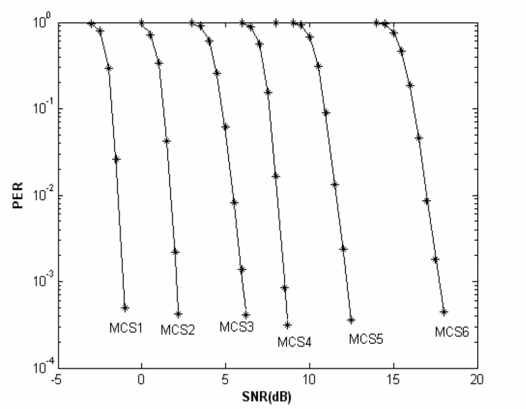

Fig. 2. PER simulation performance and fitting curves of six MCSs (star: simulation; solid line: fitting curve).

If we employ LDPC codes in the cross-layer design mentioned in Section IV, the thresholds can be obtained from (9) as follows

$$\gamma_n = \frac{\ln\{(1/P_{\mathrm{target}})^{1/a_n}-1\}}{c_n} + b_n, \quad n = 1, 2, \ldots, N,$$

$$\gamma_{N+1} = +\infty . \tag{10}$$

According to the AMC rule, each MCS n will be chosen with the following probability [1]

$$p_n = \int_{\gamma_n}^{\gamma_{n+1}} p_{\hat{\gamma}}(\hat{\gamma})d\hat{\gamma} = \frac{\Gamma(K,\lambda\gamma_n)-\Gamma(K,\lambda\gamma_{n+1})}{\Gamma(K)} . \tag{11}$$

It can be shown that the average PER for MCS n is given by [1]

$$\overline{\mathrm{PER}}_n = \int_{\gamma_n}^{\gamma_{n+1}} PER_n(\hat{\gamma})p_{\hat{\gamma}}(\hat{\gamma})d\hat{\gamma} . \tag{12}$$

Then, the average spectral efficiency of the whole system can be computed in the same way as in [1].

V. NUMERICAL RESULLTS

In this section, numerical results showing the effects of different parameters on the spectral efficiency of our cross-layer design framework are provided. At the physical layer, the MCSs were chosen from Table I, where the modulation schemes and coding rates are adopted from the IEEE 802.11a standard [7]. Here, we use RC-LDPC codes instead of convolutional codes. Rate 3/4 and rate 9/16 LDPC codes were obtained from the rate 1/2 mother code through continuous puncturing we proposed in [8].

Assume that the performance constraint at the data link layer is PER_{link} = 0.01. Let us consider three values for the maximum numbers of retransmissions, i.e., N_r =0, 1, 2. We can get the value of P_{target} from (8). Then, the thresholds can be obtained from (10) and the results are shown in Table II. When $\hat{\gamma} < \gamma_1$, which means that the channel is in deep fading and no payload bits will be sent.

TABLE I.
PARAMETERS OF MCSS AT THE PHYSICAL LAYER.

	MCS1	MCS2	MCS3	MCS4	MCS5	MCS6
Mod.	BPSK	QPSK	QPSK	16QAM	16QAM	64QAM
Coding rate	1/2	1/2	3/4	9/16	3/4	3/4
R_n (b/s)	0.50	1.00	1.50	2.25	3.00	4.50
a_n	2.07	2.46	1.39	1.59	1.20	1.20
b_n	-1.94	1.18	4.31	7.24	10.3	15.5
c_n	3.92	3.02	2.90	3.42	3.05	2.60
γ_{cf} (dB)	-3.30	-0.63	2.61	5.77	8.76	13.71

TABLE II.
THRESHOLDS γ_n (DB) FOR N_R = 0, 1, 2.

N_r	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7
0	-1.408	1.7463	5.4325	8.0757	11.58	17.003	∞
1	-1.763	1.3282	4.8042	7.5924	10.908	16.22	∞
2	-1.921	1.1361	4.5489	7.39	10.645	15.912	∞

In Fig. 3, the average spectral efficiency of the AMC-ARQ system based on RC-LDPC codes is plotted as a function of the average SNR for different values of N_r varying from 0 to 2, assuming the perfect channel estimation. Curves in Fig. 3(b) denote the average spectral efficiency of the system equipped with two transmit antennas and two receive antennas. For comparison purposes, in Fig. 3(a) we have also plotted the performance curves for the SISO system without the STBC. The average spectral efficiency gain offered by a MIMO system over a SISO system can be remarkable. By comparing Figs. 3(a) and 3(b), we conclude that compared with the SISO scenario, the MIMO system employing the STBC with 2 transmit antennas and 2 receive antennas can provide at least additional 0.5 bits/symbol spectral efficiency gain for the same average SNR and an additional 4 dB diversity gain for the same spectral efficiency. For both SISO and MIMO scenarios, the spectral efficiency improves with the increasing N_r. The spectral efficiency gain of the AMC-ARQ system with only one retransmission (N_r = 1) exceeds that of the AMC-only system (N_r = 0) by about 0.15 bits/symbol. However, the improvement degrades quickly with the increasing N_r, which implies that the maximum number of retransmissions need not to be very large. A small number of retransmissions can achieve sufficient spectral efficiency gain.

Fig. 4 and Fig. 5 illustrate the average spectral efficiency of the system considering different channel estimation qualities. It is apparent that the largest average spectral efficiency can be achieved with the perfect channel estimation, i.e., $u = 1$. With the decrease of u, the average spectral efficiency is getting smaller. As we have mentioned previously, the performance of the whole system depends to a large extent on the accuracy of the channel estimation.

VI. CONCLUSIONS

In this paper, we applied RC-LDPC codes to the cross-layer design combining the AMC at the physical layer and the ARQ at the data link layer under MIMO fading channels using the STBC. The relevant MCS is chosen based on the SNR thresholds calculated according to the LDPC PER-SNR relationship. Furthermore, the impacts of the inaccurate channel estimation on the system spectral efficiency have also been investigated. Numerical results show that our AMC-ARQ system based on RC-LDPC codes can provide better spectral efficiency than the AMC-only system. More spectral efficiency gain can be obtained when longer LDPC codes are used.

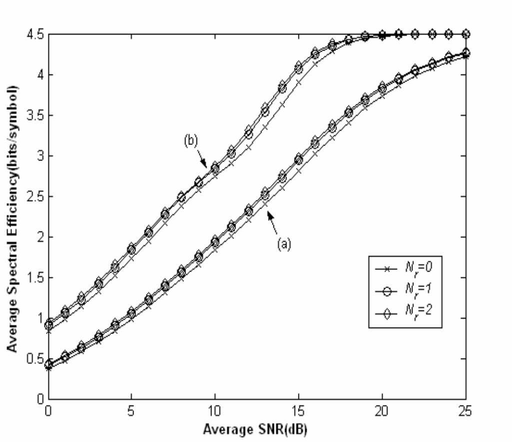

Fig. 3. Average spectral efficiency versus the average SNR for different retransmission numbers with the perfect channel estimation (a) $N_T = 1$, $N_R=1$ (b) $N_T = 2$, $N_R = 2$.

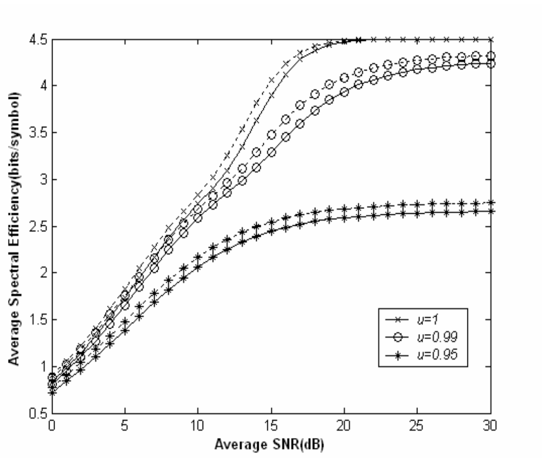

Fig. 4. Average spectral efficiency versus the average SNR for different u (dashed line: $Nr = 1$; solid line: $Nr = 0$; $N_T = 2$, $N_R = 2$).

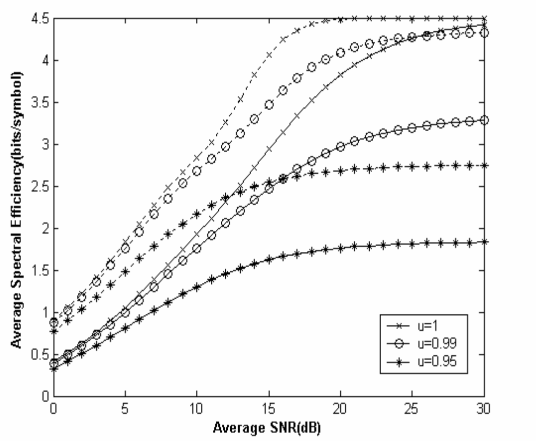

Fig. 5. Average spectral efficiency versus the average SNR for different u ($Nr = 1$, dashed line: $N_T = 2$, $N_R = 2$; solid line: $N_T = 1$, $N_R = 1$)

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