Stochastic Modeling and Simulation of Frequency Hopping Wideband Fading Channels

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***Abstract* - A channel simulator that models accurately the physical channel statistics determined by the frequency separation between two carriers in frequency hopping (FH) systems is called a FH channel simulator. In this paper, a novel stochastic FH simulation model based on Rice's sum of sinusoids is proposed for wideband mobile fading channels. It is shown that a frequency hop in the physical channel model corresponds to phase hops in the simulation model, while maintaining all other parameters unchanged. Furthermore, this simulator is capable of simulating both fast FH and slow FH fading channels by only adjusting the phase hop rate. Due to the fact that closed-form expressions are given for all model parameters, we can study the correlation properties of the proposed simulation model from both the analytical and simulation point of view. A pretty good agreement between the correlation properties of the simulation model and those of the underlying reference model has been observed in all cases.**

I. Introduction

In wireless communication systems, frequency hopping (FH) is a well-known technique to improve the system quality and spectral efficiency by taking advantage of frequency diversity and interference diversity. A channel simulator that models accurately the physical channel statistics determined by the frequency separation between two carriers in FH systems is called a FH channel simulator. Such a simulator is important for the design, performance evaluation, optimization, and test of modern FH wireless communication systems.

Typically, researchers simulating FH systems assume that the separation between the carrier frequencies is sufficiently high with respect to the coherence bandwidth of the radio channel [1, 2]. This assumption implies that the consecutive hopping channels are uncorrelated [3] and, therefore, a FH channel simulator can be implemented by using uncorrelated fading processes. However, in practical FH wireless communication networks, the channel separation is often smaller than the coherence bandwidth due to spectrum and frequency planning constraints [4, 5]. In this case, a FH channel simulator needs to take the frequency correlation of the channel into account in order to investigate the influence of the frequency correlation on the performance of realistic FH systems. In the literature, only few papers have contributed to this topic. In [6], a frequency domain FH channel simulator has been developed by using frequency transform techniques and digital filter design methods. A method of modeling frequency correlation of fast fading which can be used to emulate FH wideband channels has been presented in [7]. Based on the same reference model as described in [3], a stochastic [8] and two deterministic [9, 10] FH channel simulators have been developed for Rayleigh fading channels. These FH simulation models [8–10] are all designed according to Rice's sum of sinusoids [11, 12] modeling approach.

In this paper, we extend the applicability and the facility of the stochastic modeling procedure in [8] with two contributions. First, the narrowband reference model [3] is generalized and extended to a wideband channel model. By removing the frequency-selectivity from the proposed reference model, it can be shown that the derived correlation functions can be reduced to the results presented in [3, p. 50] for narrowband channels. Second, we propose a stochastic FH wideband channel simulator which can reproduce accurately all of the correlation properties of the derived reference model.

II. Derivation of the Correlation Functions of the Theoretical Reference Model

The derivation of the correlation functions for the reference model of FH narrowband Rayleigh fading channels is performed in [3]. There, it has been assumed that the omnidirectional receive antenna is deployed, the angles of arrival are uniformly distributed, and the echo delays τ' are negative exponentially distributed according to

$$p(\tau') = \frac{1}{\alpha}\exp(-\frac{\tau'}{\alpha}), \quad \tau' \geq 0 \tag{1}$$

where α represents the rms delay spread. An example of α for a narrowband channel is $\alpha = 0.1086\ \mu$s under the rural area (RA) environment [13] for GSM. The aim of this section is to incorporate frequency-selectivity into the reference model by assuming that the echo delays τ' are still negative exponentially distributed according to (1), but the value of α is now in the order of the symbol duration. For example, $\alpha = 1\ \mu s$ under the typical urban (TU) environment [13], which implicates a wideband channel for GSM.

Bello's wide-sense stationary uncorrelated scattering (WSSUS) model has widely been accepted as an appropriate stochastic model for wideband multipath fading channels [14]. Regarding the design of hardware or software simulation models for wideband channels, a discretization of the echo delays has to be performed and adapted to the sampling interval. A suitable and often used model is the so-called discrete $\mathcal{L}$-path tapped-delay-line model [15]. It is important to mention that the tapped-delay-line model is a WSSUS model as long as the time-variant tap gains are uncorrelated Gaussian random processes. For this model, the complex baseband impulse responses at two different carrier frequencies f_c and f_c^h can be expressed as

$$h(\tau',t) := \sum_{\ell=0}^{\mathcal{L}-1} a_\ell\, \mu_\ell(t)\, \delta(\tau'-\tau'_\ell)\ , \text{ at } f_c \quad (2a)$$

$$h^h(\tau',t) := \sum_{\ell=0}^{\mathcal{L}-1} a_\ell\, \mu_\ell^h(t)\, \delta(\tau'-\tau'_\ell)\ , \text{ at } f_c^h \quad (2b)$$

where the superscript h indicates a frequency hop, τ'_ℓ is the discrete echo delay of the ℓth propagation path, and a_ℓ is called the delay coefficient, which is a measure of the square root of the average delay power assigned to the ℓth propagation path. In (2), the stochastic processes

$$\mu_\ell(t) = \mu_{1,\ell}(t) + j\mu_{2,\ell}(t) \quad (3a)$$

$$\mu_\ell^h(t) = \mu_{1,\ell}^h(t) + j\mu_{2,\ell}^h(t) \quad (3b)$$

are zero-mean complex Gaussian processes, where $\mu_{i,\ell}(t)$ and $\mu_{i,\ell}^h(t)$ ($i = 1,2$ and $\ell = 0,1,\ldots,\mathcal{L}-1$) are both real Gaussian noise processes.

The correlation properties of $\mu_\ell(t)$ and $\mu_\lambda^h(t)$ ($\ell,\lambda = 0,1,\ldots,\mathcal{L}-1$) are completely determined by the correlation properties of the underlying real Gaussian noise processes $\mu_{i,\ell}(t)$ and $\mu_{j,\lambda}^h(t)$ ($i,j = 1,2$). Therefore, we can restrict our investigations to the following autocorrelation functions (ACFs) and cross-correlation functions (CCFs)

$$r_{\mu_{i,\ell}\mu_{j,\lambda}}(\tau) := E\{\mu_{i,\ell}(t)\mu_{j,\lambda}(t+\tau)\} \quad (4a)$$

$$r_{\mu_{i,\ell}\mu_{j,\lambda}^h}(\tau,\chi) := E\{\mu_{i,\ell}(t)\mu_{j,\lambda}^h(t+\tau)\} \quad (4b)$$

where the operator $E\{\cdot\}$ refers to statistical average, τ denotes time separation, and $\chi = f_c^h - f_c$ represents frequency separation. For the discrete $\mathcal{L}$-path tapped-delay-line model, each tap is considered as the combination of a large number of multipath components. Therefore, the echo delay interval $I = [0, \tau'_{max}]$, where τ'_{max} denotes the maximum echo delay, is required to be partitioned into $\mathcal{L}$ mutually disjoint subintervals I_ℓ, i.e., $I = \cup_{\ell=0}^{\mathcal{L}-1} I_\ell$ and $I_\ell \cap I_\lambda = \{\emptyset\}$ for every ℓ and $\lambda \neq \ell$ ($\ell,\lambda = 0,1,\ldots,\mathcal{L}-1$). Each subinterval I_ℓ corresponds to the range within which a number of multipath components are combined to the ℓth discrete propagation path. For simplicity, we set $I_\ell = [\tau'_\ell - \Delta\tau_\ell/2, \tau'_\ell + \Delta\tau_{\ell+1}/2)$ for $\ell = 0,1,\ldots,\mathcal{L}-2$ and $I_{\mathcal{L}-1} = [\tau'_{\mathcal{L}-1} - \Delta\tau_{\mathcal{L}-1}/2, \tau'_{\mathcal{L}-1} + \Delta\tau_{\mathcal{L}}/2]$, where $\Delta\tau_0 = \tau'_0 = 0$, $\Delta\tau_\ell = \tau'_\ell - \tau'_{\ell-1}$ ($\ell = 1,2,\ldots,\mathcal{L}-1$), and $\Delta\tau_{\mathcal{L}} = 2(\tau'_{max} - \tau'_{\ell-1})$. In order to find the solution of the correlation functions for narrowband channels, one has to take the statistical average over the echo delays τ'. This requires the integral over τ' with the lower limit 0 and the upper limit ∞ [3]. Now, to solve analytically (4a) and (4b) for wideband channels, the lower limit and the upper limit for the integral over τ' have to be substituted by $\tau'_\ell - \Delta\tau_\ell/2$ and $\tau'_\ell + \Delta\tau_{\ell+1}/2$, respectively. Hence, τ' will range from $\tau'_\ell - \Delta\tau_\ell/2$ to $\tau'_\ell + \Delta\tau_{\ell+1}/2$ for the ℓth discrete propagation path. Then, the following correlation functions of the wideband reference model can be obtained

$$r_{\mu_{i,\ell}\mu_{i,\ell}}(\tau) = r_{\mu_{i,\ell}^h\mu_{i,\ell}^h}(\tau) = \sigma_0^2(A-B)J_0(2\pi f_{max}\tau) \quad (5a)$$

$$r_{\mu_{1,\ell}\mu_{2,\ell}}(\tau) = r_{\mu_{1,\ell}^h\mu_{2,\ell}^h}(\tau) = r_{\mu_{2,\ell}\mu_{1,\ell}}(\tau) = r_{\mu_{2,\ell}^h\mu_{1,\ell}^h}(\tau) = 0 \quad (5b)$$

$$r_{\mu_{i,\ell}\mu_{j,\lambda}}(\tau) = r_{\mu_{i,\ell}^h\mu_{j,\lambda}^h}(\tau) = 0 \quad (5c)$$

$$r_{\mu_{i,\ell}\mu_{i,\ell}^h}(\tau,\chi) = \frac{\sigma_0^2 J_0(2\pi f_{max}\tau)}{1+(2\pi\alpha\chi)^2}\exp(-\frac{\tau'}{\alpha})\,[2\pi\alpha\chi\sin(2\pi\chi\tau') - \cos(2\pi\chi\tau')]\,\Big|_{\tau'=\tau'_\ell-\Delta\tau_\ell/2}^{\tau'=\tau'_\ell+\Delta\tau_{\ell+1}/2} \quad (5d)$$

$$r_{\mu_{1,\ell}\mu_{2,\ell}^h}(\tau,\chi) = -r_{\mu_{2,\ell}\mu_{1,\ell}^h}(\tau,\chi) = \frac{\sigma_0^2 J_0(2\pi f_{max}\tau)}{1+(2\pi\alpha\chi)^2}\exp(-\frac{\tau'}{\alpha})\,[\sin(2\pi\chi\tau') + 2\pi\alpha\chi\cos(2\pi\chi\tau')]\,\Big|_{\tau'=\tau'_\ell-\Delta\tau_\ell/2}^{\tau'=\tau'_\ell+\Delta\tau_{\ell+1}/2} \quad (5e)$$

$$r_{\mu_{i,\ell}\mu_{j,\lambda}^h}(\tau,\chi) = r_{\mu_{i,\ell}^h\mu_{j,\lambda}}(\tau,\chi) = 0 \quad (5f)$$

for $i,j = 1,2$ and $\ell,\lambda = 0,1,\ldots,\mathcal{L}-1$ with $\ell \neq \lambda$, where σ_0^2 is related to the total received power, $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind, f_{max} represents the maximum Doppler frequency and

$$A = \exp(-\frac{\tau'_\ell - \Delta\tau_\ell/2}{\alpha}) \quad (6a)$$

$$B = \exp(-\frac{\tau'_\ell + \Delta\tau_{\ell+1}/2}{\alpha})\,. \quad (6b)$$

In (5d) and (5e), $f(x)|_{x=a}^{x=b}$ is an abbreviation for $f(b) - f(a)$. The expressions (5c) and (5f) state that there are no cross-correlations between different propagation paths due to the uncorrelated scattering (US) condition imposed on the model. The uncorrelatedness between the inphase and quadrature components is shown in (5b). Also, the expressions (5a), (5d), and (5e) allow further observations: when $\tau'_\ell - \Delta\tau_\ell/2 \to 0$ and $\tau'_\ell + \Delta\tau_{\ell+1}/2 \to \infty$, they will tend to the results for narrowband channels already presented in [3, p. 50], as well as in [8–10]. The above equations (5a)–(5f) provide the basis for the performance evaluation of the simulation model in Section IV.

III. Stochastic FH Simulation Model

After successfully deriving the correlation functions of a reference model for FH wideband channels, we are now confronted with the task to find a proper simulation model that can approximate as close as possible the desired correlation properties. The stochastic simulation model proposed here has a similar structure as that of the reference model, but the manner how the complex random processes will be modeled is completely different. The impulse responses of the simulation model are again composed of $\mathcal{L}$ discrete propagation paths according to

$$\hat{h}(\tau', t) := \sum_{\ell=0}^{\mathcal{L}-1} a_\ell \, \hat{\mu}_\ell(t) \, \delta(\tau' - \tau'_\ell) \ , \text{ at } f_c \tag{7a}$$

$$\hat{h}^h(\tau', t) := \sum_{\ell=0}^{\mathcal{L}-1} a_\ell \, \hat{\mu}^h_\ell(t) \, \delta(\tau' - \tau'_\ell) \ , \text{ at } f^h_c \tag{7b}$$

where the quantities a_ℓ and τ'_ℓ are identical to those of the reference model. In (7a) and (7b), the stochastic processes $\hat{\mu}_\ell(t)$ and $\hat{\mu}^h_\ell(t)$ are modeled by a finite sum of properly weighted exponential functions .

$$\hat{\mu}_\ell(t) = \hat{\mu}_{1,\ell}(t) + j\hat{\mu}_{2,\ell}(t) = \sum_{n=-N_\ell+1}^{N_\ell} \sum_{m=1}^{M_\ell} c_{n,m,\ell} \cdot \exp[j(2\pi f_{n,\ell} t - \theta_{m,\ell} - \hat{\theta}_{m,\ell})] \tag{8a}$$

$$\hat{\mu}^h_\ell(t) = \hat{\mu}^h_{1,\ell}(t) + j\hat{\mu}^h_{2,\ell}(t) = \sum_{n=-N_\ell+1}^{N_\ell} \sum_{m=1}^{M_\ell} c_{n,m,\ell} \cdot \exp[j(2\pi f_{n,\ell} t - \theta^h_{m,\ell} - \hat{\theta}_{m,\ell})] \tag{8b}$$

respectively, with

$$\theta_{m,\ell} = 2\pi f_c \varphi_{m,\ell} \ , \quad \theta^h_{m,\ell} = 2\pi (f_c + \chi) \varphi_{m,\ell} \tag{9a,b}$$

where $2N_\ell$-by-M_ℓ defines the number of exponential functions of the ℓth propagation path. In (8) and (9), the so-called Doppler coefficients $c_{n,m,\ell}$, discrete Doppler frequencies $f_{n,\ell}$ and the quantity $\varphi_{m,\ell}$ will be determined during the simulation setup phase and kept constant during the simulation run. The phases $\hat{\theta}_{m,\ell}$ are random variables uniformly distributed in the interval $(0, 2\pi]$. For each simulation run, a different set of phases $\hat{\theta}_{m,\ell}$ will be used. Consequently, one realization of the processes $\hat{\mu}_\ell(t)$ and $\hat{\mu}^h_\ell(t)$ will be obtained. However, the final intended result has to be calculated by taking the statistical average over many realizations of the processes. This is also the reason why the simulation model is of stochastic nature. The expressions (8) and (9) provide us with further observations: when a frequency hop $f_c \to f^h_c$ of size $\chi = f^h_c - f_c$ takes place in the physical channel model, we only have to perform phase hops $\theta_{m,\ell} \to \theta^h_{m,\ell}$ of sizes $2\pi\chi\varphi_{m,\ell} = \theta^h_{m,\ell} - \theta_{m,\ell}$ $(m = 1, 2, \ldots, M_\ell)$ in the simulation model, while maintaining all other parameters unchanged. Therefore, this simulator can easily realize the characteristics of FH.

Since the stochastic processes $\hat{\mu}_{i,\ell}(t)$ and $\hat{\mu}^h_{j,\lambda}(t)$ $(i, j = 1, 2$ and $\ell, \lambda = 0, 1, \ldots, \mathcal{L}-1)$ in the simulation model include both constant parameters and random parameters, their temporal correlation properties, the counter parts of (4a) and (4b), have to be calculated analytically by applying both time averages and statistical averages according to the following definitions

$$\hat{r}_{\mu_{i,\ell}\mu_{j,\lambda}}(\tau) := E_{\hat{\theta}_{m,\ell}}\{\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \hat{\mu}_{i,\ell}(t) \cdot \hat{\mu}_{j,\lambda}(t+\tau) dt\} \tag{10a}$$

$$\hat{r}_{\mu_{i,\ell}\mu^h_{j,\lambda}}(\tau, \chi) := E_{\hat{\theta}_{m,\ell}}\{\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \hat{\mu}_{i,\ell}(t) \cdot \hat{\mu}^h_{j,\lambda}(t+\tau) dt\} \ . \tag{10b}$$

Here, the operator $E_{\hat{\theta}_{m,\ell}}\{\cdot\}$ denotes that the statistical averages are taken with respect to the random phases $\hat{\theta}_{m,\ell}$. If we substitute (8) and (9) in (10), the following relations can be obtained

$$\hat{r}_{\mu_{i,\ell}\mu_{i,\ell}}(\tau) = \hat{r}_{\mu^h_{i,\ell}\mu^h_{i,\ell}}(\tau) = \sum_{n=-N_\ell+1}^{N_\ell} \sum_{m=1}^{M_\ell} \frac{c^2_{n,m,\ell}}{2} \cos(2\pi f_{n,\ell}\tau) \tag{11a}$$

$$\hat{r}_{\mu_{1,\ell}\mu_{2,\ell}}(\tau) = \hat{r}_{\mu^h_{1,\ell}\mu^h_{2,\ell}}(\tau) = \hat{r}_{\mu_{2,\ell}\mu_{1,\ell}}(\tau) = \hat{r}_{\mu^h_{2,\ell}\mu^h_{1,\ell}}(\tau) = \sum_{n=-N_\ell+1}^{N_\ell} \sum_{m=1}^{M_\ell} \frac{c^2_{n,m,\ell}}{2} \sin(2\pi f_{n,\ell}\tau) \tag{11b}$$

$$\hat{r}_{\mu_{i,\ell}\mu_{j,\lambda}}(\tau) = \hat{r}_{\mu^h_{i,\ell}\mu^h_{j,\lambda}}(\tau) = 0 \tag{11c}$$

$$\hat{r}_{\mu_{i,\ell}\mu^h_{i,\ell}}(\tau, \chi) = \sum_{n=-N_\ell+1}^{N_\ell} \sum_{m=1}^{M_\ell} \frac{c^2_{n,m,\ell}}{2} \cdot \cos(2\pi f_{n,\ell}\tau - 2\pi\varphi_{m,\ell}\chi) \tag{11d}$$

$$\hat{r}_{\mu_{1,\ell}\mu^h_{2,\ell}}(\tau, \chi) = -\hat{r}_{\mu_{2,\ell}\mu^h_{1,\ell}}(\tau, \chi) = \sum_{n=-N_\ell+1}^{N_\ell} \sum_{m=1}^{M_\ell} \frac{c^2_{n,m,\ell}}{2} \cdot \sin(2\pi f_{n,\ell}\tau - 2\pi\varphi_{m,\ell}\chi) \tag{11e}$$

$$\hat{r}_{\mu_{i,\ell}\mu^h_{j,\lambda}}(\tau, \chi) = \hat{r}_{\mu^h_{i,\ell}\mu_{j,\lambda}}(\tau, \chi) = 0 \tag{11f}$$

for $i, j = 1, 2$ and $\ell, \lambda = 0, 1, \ldots, \mathcal{L}-1$ with $\ell \neq \lambda$.

In the sequel, our aim is to find proper simulation model parameters in such a way that all of the correlation properties of the simulation model described by the expressions (11a)–(11f) will approximate as close as possible those of the given reference model described by the expressions (5a)–(5f). The Doppler coefficients $c_{n,m,\ell}$ and discrete Doppler frequencies $f_{n,\ell}$ are derived by using the method of exact Doppler spread [16], which results in the following closed-form expressions

$$c_{n,m,\ell} = \frac{\sigma_0}{\sqrt{N_\ell M_\ell}} \sqrt{A - B} \tag{12a}$$

$$f_{n,\ell} = f_{max} \sin\left[\frac{\pi}{2N_\ell}(n - \frac{1}{2})\right] \tag{12b}$$

for $n = -N_\ell + 1, -N_\ell + 2, \ldots, N_\ell$, $m = 1, 2, \ldots, M_\ell$, and $\ell = 0, 1, \ldots, \mathcal{L} - 1$. After substituting (12a) into (11a) and (11b), we realize that M_ℓ has no influence on $\hat{r}_{\mu_{i,\ell}\mu_{i,\ell}}(\tau)$ and $\hat{r}_{\mu_{1,\ell}\mu_{2,\ell}}(\tau)$. Equation (12b) states that the US condition, which requires $f_{n,\ell} \neq \pm f_{k,\lambda}$ for $\ell \neq \lambda$, can always be fulfilled in case that $N_\ell / N_\lambda \neq (2n-1)/(2k-1)$ holds, where $n = 1, 2, \ldots, N_\ell$ and $k = 1, 2, \ldots, N_\lambda$ [17]. Concerning the computation of $\varphi_{m,\ell}$ appearing in (11d) and (11e), a similar procedure to that proposed in the Appendix of [8] is applied and the following expression is obtained

$$\varphi_{m,\ell} = \alpha \ln \left(\frac{1}{A - \frac{m}{M_\ell}(A - B)} \right) . \qquad (13)$$

Note that $\varphi_{m,\ell}$ are located in the interval $(\tau'_\ell - \Delta\tau_\ell/2, \tau'_\ell + \Delta\tau_{\ell+1}/2]$, in which $\tau'_\ell - \Delta\tau_\ell/2$ and $\tau'_\ell + \Delta\tau_{\ell+1}/2$ correspond to $\varphi_{0,\ell}$ and $\varphi_{M_\ell,\ell}$, respectively. Therefore, $\varphi_{m,\ell}$ actually represent echo delays τ', since the echo delays τ' also range from $\tau'_\ell - \Delta\tau_\ell/2$ to $\tau'_\ell + \Delta\tau_{\ell+1}/2$ for the ℓth discrete propagation path. It is important to mention again that when $\tau'_\ell - \Delta\tau_\ell/2 \to 0$ and $\tau'_\ell + \Delta\tau_{\ell+1}/2 \to \infty$, all of the above FH wideband channel simulator parameters will be reduced to the results obtained for the narrowband channel simulator presented in [8].

It should be clarified at this point that this simulator is capable of simulating both slow FH (SFH) and fast FH (FFH) wireless channels. The difference between SFH and FFH is mainly that the hop rate is less than the symbol rate in SFH systems, while it is larger than or equal to the symbol rate in FFH systems [18]. If we assume that the transmitted symbol rate is fixed, then, whether it is a SFH or FFH scheme will rely on the hop rate. As mentioned above, a frequency hop in the physical channel model corresponds to phase hops in the simulation model. Consequently, how fast the phases variate will determine directly whether this simulator is employed for SFH or FFH schemes. If we substitute a given set of phases $\theta_{m,l}$ by another set of phases $\theta^h_{m,l}$ with a rate which is less than the symbol rate, then, this model is a SFH channel simulator; on the other hand, if we substitute $\theta_{m,l}$ by $\theta^h_{m,l}$ at least once per symbol duration, then, this is a FFH channel simulator.

IV. Performance Evaluation

In this section, the performance of the proposed FH wideband channel simulator will be investigated from both the analytical and simulation point of view. The corresponding correlation functions of the simulation model [see (11a)–(11f)] will be compared with those of the reference model [see (5a)–(5f)].

By substituting (12a) and (12b) in (11a), it can be shown that the ACF $\hat{r}_{\mu_{i,\ell}\mu_{i,\ell}}(\tau)$ of the simulation model approaches the desired one $r_{\mu_{i,\ell}\mu_{i,\ell}}(\tau)$ if the number of exponential functions tends to infinity, i.e., $\hat{r}_{\mu_{i,\ell}\mu_{i,\ell}}(\tau) \to r_{\mu_{i,\ell}\mu_{i,\ell}}(\tau)$ as $N_\ell \to \infty$. By analogy, the substitution of (12a), (12b), and (13) in (11d) and (11e) results for $N_\ell \to \infty$ and $M_\ell \to \infty$ in $\hat{r}_{\mu_{i,\ell}\mu^h_{i,\ell}}(\tau, \chi) \to r_{\mu_{i,\ell}\mu^h_{i,\ell}}(\tau, \chi)$ and $\hat{r}_{\mu_{1,\ell}\mu^h_{2,\ell}}(\tau, \chi) \to r_{\mu_{1,\ell}\mu^h_{2,\ell}}(\tau, \chi)$, respectively. For brevity of presentation, the proofs are not shown here.

The substitution of (12a) and (12b) in (11b) makes clear that $r_{\mu_{1,\ell}\mu_{2,\ell}}(\tau)$ and $\hat{r}_{\mu_{1,\ell}\mu_{2,\ell}}(\tau)$ are exactly the same for any given number of N_ℓ and M_ℓ, since M_ℓ has no influence on $\hat{r}_{\mu_{1,\ell}\mu_{2,\ell}}(\tau)$ and the $f_{n,\ell}$'s fulfill the following symmetry condition: $f_{-N_\ell+1,\ell} = -f_{N_\ell,\ell}, \ldots, f_{0,\ell} = -f_{1,\ell}$. Therefore, it follows that $\hat{r}_{\mu_{1,\ell}\mu_{2,\ell}}(\tau) = r_{\mu_{1,\ell}\mu_{2,\ell}}(\tau) = 0$ holds for all τ. By observing (11c) and (11f), it turns out that this simulation model has the same distinctive US feature as the reference model, as shown in (5c) and (5f).

In the next, the 6-path COST 207 TU channel model [13] with a rms delay spread of $\alpha = 1$ μs will be applied to investigate the degradation effects of the simulation model with a practical limited number of exponential functions. The parameters of the TU model are summarized in Table I. The discrete echo delays τ'_ℓ of the reference model and simulation model can then directly be equated with the values given in the table, while the corresponding delay coefficients a_ℓ have to be calculated in terms of the square root of the path power. For the proposed simulation model, $M_\ell = 20$ ($\ell = 0, 1, \ldots, 5$), $N_0 = 15$, $N_1 = 16$, $N_2 = 18$, $N_3 = 20$, $N_4 = 24$, and $N_5 = 32$ are selected as a good compromise between the model's precision and complexity. Fig. 1 impressively shows the excellent accordance between $r_{\mu_{i,\ell}\mu_{i,\ell}}(\tau)$ and $\hat{r}_{\mu_{i,\ell}\mu_{i,\ell}}(\tau)$ for $\ell = 0$. In order to check the validity of the analytical expressions, the simulation result is also illustrated in the figure. To demonstrate the excellent approximation quality of $r_{\mu_{i,\ell}\mu^h_{i,\ell}}(\tau, \chi) \approx \hat{r}_{\mu_{i,\ell}\mu^h_{i,\ell}}(\tau, \chi)$, the numerical results of (5d) and (11d) are plotted in Figs. 2(a) and 2(b), respectively. A good agreement between $r_{\mu_{1,\ell}\mu^h_{2,\ell}}(\tau, \chi)$ and $\hat{r}_{\mu_{1,\ell}\mu^h_{2,\ell}}(\tau, \chi)$ for the given number of exponential functions is also observed, which are not shown here.

TABLE 1. Specification of the 6-path COST 207 TU channel model

Path no. ℓ	Propagation delay (τ'_ℓ)	Path power (lin.)	(dB)
0	0.0 μs	0.5	-3
1	0.2 μs	1	0
2	0.5 μs	0.63	-2
3	1.6 μs	0.25	-6
4	2.3 μs	0.16	-8
5	5.0 μs	0.1	-10

V. Conclusion

A novel stochastic FH wideband fading channel simulator has been designed and all parameters have been determined in terms of closed-form expressions. It can be proved that all correlation functions of the simulation model will converge to those of the underlying reference model if the number of exponential functions tends to infinity. However, even a limited number of parameters give also excellent approximation results. The proposed channel simulator is capable of emulating both SFH and FFH schemes by only adjusting the phase hop rate. Therefore, this simulator is expected to play an important role in realistic FH system simulations.

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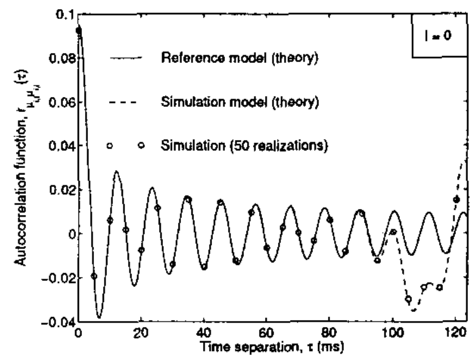
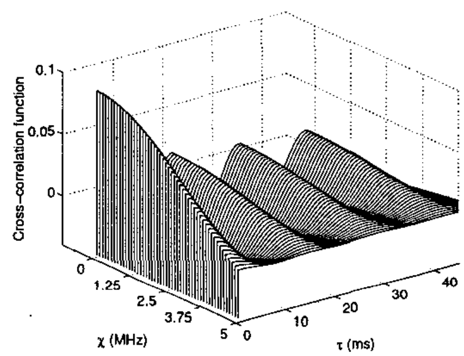


Fig. 1. ACF $r_{\mu_{i,\ell}\mu_{i,\ell}}(\tau)$ (reference model) in comparison with $\hat{r}_{\mu_{i,\ell}\mu_{i,\ell}}(\tau)$ (simulation model, $N_\ell = 15$, $M_\ell = 20$) for $\ell = 0$, $\sigma_0^2 = 1$, and $f_{max} = 91$ Hz.

(a) Reference model ($\ell = 0$)



(b) Simulation model ($\ell = 0$, $N_\ell = 15$, $M_\ell = 20$)

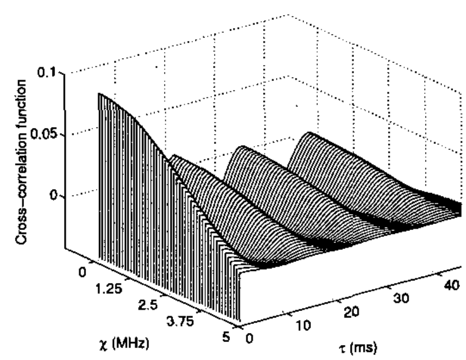


Fig. 2. CCFs for the COST 207 TU channel ($\sigma_0^2 = 1$, $f_{max} = 91$ Hz, and $\alpha = 1$ μs): (a) $r_{\mu_{i,\ell}\mu^h_{i,\ell}}(\tau, \chi)$ (reference model, cf. (5d)) and (b) $\hat{r}_{\mu_{i,\ell}\mu^h_{i,\ell}}(\tau, \chi)$ (simulation model, cf. (11d)).