

# A Modified Iterative Packet Combining Scheme for Intersymbol Interference Channels

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**Abstract**—We study Iterative Combining (IC) technique for hybrid Automatic Repeat reQuest (ARQ) scheme over Intersymbol Interference (ISI) channels. By utilizing the EXtrinsic Information Transfer (EXIT) chart, we study the performance of the IC and find that the IC technique has a very good convergence behavior in the first little iteration, but it has a bad error floor, especially at the low Signal-to-Noise Ratio (SNR). In this paper, we propose a modified IC scheme to overcome the shortcoming of the traditional IC. We study the receiver from another aspect in order to acquire a serial concatenation turbo-like scheme, and then we can combine the parallel (IC) and the serial concatenation turbo-like scheme together to propose a modified IC scheme. In this modified IC scheme, the EXIT chart as a balance switch between the two turbo-like schemes. It is shown that the proposed modified IC scheme provides excellent performance both analytically and through simulations without any computational complexity increase.

## I. INTRODUCTION

In communication systems, there are two fundamental techniques used for error control: Forward Error Correction (FEC) and ARQ. In FEC schemes, the receiver have the ability to correct erroneous data, while in pure ARQ schemes the receivers can only detect if there are errors in the transmission. ARQ schemes are widely used in data communication where transmission delay is acceptable. In Type I ARQ schemes, uncoded data is transmitted in packets, and, when an erroneous packet is detected, the receiver can request the transmitter to resend the same packet of data until no error is detected. Hybrid ARQ schemes (Type II or Type III ARQ schemes) which combine FEC and ARQ are also commonly used [9]. In this paper, we study the hybrid ARQ schemes.

It is well known that the throughput of an ARQ scheme can be improved by using the packet combining. In packet combining, all received copies of the data packet are combined to form a more reliable estimate of the transmitted data. Inspired by the turbo codes [1], in recent years, there has been a growing interest in applying turbo principle to enhance performance in communication systems [2] [3]. The application of the turbo principle in ARQ schemes for ISI channels was pioneered by Balachandran and Anderson [4], in their paper, they proposed a turbo-like scheme where component encoders of the original turbo code were replaced by two equivalent discrete-time ISI channels. In the contrast to [4], [5] give the exactly concept of the parallel scheme IC and consider the multiple retransmissions

and precoding. In this paper, we study the IC for hybrid ARQ scheme over ISI channel, for simple, here we just consider one retransmission without precoding.

In this paper, we utilize the EXIT chart to study the performance of IC. We can find that the IC has a very good convergence behavior and a very bad error floor, which means this method typically works well in the first little iteration but fails to improve. For overcoming the disadvantage of the traditional IC scheme, we study the receiver from another aspect, which means here we regard one equalizer, the interleaver and the de-interleaver belonging to the IC scheme and also including the decoder as one group, and then we can find that this group just can structure a serial concatenation turbo-like scheme. So based on the above, we can acquire a parallel and a serial concatenation turbo-like scheme at the receiver to propose a modified IC scheme. In this proposed modified IC scheme, we can use the EXIT chart to combine the parallel and serial concatenation turbo-like scheme together neatly, which means we can utilize the EXIT chart to investigate the possibility to match the two different schemes over different ranges of the iteration process. Because of introducing a serial concatenation turbo-like scheme in the traditional parallel IC scheme, the modified IC scheme can overcome the shortcoming of the traditional IC scheme, which means this modified method can provide excellent performance without increasing any computational complexity.

## II. SYSTEM MODEL

The system model is shown in Fig. 1. A block of  $N$  bits of data is encoded by an optional outer code and then modulated and transmitted  $K$  times over a channel corrupted by ISI ( $h[n]$ ) and Additive White Gaussian Noise (AWGN) ( $w[n]$ ). For simple and ease of exposition, we assume that  $K = 2$  and the

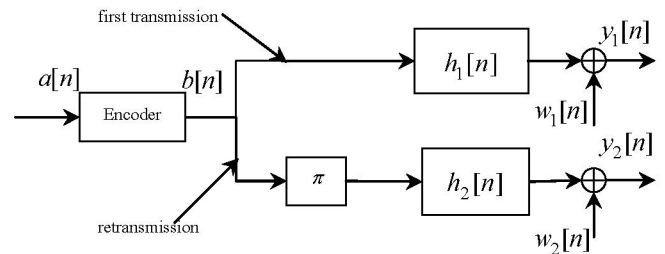


Fig. 1. Coded data transmission model, where  $\pi$  represents interleaver.

modulation technique is Binary Phase Shift Keying (BPSK), and notes that extension to the larger  $K$  and the higher order of modulation is straightforward. In the one retransmission case, prior to transmission, the data over one of the channels is segmented into blocks, and the data within each block is then interleaved to introduce additional temporal diversity. The data BPSK-encoded and that the overall effect of each of the transmitter-channel-receiver paths can be represented as a linear time-invariant discrete-time channel,  $h_1[n]$  and  $h_2[n]$ . In Fig. 1, the data  $b[n]$  is interleaved prior to transmission over  $h_2[n]$ . The AWGN  $w_i[n]$  is present in each channel and that the noise in each of the channels is uncorrelated. The discrete-time baseband model of the two channel outputs for the transmitted data sequence  $b[n]$  is then

$$y_1[n] = \sum_{k=0}^{M_1} h_1[k] b[n-k] + w_1[n] \quad (1)$$

$$y_2[n] = \sum_{k=0}^{M_2} h_2[k] \tilde{b}[n-k] + w_2[n] \quad (2)$$

where  $i_n = \pi(1, \dots, N)$  is the data permutation index,  $b[n]$  is the input symbol stream, and  $\tilde{b}[n] = b[i_n]$  is the permuted symbol stream.

### III. IC TECHNIQUE

During the first transmission, the received packet is equalized and decoded using the outer code and then checked for errors using an error detection code. A Cyclic Redundancy Check (CRC) or the outer code itself may serve as the error detection code. When the packet is determined to be in error, a retransmission is requested. What is retransmitted during the retransmissions and how the retransmissions are combined depend on the exact combining technique used at the receiver. In the following, we will give a simple introduction of the IC technique.

In the case of IC, during a retransmission, an interleaver is placed at the input to the ISI channel as shown in Fig. 2. Different interleavers are used for different retransmissions.

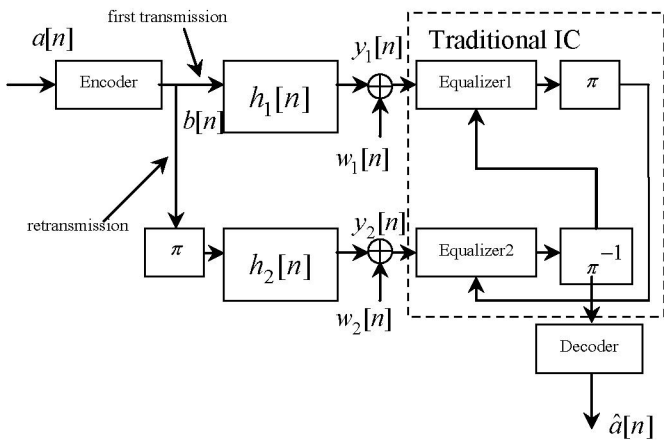


Fig. 2. IC scheme for coded data transmission system.

This is only the modification that is required at the transmitter. Multiple retransmissions through the ISI channel can be considered as a concatenation of convolutional codes. When an interleaver is used in between the transmissions, it represents a parallel concatenated turbo-like scheme where the ISI channels are the component codes. Multiple transmissions are combined using turbo decoder that iteratively forms estimates for the transmitted data packet  $\mathbf{b}$  based on the received  $\mathbf{y}_1, \mathbf{y}_2$ . The receiver consists of  $K$  component soft-in soft-out (SISO) equalizer (here  $K = 2$ ). During the completion of the  $(m+1)$ th iteration, the soft information (extrinsic information) produced by the  $i$ th SISO is given by

$$\mathbf{L}_{Ei}^{(m+1)} = f(\mathbf{y}_i, \mathbf{L}_{Ej}^m) \quad (3)$$

where  $f(\cdot)$  is the MAP equalization function which can be implemented using either the Bahl, Cocke, Jelinek, and Raviv (BCJR) algorithm or the Soft decision Viterbi Algorithm (SOVA) or any variation of these algorithms. In this paper, we consider only the MAP implementation [1]. The equalizer output is fed to the MAP decoder, which computes the transmitted data packet estimates  $\hat{\mathbf{a}}$ .

### IV. EXIT CHART

The EXIT chart is a convenience tool to visualize the performance by means of mutual information between transmitted bits and LLR values used within the turbo-like scheme. A comprehensive overview of this method can be found in [6] [7]. Here, we briefly discuss the basic principle of the EXIT chart tool.

Let us start with the equalizer. We assume perfect interleaving. Therefore, the input LLR values of the equalizer are modeled by independent and identically distributed (IID) random variables  $A_{Equ}$ . The pdf  $p(l|x)$  of  $A_{Equ}$  conditioned on the transmitted bits  $x$  is assumed to be Gaussian with mean  $\sigma_A^2 / 2(1-2k)$  (with  $k = 0, 1$ ) and variance  $\sigma_A^2$ . The information content  $I_A(Equ) \triangleq I(x, A_{Equ})$  between  $x$  and  $A_{Equ}$  has a one-to-one relationship to  $\sigma_A$ .  $I_A(Equ)$  may be evaluated numerically [4]:

$$I_A(Equ) = \frac{1}{2} \sum_{k=0,1} \int_{-\infty}^{\infty} p(l|x=k) \log_2 \left( \frac{2p(l|x=k)}{p(l|x=0) + p(l|x=1)} \right) dl \quad (4)$$

Similarly, the mutual information  $I_E(Equ) \triangleq I(x, E_{Equ})$  between  $x$  and the extrinsic equalizer output (random variables  $E_{Equ}$ ) may be determined by observing the pdf of  $E_{Equ}$  conditioned on the transmitted bits  $x$ . The mutual output information  $I_E(Equ)$  as a function of the mutual input information  $I_A(Equ)$  is denoted as the extrinsic information transfer characteristics of the equalizer. Since (4) is

monotonically increasing,  $I_A(Equ)$  as a function of  $\sigma_A^2$  is invertible. Therefore, artificial LLR values with variance  $\sigma_A^2$  and mean  $\sigma_A^2/2(1-2k)$  may be generated for a given input information  $I_A(Equ)$ . These LLR values and the received symbol sequence  $\mathbf{y}$  are then fed to the equalizer and the output pdf of  $E_{Equ}$  conditioned on the transmitted bits is measured by means of Monte Carlo simulations (histogram measurements) or the following equation (4) to determine  $I_E(Equ)$ .

$$I_E(Equ) \approx 1 - \frac{1}{N} \sum_{n=0}^{N-1} \log_2(1 + \exp(-x_n L(x_n))) \quad (5)$$

where  $x_n = \pm 1$  and the pdf  $p(l|x)$  satisfies both the symmetry and the consistency constraint. This procedure is also applied to characterize the decoder. We denote the corresponding mutual input information of the decoder as  $I_A(Dec)$  and the output information as  $I_E(Dec)$ , respectively.

## V. MODIFIED IC SCHEME

In this section, firstly, we will utilize the EXIT chart discussed at the above section to study the performance of the traditional IC scheme and then give the motivation for the modified IC scheme, at last we will propose the modified IC scheme.

### A. Motivation for the Modified IC Scheme

We can utilize the EXIT chart to analyze the performance of the traditional IC scheme (as shown in Fig.3). Fig. 3 depicts the EXIT charts of a MAP decoder for the rate 1/2 convolutional code  $(7, 5)_8$  and two MAP equalizers, which process symbols transmitted and one retransmitted over the same ISI channels ( $h_1[n] = h_2[n] = [0.407, 0.815, 0.407]$ ) at a fixed SNR (here is at 4 dB). Also included is the trajectory (measurement of the

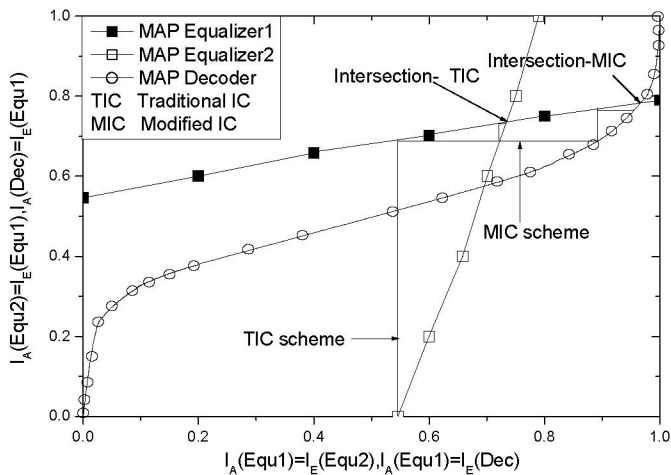


Fig. 3. Motivation for the modified IC scheme at 4 dB.

mutual information after each equalization and equalization step) of an actual system using length  $10^5$  data bit blocks (the block length is 512). We can observe that the traditional IC scheme has a very good convergence behavior, which means the traditional IC scheme needs short zigzag trajectory to get good mutual information. But because the intersection between the two equalizer transfer functions---Intersection-TIC is far away from the mutual information 1 (the transmitted bits are known), the traditional IC scheme has a very bad error floor, which means the traditional IC scheme will not get any the improvement of the performance after the more iteration. In general, based on the EXIT chart, we can observe that the traditional IC scheme typically works well in the first little iteration but fails to improve.

Based on the aforementioned discussion, we know the essence of the shortcoming of the traditional IC scheme is that the Intersection-TIC is far away from the mutual information 1. The direct way to overcome the disadvantage of the traditional IC scheme is to find some methods to make the Intersection-TIC near to 1. In [5], the author consider the precoding in the ISI channels, which can make the Intersection-TIC near to 1 but also increase the computational complexity. In this paper, we use another way to solve this problem. The main idea of our solution is that we do not try our best to improve the Intersection-TIC, otherwise, to utilize another iterative scheme to replace the traditional IC scheme at the appropriate range of iteration, in order to hop over the limit of the Intersection-TIC and to get the Intersection-MIC, which is very near to 1 (just as Fig. 3 shown). In Fig. 3, we can find that in the modified IC scheme, we utilize the serial concatenation turbo-like scheme to replace the traditional IC scheme after the first iteration at 4 dB.

### B. Modified IC Scheme

For realizing our solution, we need a serial concatenation turbo-like scheme consisting of the equalizer and the decoder as Fig.3 shown. Fortunately, if we carefully study the receiver of the traditional IC scheme (as shown in Fig. 2) from another aspect, we can find that the receiver of Fig. 2 includes a serial concatenation turbo-like scheme. The included serial turbo-like scheme consists of one equalizer, interleaver, de-interleaver and decoder. As Fig.4 shown, flexible choosing the component through EXIT chart at the receiver we can get the modified IC scheme without increasing any complexity. Note that the interleaver and de-interleaver in the modified IC scheme are not only used to compose of a serial concatenation turbo-like scheme but also used to compose of a parallel concatenation turbo-like scheme.

## VI. SIMULATION RESULTS

In this section, we consider a system employing a rate 1/2 systematic convolutional code with generator polynomials  $[7, 5]_8$ . The information block length is 512 bits. An S-random ( $S=16$ ) interleaver is used before transmitting the coded bits in the BPSK format. We employ tap-coefficients  $h_1[n] = h_2[n] = [0.407, 0.815, 0.407]$  (ISI-3 channel) [8]. It is assumed that the receiver has exact knowledge of the channel fading coefficients. The performance in an AWGN channel without any ISI is

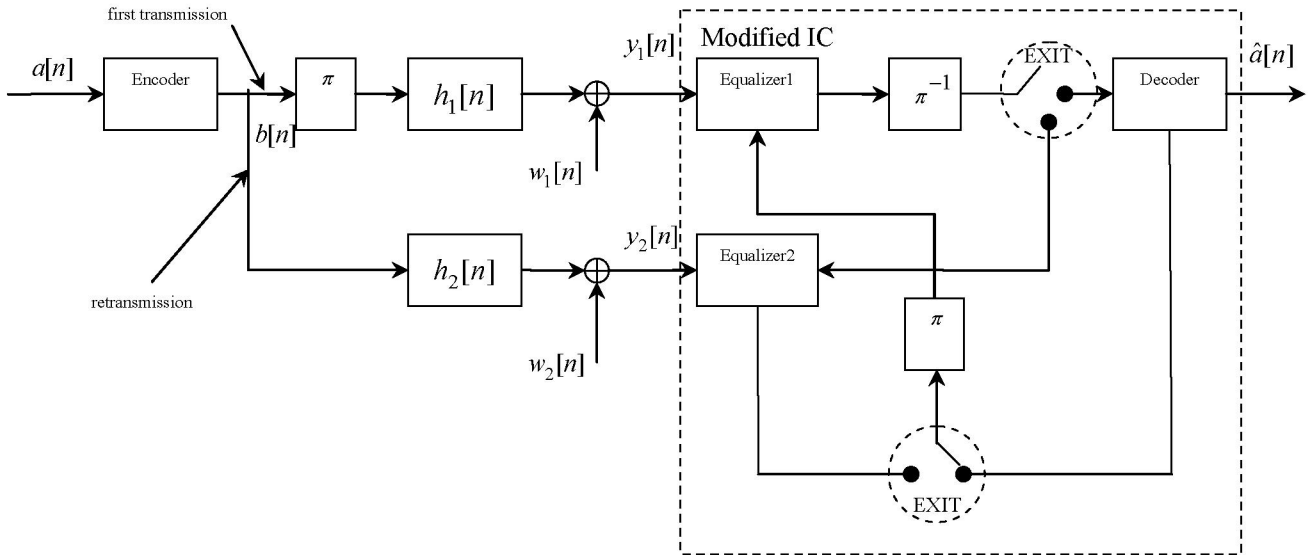


Fig. 4. Modified IC scheme for coded data transmission system.

included as a reference. The “it” in the Fig. 5 and Fig. 6 is the abbreviation for iteration.

Fig. 5 illustrates the BER performance of the traditional IC scheme. We can find that the traditional IC scheme typically works very well in early iterations, here after the first iteration it can get very good BER performance. But through increasing the number of iteration there is very a little improvement of BER performance, even without any improvement. So even after much iteration, the BER performance is also far away from the lower bound (corresponding to the dashed line in Fig. 5).

Fig. 6 depicts the comparison of the BER performance between the traditional IC scheme and the modified IC scheme. We can find this modified IC scheme typically works very well in all iteration. Based on the EXIT charts we choose the different iterative scheme from the traditional IC and the modified IC to get the better BER performance at different iteration and SNR. In Fig. 6 we can find after about three iterations the BER performance of the modified IC scheme can

reach the lower bound very closely.

## VII. CONCLUSIONS

Based on the understanding of the traditional IC scheme and the EXIT chart, we utilize another way to overcome the shortcoming of the traditional IC scheme, which means we consider the decoder into the IC scheme in order to combine a serial and a parallel concatenation turbo-like scheme together to propose the modified IC scheme. In the modified IC scheme, we utilize the EXIT chart as a balance switch between the two turbo-like schemes. Because of considering the decoder into the IC scheme, we can get the modified IC scheme without any computational complexity. In general, the main idea of the modified IC scheme is to select suitable iterative scheme for every iterations and different SNR according to their EXIT charts. From both the analysis and the simulation, we can find that the modified IC scheme can get excellent performance while without any computational complexity increase.

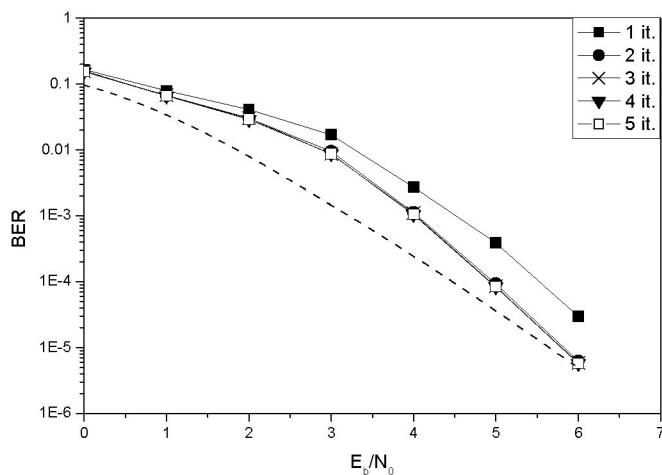


Fig. 5. BER performance of the traditional IC scheme.

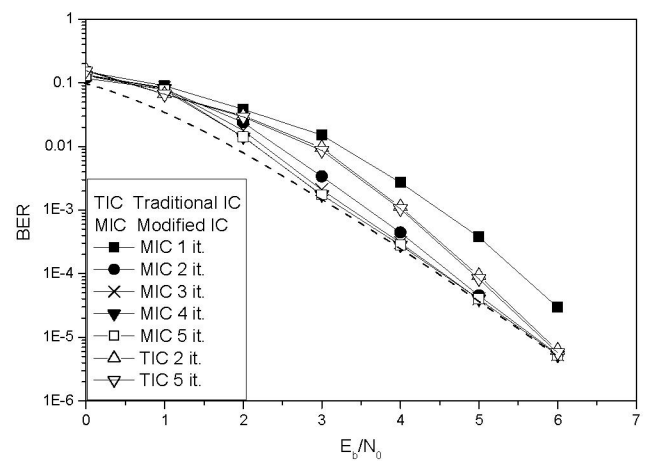


Fig. 6. Comparison of the BER performance between the traditional IC scheme and the modified IC scheme.

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