Comparison of Multilevel Coded Modulations with Different Decoding Methods for AWGN and Rayleigh Fading Channels

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***Abstract*-Multilevel coding (MLC) schemes based on channel capacity with multistage decoding (MSD) and parallel decoding on levels (PDL) are considered and compared. The channel models AWGN and Rayleigh fading are used in order to study the performance of MLC systems under different conditions. The investigation is done for 8ASK modulation and three set partitioning strategies. In each scheme BCH codes with different code lengths are used as component codes. Numerical results indicate that MSD is a sub-optimal decoding method of MLC for both channels, while PDL is most robust to varying channels if block partitioning (BP) is used. For Ungerboeck partitioning(UP) and Mixed partitioning(MP) strategy, MSD method is strongly recommended to use for MLC system, while for BP strategy, PDL is suggested to use as a simple decoding method compared with MSD.**

I. INTRODUCTION

A sub-optimal decoding technique called multi-stage decoding (MSD) was introduced in [2] for the decoding of multi-level codes. This decoding procedure is done stage by stage and is accomplished by decoding the component codes one at a time. The reliability of MLC system can be improved greatly by using MSD method in which each component code is decoded individually starting from the lowest level and using decisions of previous decoding stages. Because of the advantage of MSD, many publications have concentrated on it[3-8].

Another decoding method for MLC proposed by P. Schramm in 1997 is parallel decoding on levels (PDL)[10]. The complexity and time delay of this decoding method is lower than MSD and it has robustness to different channels[10].

In this paper, we are focusing on the comparison of two decoding methods for MLC. Based on the calculation for capacities of equivalent channels[1], the performance of MLC/MSD and MLC/PDL schemes with three set partitioning strategies in AWGN and Rayleigh fading channels is investigated, in which BCH codes with different code lengths are chosen as component codes, and 8ASK signal constellation is used.

II. EQUIVALENT CHANNELS AND CAPACITY RULE

Fig.1 shows the structure of multilevel coding system.

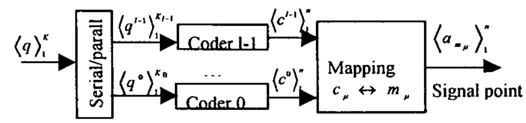

Fig.1 Multilevel coding scheme

Since the mapping M is bijective and hence lossless in the sense of information theory, the mutual information I(Y;A) between the transmitted signal point a∈A and the received signal y∈Y equals the mutual information $I(Y;X^0,X^1,...X^{l-1})$ between the address vector $X \in \{0,1\}^l$ and the received signal point:

$$I(Y;A) = I(Y;X^0,X^1,...X^{l-1}) \tag{1}$$

The random variables corresponding to the transmitted and received signal point, to the binary address vector, and to its components are denoted by capital letters.

Applying the chain rule to the mutual information yields[11]

$$I(Y;X^0,X^1,...X^{l-1}) = I(Y;X^0) + I(Y;X^1 \mid X^0) + ... + I(Y;X^{l-1} \mid X^0X^1...X^{l-2}) \tag{2}$$

This equation may be interpreted in the following way: the transmission of vectors with binary digits $x^i, i=0,1,...l-1$, over the physical channel can be virtually separated into the parallel transmission of the digits x^i over ℓ equivalent channels. The equivalent channel i consists of the equivalent mapper i, provided that the digits $x^0...x^{i-1}$ are known, and the noisy channel. The binary symbol x^i is multiply represented in the signal set of the equivalent mapper i for $i < \ell - 2$.

The capacities of MLC/MSD scheme and of its equivalent channels are proposed and derived by [12, 13] which directly leads to the capacity rule. Given a 2 -ary digital modulation scheme, choose the rate R_i at the individual coding level i of a MLC scheme equal to the capacity C_i of the equivalent channel i :

$$R^i = C^i \qquad i = 0,1,..., l-1 \tag{3}$$

The basis of the capacity rule is to characterize the transmission properties of the equivalent channels by its capacities. Operating at the capacity limit of MLC scheme, the capacity rule provides the maximum individual rates to be transmitted with arbitrarily low error probability. Thus, the design of MLC scheme with an optimum trade-off between power and bandwidth efficiency has to be based on the

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capacity rule.

The capacities of the equivalent channels can be calculated very efficiently by using the following form of the chain rule for mutual information:

$$I(Y;X^i \dots X^{l-1}/X^0 \dots X^{i-1}) = I(Y;X^i/X^0 \dots X^{i-1}) + I(Y;X^{i+1} \dots X^{l-1}/X^0 \dots X^i) \quad (4)$$

The capacity C_i for given a-priori probabilities of signal points yields:

$$C^i = I(Y;X^i/X^0 \dots X^{i-1}) = I(Y;X^i \dots X^{l-1}/X^0 \dots X^{i-1}) - I(Y;X^{i+1} \dots X^{l-1}/X^0 \dots X^i) \quad (5)$$

The mutual information $I(Y;X^i \dots X^{l-1}/X^0 \dots X^{i-1})$ is calculated by averaging with respect to all possible combinations of $x^0, \dots x^{i-1}$:

$$I(Y;X^i \dots X^{l-1}/X^0 \dots X^{i-1}) = E_{x^0 \dots x^{i-1}}\{I(Y;X^i \dots X^{l-1}/X^0 \dots X^{i-1})\} \quad (6)$$

Thus, Ci is given by:

$$\begin{cases} C^i = E_{x^0 \dots x^{i-1}}\{C(A(x^0 \dots x^{i-1}))\} - E_{x^0 \dots x^i}\{C(A(x^0 \dots x^i))\}, & i = 1, \dots, l-1 \\ C^0 = C(A) - E_{x^0}\{C(A(x^0))\} & i = 0 \end{cases} \quad (7)$$

III. Different Rates Of Three Mapping Strategies

There are three kinds of mapping(set partitioning) strategies for the signal constellations. Traditional Ungerboeck partitioning (UP) is aimed at maximizing the intra subset minimum Euclidean distance. As a different strategy, we call block partitioning (BP). This scheme maintains the intra subset minimum Euclidean distance. Last strategy is a kind of combination of UP and BP strategy called mixed partitioning (MP).

BP [12] is shown in Fig.2. Just because it goes absolutely inverse direction comparing with UP and UP has been proved not to be a good and efficient method to Rayleigh fading channels, BP will be worthily considered as an efficient set partitioning strategy and a better criteria in designing an optimal MLC system for Rayleigh fading channels. This assumption has been proved by our calculations of capacities for different set partitioning strategies in both of Rayleigh and AWGN channels[1].

MP results from a combination. In this letter, it is defined in this way: BP-UP-UP which means the first partitioning step is done by the rule of BP and follow by UP and UP.

From the results of capacity for three level MLC scheme, the rate design values of MLC/MSD with three set partitioning strategies and 8ASK modulation over AWGN and Rayleigh fading channels when total rate is 2bits/symbol or 2.5bits/symbol are obtained and listed in Table 1 and Table 2, respectively..

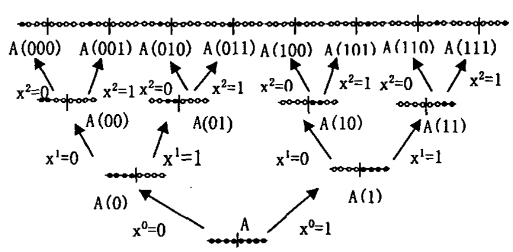


Fig.2 BP of 8ASK signal set

IV. Comparison Of Decoding Methods For Multilevel Coding

For both of AWGN and Rayleigh fading channels, optimal decoding of multilevel codes can be performed by a maximum-likelihood(ML) decoder that finds the best input sequence that maximizes the probability of receiving the observed sequence. But this decoder has to work with a huge complexity since the number of states becomes quite large. In this case the ratio between performance and decoding complexity is poor even for very simple codes in each level. Thus, good sub-optimal decoding techniques are needed to obtain the good trade-off between performance and complexity.

A. Multistage Decoding for Multilevel Codes(MLC/MSD)

The right side of the chain rule (2) suggests the rule for a low complex staged decoding procedure that is well-known as MSD[2]. The component codes C_i are successively

TABLE 1

Different Rates Of MLC/MSD Scheme With 8ASK Modulation Based On Capacity Rule (R=2.5bits/symbol)

	AWGN Channel	Rayleigh Fading Channel
UP	$C_0=R_0=0.5$ $C_1=R_1=1$ $C_2=R_2=1$	$C_0=R_0=0.59$ $C_1=R_1=0.91$ $C_2=R_2=1$
BP	$C_0=R_0=0.95$ $C_1=R_1=0.85$ $C_2=R_2=0.7$	$C_0=R_0=0.9$ $C_1=R_1=0.85$ $C_2=R_2=0.75$
MP	$C_0=R_0=0.875$ $C_1=R_1=0.625$ $C_2=R_2=1$	$C_0=R_0=0.925$ $C_1=R_1=0.65$ $C_2=R_2=0.925$

TABLE 2

Different Rates Of MLC/MSD Scheme With 8ASK Modulation Based On Capacity Rule (R=2bits/symbol)

	AWGN Channel	Rayleigh Fading Channel
UP	C0=R0=0.1818 C1=R1=0.8182 C2=R2=1	C0=R0=0.3125 C1=R1=0.75 C2=R2=0.9375
BP	C0=R0=0.85 C1=R1=0.7 C2=R2=0.45	C0=R0=0.8125 C1=R1=0.6875 C2=R2=0.5
MP	C0=R0=0.85 C1=R1=0.25 C2=R2=0.9	C0=R0=0.8475 C1=R1=0.35 C2=R2=0.8025

decoded by the corresponding decoders D_i, see Fig.3. At stage i, decoder D_i processes not only the block $y=(y[1],\ldots,y[N])$, $y[\mu]\in Y$, of received signal points, but also decisions $\hat{x}^j$, j=0, …,i-1, of previous decoding stages j. The use of previous decoding decisions accomplishes the selection of the current subsets of the equivalent mapper i for the different time instants $\mu=1,\ldots,N$.

Actually, the staged decoding according to the chain rule in (2) would require the transmitted symbol x^j instead of the estimate $\hat{x}^j$. But if we assume error-free decisions $\hat{x}^j=x^j$ of decoder D_j, MSD can be interpreted as an implementation of the chain rule. Clearly, in practice, errorneous decisions occur and errors propagate from low levels to higher ones. But it is shown later that error propagation in MSD does not significantly influence the performance of the total scheme.

Obviously, multistage decoding is not identical to ML decoding, although each level can be ML decoded. Therefore, we will loose performance compared to the super-decoder. However, the decoding complexity is significantly reduced because now the complexity is the sum of decoding complexity of each level instead of the product. Of course, additional delay is imposed on the decoding process, because the single decoders cannot work in parallel.

B. Parallel Decoding for Multilevel Codes(MLC/PDL)

The use of estimates on lower levels may be unsuitable in practice, e.g. due to memory requirements. In this case, the codes on the levels could be decoded independently, i.e. without feedback of estimates. Therefore, an alternative decoding strategy for multilevel coded transmission is parallel decoding of the individual levels(PDL)[9]. Thereby, in contrast to the MSD approach, decoder D_i makes no use of decisions of other levels $i\neq j$. All decoders D_i, i=0,1, …,l-1, are working in parallel. The PDL approach is sketched in Fig.4. For MLC/PDL the transmission of each address symbol x^i, i=0,1,…,l-1, over the equivalent channel i is based on the entire signal constellation, since there is no preselection of signal points at higher levels due to decoding decisions of other levels. Of course, information is lost by not using estimates from lower levels. Thus, the sum of the capacities C_{sum} of all levels is less than(or equal to) the total capacity of the signal set, i.e.

$$C_{sum} = \sum_i C_i \le C_{set} \tag{8}$$

To be more accurate, the concept of the equivalent channel

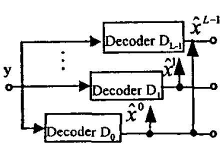


Fig.3 Multistage decoder

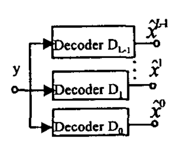


Fig.4 Parallel decoding of levels

and its characterizing pdf has to be adopted appropriately for an MLC/PDL scheme. While in the case of MLC/MSD the signal set of the equivalent mapper i is time variant for i>0 depending on the binary digits x^j of lower levels j, j=0, …,i-1, the equivalent mapper i for the MLC/PDL scheme is time invariant for all i=0, …,l-1. Since the decoding at one level is done independently of other levels, the equivalent mappers for MLC/PDL comprise the entire signal set **A** in every case. In the signal set of equivalent mapper i the binary symbol b^i is multiply represented by all signal points with address digit $x^i=b^i$, $b^i\in\{0,1\}$.

An advantage of the PDL decoding approach is certainly that error propagation from low to higher levels can be avoided since the levels are decoded independently. Additionally, PDL is favorable in terms of decoding delay since the individual decoders are working in parallel instead of serial in the staged decoding approach MSD. If transmission over time-variant channels is considered, where both the static AWGN and the Rayleigh fading channel are present, MLC/PDL turned out to provide the best robustness to both channel situation among other competing coded modulation schemes[9].

V. RESULTS AND DISCUSSIONS

According to the discussion of channel capacity in the previous section and the found code rates in Table 1 and Table 2, the performance comparison of MLC/MSD and MLC/PDL over AWGN and Rayleigh fading channels was performed by means of simulations. The presented results are bit error rates(BERs or Pb) as a function of E_b/N_0, where E_b denotes average energy per information bit.

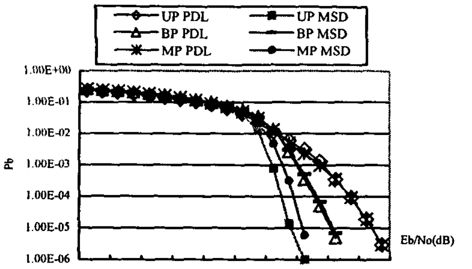
In order to allow for a fair comparison, some parameters of the schemes are identical for all cases: the modulation scheme is 8ASK, BCH codes with different code lengths and different code rates are chosen as component codes on three levels.

A. Comparison of MLC/MSD and MLC/PDL for AWGN Channels

Simulation results of MLC scheme using two different decoding methods with UP, BP or MP strategy over AWGN channels are shown in Fig.5. In each scheme code rates of component codes are designed according to "capacity rule" shown in Table 1. For performance comparison, the total rates of scheme are all chosen as: 2.5bits/symbol. From the results, we can see:

(a) As shown in Fig.5, MSD is superior to PDL for MLC scheme with any the same set partitioning strategy over AWGN channels.

(b) For UP and MP strategies, the performance of MLC



UP: C0(127,57,11); C1(127,127); C2(127,127)
BP: C0(127,120,1); C1(127,106,3); C2(127,85,6)
MP: C0(127,106,3); C1(127,78,7); C2(127,127,0)

Fig.5 Comparison of MSD and PDL with 8ASK over AWGN channel

/PDL and MLC/MSD is nearly the same at lower signal-to-noise ratio(SNR) when E_b/No is lower than 12dB; With the increase of SNR (E_b/No>12dB), the performance difference will be larger. For UP and MP mapping strategy, the power efficiency of MLC/MSD scheme is higher than MLC/PDL with 3~4dB coding gain for UP strategy, while with 2~3dB coding gain for MP strategy when Pb=10^{-3}.

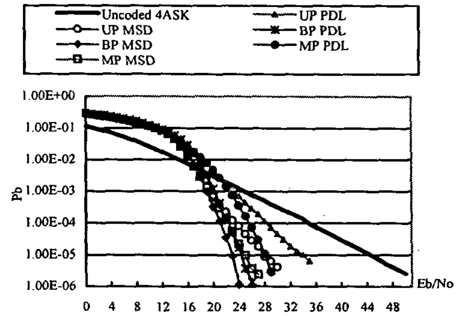
(c) For BP mapping strategy, the performance of MLC/PDL and MLC/MSD schemes is nearly the same at any SNR.

B. Comparison of MLC/MSD and MLC/PDL for Rayleigh Fading Channels

Another set of simulations was performed to provide a comparison of MLC/MSD and MLC/PDL for Rayleigh fading channels. For any set partitioning strategy, code rates of component codes are designed based on "capacity rule" listed in Table 2. Numerical results are depicted in Fig.6 and Fig.7. Fig.6 shows the results for BCH codes with code lengths of 127, while Fig.7 is the results for 255. The total rates of all schemes are all R=2 bits/symbol. For comparison, the performance of uncoded 4ASK modulation scheme is also given. From the simulation results, we can see:

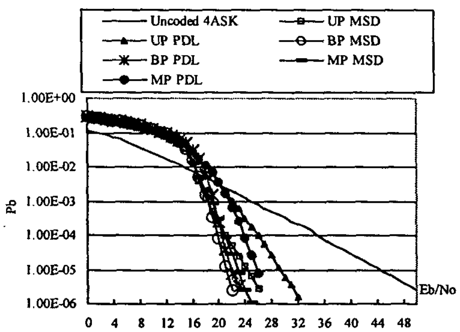
(a) For any the same set partitioning strategy, MSD is superior to PDL for MLC scheme over Rayleigh fading channels. Therefore, the MLC/MSD scheme is shown to be an asymptotically optimum approach to coded modulation in terms of channel capacity for both AWGN and Rayleigh fading channels. The condition for this optimality is that the rates of the component codes are chosen to be equal to the capacities of the equivalent channels which are provided by the individual levels.

(b) For each the same mapping strategy with different code length, we can see that the performance of MLC/MSD and MLC/PDL schemes will be better using component codes with longer code lengths when code rates are designed



UP: C0(127,36,15);C1(127,99,4); C2(127,120,1)
BP: C0(127,99,4); C1(127,92,5); C2(127,64,10)
MP:C0(127,106,3);C1(127,43,14);C2(127,106,3)

Fig.6 Comparison of MSD and PDL with 8ASK over Rayleigh channel



UP:C0(255,79,27);C1(255,191,8);C2(255,239,2)
BP:C0(255,207,6);C1(255,171,11);C2(255,131,18)
MP:C0(255,215,5);C1(255,87,26);C2(255,207,6)

Fig.7 Comparison of MLC/MSD and MLC/PDL with 8ASK, over Rayleigh channel

according to "capacity rule". Compared with uncoded 4ASK scheme, they all have high coding gains.

(c) As shown in Fig.6 and Fig.7, for UP and MP strategies, the power efficiency of MLC/MSD and MLC/PDL is nearly the same at lower SNR when E_b/N_0 is lower than 16dB. At high SNR, the performance difference will be larger. When Pb=10^{-3}, MLC/MSD scheme is superior to MLC/PDL with 3~5dB for UP strategy using BCH codes with code lengths of 127, as shown in Fig.6, while with 2~4dB for MP strategy using BCH codes with code lengths of 255, as shown in Fig.7.

(d) As Fig.6 and Fig.7 show that PDL and MSD lead to approximately the same performance for MLC scheme as long as BP strategy is used. Therefore, PDL, the simple and pragmatic decoding method, can be used instead of MSD, the complex and iterative decoding method with long time delay, for MLC system over Rayleigh fading channels when BP strategy is employed. This conclusion has great importance and significance for multilevel coder with more levels, e.g.

coded 64QAM MLC scheme with six levels, because the complexity and time delay of MSD will increase greatly.

(e) Comparing Fig.5 and Fig.6, for BP strategy, we can see that the robustness to channel variations is the key feature of PDL which makes this decoding method attractive for mobile fading channels with both of AWGN and Rayleigh fading noise together.

VI. CONCLUSIONS

From simulation results and discussions, some conclusions can be got:

(a) For any set partitioning strategy, MLC/MSD scheme is superior to MLC/PDL over AWGN and Rayleigh fading channels. In each scheme the code rates of component codes with different code lengths are all designed based on "capacity rule". Therefore, MSD is the sub-optimal decoding method for multilevel coding system.

(b)As long as BP strategy is used, the performance of MLC/PDL is nearly the same with that of MLC/MSD scheme for both AWGN and Rayleigh fading channels. Therefore, for mobile fading channels with AWGN and Rayleigh fading noise together, PDL can be used as a more attractive decoding emthod instead of MSD for MLC systems. This conclusion has great significance for designing the MLC scheme with higher bandwidth efficiency, e.g. there is more than three levels in MLC system.

(c)The performance of MLC scheme with different decoding methods is related to set partitioning strategies. For UP and MP strategy, MSD method is strongly recommended to use because the performance of MLC/MSD scheme is much better than that of MLC/PDL. For BP strategy, PDL is suggested to use as a simple decoding method because the performance of MLC scheme with two decoding methods is nearly the same.

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