A PAPR Reduction Method using Constraint Code in Multicode CDMA System

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Abstract - ***Multicode – Code Division Multiple Access (MC-CDMA) system is very good for the several kinds of service rates with various quality of service requirement because it can affordable by assigning multiple codes and controlling the flexible capacity. However, the so many multi-code signals are summed in the time domain so that it suffers from the serious problem of high peak to average power ratio (PAPR). Therefore, the expensive linear amplifier or large input back-off is necessary to overcome the problem but it makes power consumption of high power amplifier (HPA) very poor. In this paper, we propose a new method that can reduce PAPR efficiently by constraint codes based on the opposite correlation property of the incoming information data. PAPR reduction performance depends on the length and indices of constraint codes in the MC-CDMA system. There is a trade-off between PAPR reduction and the length of constraint codes. We also study the BER improvement in AWGN channel. The simulation results show that BER performance can be very close to the linear amplifier in two cases: 1) when the exact constraint codes are used without input back-off and 2) when a few constraint codes are used with small input back-off.***

Index Terms: Multi-code CDMA; PAPR; HPA and correlation

I. INTRODUCTION

The wireless communication system will offer significant multimedia and Internet related services. To serve for large number users with very different, and even time-varying, rate and quality of service (QoS), efficient multirate transmission techniques are necessary. Two types of multirate systems based on CDMA are proposed. The first system is multicode CDMA (MC-CDMA) and the second is variable spreading gain CDMA (VSG-CDMA). Furthermore, MC-CDMA is more convenient for adaptive resource allocation technique because the code of the user can be adjusted independently [1]. Multicode CDMA has been proposed in [2], which achieves variable transmission rates by assigning a data user multiple orthogonal Walsh codes.

Unfortunately, MC-CDMA has inherent drawback that output of WHT is not a binary signal. The transmitted signals typically result in multilevel amplitude variation with wide dynamic range. This situation of MC-CDMA is the same with OFDM system in which the high PAPR is a serious problem. Therefore, the system requires highly linear amplification at the transmitter, or a high power amplifier (HPA) becomes inefficient because of the large input back-off

Since PAPR problem in MC-CDMA is similar to that of OFDM, many techniques to reduce PAPR in OFDM are also applied for MC-CDMA. In [3], [4] precoding techniques to reduce the signal envelope variation was developed. But there were undesirable autocorrelation characteristics that can cause performance degradation in a multipath delay spread channel. PAPR reduction codes are systematically studied in [6], [7], and [8]. Along with PAPR reduction, error-correcting capability of codes for MC-CDMA was also derived. In [9], the authors have analyzed statistical characteristics of MC-CDMA and proposed two types of multiple signal representation technique, which are SLM and PTS to reduce PAPR in MC-CDMA. PTS and SLM can reduce PAPR efficiently, but the additional side information about the phase rotation is corrupted by noise or interference in channel. In [10] and [11], the impact of amplifier nonlinearity was investigated in various multicode systems. Some techniques for combating the nonlinearity like predistortion and reduction of parallel-transmitted sequences were also proposed. All above were the contributions to reduce PAPR of MC-CDMA efficiently.

In this paper, we propose a method to reduce PAPR in MC-CDMA by using constraint codes based on opposite correlation to the incoming information data. This technique has been applied to reduce PAPR of OFDM signal in [12] as the name of tone reservation. The proposed method can efficiently reduce PAPR in MC-CDMA and does not need side information. We also investigate the BER improvement of this method in AWGN channel when HPA is included. PAPR OF MC-CDMA

II. PAPR OF MC-CDMA

A. System description and PAPR definition

A simplified block diagram of the transmitter in MC-CDMA system is shown in Fig.1.

High-speed stream data of user is divided into N basic rate stream in serial-to-parallel converter (S/P). Each stream is encoded with a different Walsh code. Encoding is equivalent to Walsh-Hadamard Transform (WHT). The output of the WHT can be represented as:

$$\mathbf{s} = H_N(S_0, S_1, ..., S_{N-1})^T \quad (1)$$

where $S_k \in \{+1, -1\}$ is a data symbol.

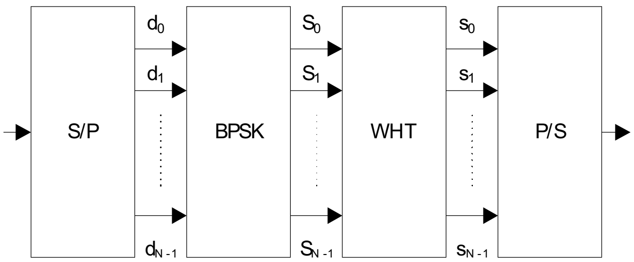

Fig. 1. Simplified block diagram of the MC-CDMA transmitter.

Output of parallel-to-serial converter is $\mathbf{s} = (s_0, s_1, ..., s_{N-1})$. This sequence $\{s_k\}$ is a multilevel sequence, with levels varying from –N to N. After multicode conversion, the transmitted signal is obtained by multiplying s_k with a spreading sequence, filtering and frequency up conversion. We do not consider these operations, because they do not affect the PAPR greatly. PAPR of MC-CDMA is defined as

$$PAPR(s) = \frac{1}{N} \max_{0 \le k \le N-1} |s_k|^2 \quad (2)$$

According to [9], we have

$$1 < PAPR(s) \le N \quad (3)$$

When N is large, PAPR is very high. For example, for N=256, the maximum PAPR can be as high as 24dB ($10\log_{10}256$). The PAPR of MC-CDMA signal is so high that make impairments when system has nonlinear element like power amplifier. The situation is serious; especially when N increases. Therefore, PAPR reduction of MC-CDMA signal in front of power amplifier is necessary.

B. CCDF of PAPR

Since input data is randomly distributed, PAPR is also a random variable. To show the statistical characteristics of PAPR, we can use complementary cumulative distribution function (CCDF), which is the probability that the PAPR of MC-CDMA symbol exceeds a certain threshold. The CCDF is defined as

$$CCDF(PAPR_0) = \Pr(PAPR(s) > PAPR_0) \quad (4)$$

This distribution has been estimated in [9] as

$$CCDF(\varsigma) = \Pr(PAPR(s) > \varsigma) = 1 - [F(\varsigma)]^N \quad (5)$$

where

$$F(\varsigma) = \begin{cases} \sum_{l=0}^{m} \Pr(|S_k| = 2l) & 2m \le \varsigma < 2m+2 \\ 1 & N < \varsigma \end{cases} \quad (6)$$

and

$$\Pr(|S_k| = 2l) = \begin{cases} \frac{1}{2^N}\binom{N}{N/2} & l = 0 \\ \frac{1}{2^{N-1}}\binom{N}{\frac{N+2l}{2}} & 0 < l \le N/2 \end{cases} \quad (7)$$

III. PROPOSED PAPR REDUCTION METHOD

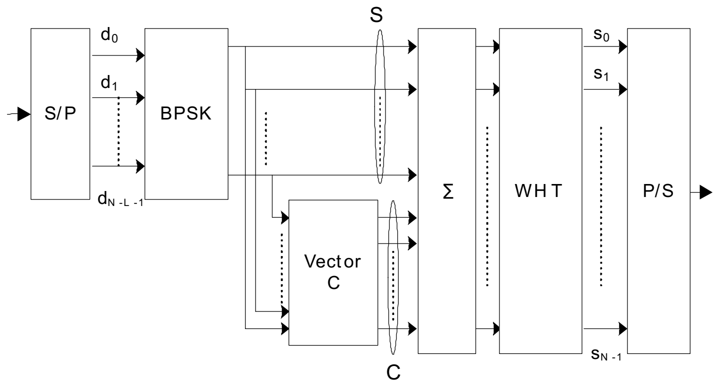

Fig. 2. Block diagram of proposed method.

The block diagram of proposed PAPR reduction method is shown in Fig.2. The difference of proposed system with conventional MC-CDMA system in Fig.1 is the computation vector C from incoming information data. We use it to reduce peak value. Based on the proposed system in Fig.1, the output of WHT $\mathbf{s} = (s_0, s_1, ..., s_{N-1})^T$ is a multilevel signal. In matrix,

$$\mathbf{s} = H \cdot \mathbf{S} \quad (8)$$

where $\mathbf{S} = (S_0, S_1, ..., S_{N-1})^T$ and H is a N×N Hadamard matrix defined as above

$$H = \begin{bmatrix} h_{11} & h_{12} & ... & h_{1N} \\ h_{21} & h_{22} & ... & h_{2N} \\ ... \\ h_{N1} & h_{N2} & ... & h_{NN} \end{bmatrix} \quad (9)$$

To reduce PAPR, we will find out the vector $\mathbf{c} = (c_0, c_1, ..., c_{N-1})^T$, so that new vector $\mathbf{v} = \mathbf{s} + \mathbf{c}$ has PAPR(v) which is less than PAPR(s).

We have

$$\mathbf{v} = \mathbf{s} + \mathbf{c} = HV = H \cdot (\mathbf{S} + \mathbf{C}) \quad (10)$$

where $\mathbf{C} = (C_0, C_1, ..., C_{N-1})^T$. We do not use all N Walsh codes to encode user data. Let $I = \{i_0, i_1, ..., i_{L-1}\}$ denote the ordered subset of unused codes to encode user data.

The indices satisfy $0 \le i_0 \le ... \le i_{L-1} < N$ where L < N is the number of unused codes. Vector **C** is chosen to satisfy

$$V_k = S_k + C_k = \begin{cases} C_k & if \quad k \in I \\ S_k & if \quad k \notin I \end{cases} \quad (11)$$

It is equivalent to $C_k \begin{cases} \ne 0 & if \quad k \in I \\ = 0 & if \quad k \notin I \end{cases}$

and

$$S_k \begin{cases} = 0 & if \quad k \in I \\ \ne 0 & if \quad k \notin I \end{cases}. \quad (12)$$

In Fig.2, the data vector S has only N-L nonzero elements $\hat{S} = S_k\,(k \notin I)$ and vector C has L nonzero elements

$\hat{C} = C_k\ (k \in I)$. The column vector with length L which is selected the values with index $k \in I$ $\hat{\mathbf{C}} = (C_{i_0}, C_{i_1}, ..., C_{i_{L-1}})^T$ is called constraint code, and $\hat{H}$ is the sub-matrix of H constructed by choosing its columns with index $k \in I$, i.e.

$$\hat{H} = \begin{bmatrix} h_{1i_0} & h_{1i_1} & \dots & h_{1i_{L-1}} \\ h_{2i_0} & h_{2i_1} & \dots & h_{2i_{L-1}} \\ \dots & & & \\ h_{Ni_0} & h_{Ni_1} & \dots & h_{Ni_{L-1}} \end{bmatrix} \quad (13)$$

$$\mathbf{c} = H \cdot \mathbf{C} = \hat{H} \cdot \hat{\mathbf{C}} \quad (14)$$

The PAPR of MC-CDMA symbol prior to PAPR reduction

$$PAPR(x) = \frac{1}{N} \max_{0 \le k \le N-1} |s_k|^2$$

and the PAPR of new MC-CDMA symbol $\mathbf{s} + \mathbf{c}$

$$PAPR(s+c) = \frac{1}{N} \max_{0 \le k \le N-1} |s_k + c_k|^2 \quad (15)$$

The problem minimize PAPR of combined signal is equivalent to find out the value of $\mathbf{c}^{opt}$ or $\mathbf{C}^{opt}$ that minimize the maximum peak value. That is

$$\min_{\mathbf{c}} |\mathbf{s} + \mathbf{c}|_\infty = \min_{\hat{\mathbf{C}}} \left| \mathbf{s} + \hat{H} \cdot \hat{\mathbf{C}} \right|_\infty \quad (16)$$

where $|\mathbf{s} + \mathbf{c}|_\infty$ is the ∞-norm (the maximum of the absolute value of its components) of vector $\mathbf{s} + \mathbf{c}$. This optimization is linear process to the variables $\hat{\mathbf{C}} = (C_{i_0}, C_{i_1}, ..., C_{i_{L-1}})^T$.

Equation (16) is equivalent to

$$\min_{\hat{\mathbf{C}}} \quad t \quad (17)$$

$$\text{Subject to: } \left| s_k + \hat{H}_k \cdot \hat{\mathbf{C}} \right| \le t \qquad k = 0, 1, \dots, N-1 \quad (18)$$

These constraints can be written in vector form

$$\min_{\hat{\mathbf{C}}} \quad t \quad (19)$$

$$\text{Subject to: } \begin{aligned} \mathbf{s} + \hat{H} \cdot \hat{\mathbf{C}} &\le t \cdot \mathbf{1}_N \\ \mathbf{s} + \hat{H} \cdot \hat{\mathbf{C}} &\ge -t \cdot \mathbf{1}_N \end{aligned} \quad (20)$$

where $\mathbf{1}_L$ is the vector with N ones.

Moving the unknowns t and $\hat{\mathbf{C}}$ to the left hand and grouping the constraints, the optimization problem can be written as eqs.(21), (22).

This linear process has 2L+1 unknown $\{\hat{\mathbf{C}}, t\}$ and 2N inequalities and is expressed in the standard form as eq.(23).

$$\min_{\hat{\mathbf{C}}} \quad t \quad (21)$$

$$\text{Subject to: } \begin{pmatrix} \hat{H} & -\mathbf{1}_N \\ -\hat{H} & -\mathbf{1}_N \end{pmatrix} \begin{pmatrix} \hat{\mathbf{C}} \\ t \end{pmatrix} \le \begin{pmatrix} -\mathbf{s} \\ \mathbf{s} \end{pmatrix} \quad (22)$$

$$\min \quad \mathbf{c}^T \mathbf{y} \quad (23)$$

$$\text{Subject to: } A\mathbf{y} \le \mathbf{b}$$

where $\mathbf{y}$ are the optimization variables. The matrix A, the vector $\mathbf{b}$ and $\mathbf{c}$ are known parameters.

IV. NUMERICAL RESULTS AND DISCUSSIONS

The positions of elements in constraint codes are chosen from subset $I = \{i_0, i_1, ..., i_{L-1}\}$. We consider for two cases, in the first case $\{i_k\}$ are adjacent and the second $\{i_k\}$ are randomly chosen. The optimization problem above must be solved over all possible discrete sets I and not be solved only according to specific indexes so that the good results can be obtained by randomly generated subset I. In the first case for adjacent indices, PAPR decreases slowly to the floor and does not reduce anymore even when L increases.

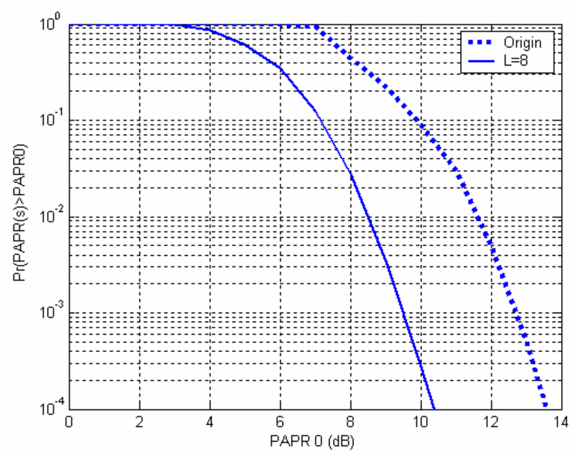

Fig. 3. CCDF of PAPR with N=64 and L=8 (adjacent indices)

In Fig.3, when N=64, length is L=8, and constraint codes are adjacently chosen, PAPR can be reduced from 13.7dB to 10.4dB, there is about 3.4dB gain at 10^{-4} compared to original method. However, PAPR can be reduced more than the first case if indices are randomly chosen. As the length of constraint codes increases, the PAPR can decrease. In Fig.4, when L=24, PAPR can be improved 9.2dB at 10^{-4}, from 13.7dB to 4.5dB and PAPR reduces proportionally with L. There is a trade-off between PAPR reduction and length of constraint codes to get the best performance. Fig. 5 shows the BER of MC-CDMA signal subjected by nonlinear SSPA. The system needs about 9dB IBO to get close to the linear BER. With large input back-off, HPA operates inefficiently. Fig.6 shows BER performance of MC-CDMA before and after using PAPR reduction method with constraint code length L=18 and adjacent indices. Evidently, BER performance has been improved with PAPR reduction method. At BER=10^{-3}, using PAPR reduction with IBO=0, we can achieve 5dB SNR gain (from 17dB down to 12dB). BER performance can be better in this method with random indices.

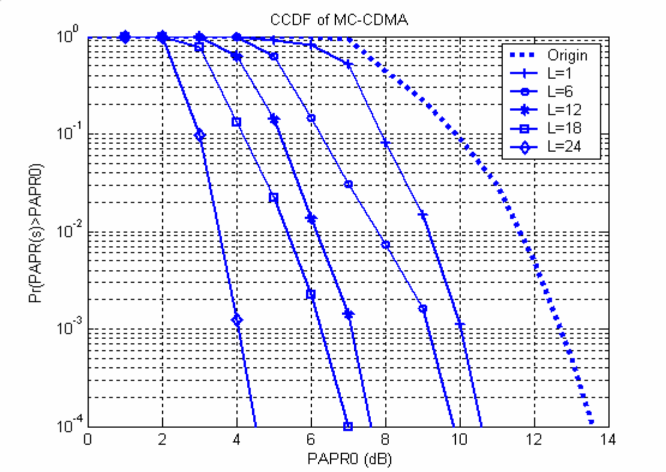

Fig.4. CCDF of PAPR with N=64 and different value of L .

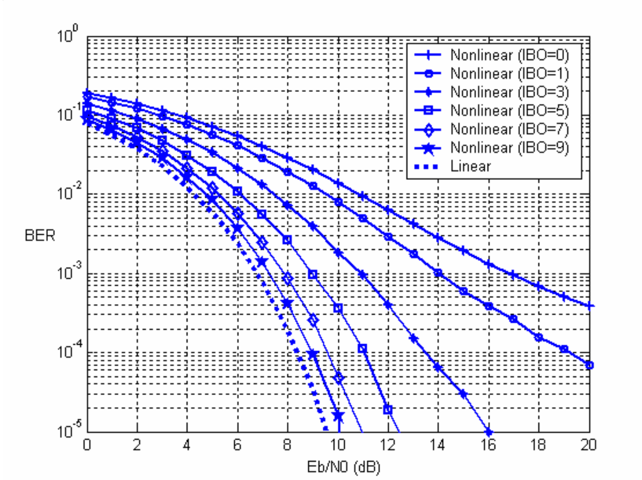

Fig. 5. BER of MC-CDMA using SSPA with different IBO

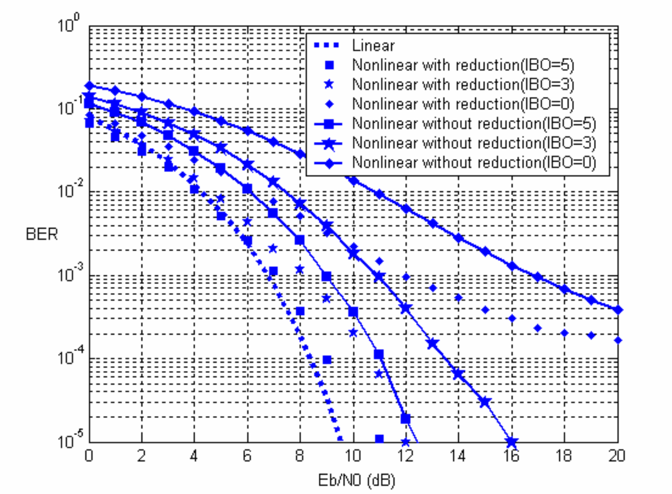

Fig. 6. BER before and after PAPR reduction in SSPA

V. CONCLUSION

We also investigate the effect of high power amplifier on BER performance with and without PAPR reduction method when nonlinear HPA is included. This method can efficiently reduce PAPR in MC-CDMA and there is a trade-off between PAPR reduction and length of constraint codes. When the constraint codes length is L=8 and indices are randomly chosen, PAPR can reduce 6.2dB at 10^{-5}. Because this technique needs no additional information, it is not necessary to worry about the effects of channel on side information. Clearly, BER performance has been improved when using this method in nonlinear environment. The simulation results show that BER performance can be similar to the linear amplifier case with L=24 (random indices) and without input back-off.

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