Optimal Selection of Pilot Positions for Frequency Domain Pilot Multiplexing Channel Estimation in SC-FDE Systems

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***Abstract*—In this paper, we investigate the pilot position selection (PPS) problem in the frequency domain pilot multiplexing technique (FDPMT) for channel estimation of single-carrier block transmission with frequency domain equalization (SC-FDE). Unlike the widely accepted point of view that the conventional PPS technique is the optimal one, this paper for the first time questions this viewpoint and demonstrates the suboptimal property of the conventional PPS technique. To further improve the performance of the FDPMT, an optimal PPS technique is proposed based on the minimization of the average bit error rate (BER). Compared with the conventioanl PPS technique, the proposed PPS technique exhibits better performance with similar complexity.**

I. Introduction

Unlike orthogonal frequency division multiplexing (OFDM) systems, the SC-FDE transmits the data block-wisely in the time domain while performs the channel estimation and equalization in the frequency domain. Compared with the OFDM, the SC-FDE has similar overall implementation complexity but exhibits superior system performance [1]. To improve the data transmission performance, coherent demodulation is commonly applied for both OFDM and SC-FDE systems. The successful application of such technique critically depends on the accuracy of the channel estimation. For the past years, plenty of investigations have preferred utilizing the frequency domain multiplexed (FDM) pilots insertion technique for the channel estimation of OFDM systems while preferred using the time domain multiplexed (TDM) pilots insertion techniques for the channel estimation of SC-FDE systems [1][2].

In [3], two types of FDM pilot insertion techniques, i.e., frequency expanding technique (FET) and frequency domain superimposed pilot technique (FDSPT), were proposed for the first time for SC-FDE systems to track the channel state information (CSI). Unlike the FET, the FDSPT does not need extra spectrum for the pilot tones. Instead, the equally spaced pilots are superimposed on the data tones starting from the first tone for all SC-FDE blocks and thus the FDSPT keeps the spectral efficiency as the original SC-FDE systems. However, the simulation results in [3][4] have shown that the introduction of the mutual interference between data symbols and pilots in the FDSPT degrades the BER performance.

To improve the performance of FDSPT, a new FDPMT was proposed in [5]. Unlike the FDSPT, the FDPMT first eliminates some equally spaced data tones whose initial position is selected based on the proposed PPS technique at the transmitter and then inserts pilots block by block. Based on such PPS technique and by iteratively reconstructing the distorted data symbols at the receiver, the FDPMT provides better BER performance than the FDSPT and even approaches the lower bound of SC-FDE systems after many iterations. Note that compared with the FDSPT, the essential innovation of the FDPMT is the development of the PPS technique. Up to the present, it has been widely accepted in academia [6]-[10] that the proposed PPS technique in [5] is the optimal one in achieving the ultimate system performance.

In this paper, we reinvestigate the principle of the FDPMT and for the first time question the aforementioned widely accepted viewpoint based on our observation. Specifically, different from the superimposed pilot scheme in an OFDM system where the distortion caused by the insertion of pilots can be treated as noise [11], the loss of data tones in the superimposed pilot scheme for a SC-FDE system actually results in the deterministic interference. This is because that the decision for SC-FDE signals is performed in the time domain. The above mentioned observation indicates that the current PPS technique based on the minimization of the distortion of SC-FDE signals [5] would not be equivalent to the optimization for the symbol decision. To prove the suboptimal property of the PPS technique in [5], 16-QAM modulation is adopted as an example for the derivation and analysis throughout this paper.

The suboptimal property of the current PPS technique motivates us to propose an optimal one. Therefore, in this paper, an optimal PPS technique is proposed, where the selection of the initial position of pilots is based on the minimization of the average BER in each SC-FDE block with pilots. Compared with the significant performance gain of the proposed PPS technique over the current one, the increase of the complexity of the proposed PPS technique over the current one is marginal. Moreover, the proposed PPS technique with no iteration at the receiver even expresses better performance than the current one with one iteration for higher signal-to-noise (SNR). This means that for higher SNR the proposed PPS technique can make the SC-FDE system express similar complexity to the system applying the current PPS technique while still preserve the performance gain.

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II. System Models

Fig. 1 depicts the block diagram of the SC-FDE transmitter with FDM pilots [5][8]. The SC-FDE transmit block is of length N and denoted by $\mathbf{s} = [s_0, \ldots, s_{N-1}]$ with each symbol drawn from a complex valued alphabet. The N-point discrete Fourier transform (DFT) of $\mathbf{s}$ is given by

$$S_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} s_n e^{-j\frac{2\pi nk}{N}}, \quad k = 0, \ldots, N-1. \tag{1}$$

For the FDPMT, some data tones are deliberately eliminated to insert pilots in substitution, and thus the actual frequency domain transmit sequence becomes

$$\mathbf{X}_{d_0} = [S_0, \ldots, S_{d_0-1}, C_0, \ldots, S_{M+d_0-1}, C_1, \ldots, S_{N-1}] \tag{2}$$

where $\mathbf{C} = \left[C_0, \ldots, C_{N_p-1}\right]$ is the pilot sequence of length N_p, $M = N/N_p$ is the spacing between two adjacent pilot tones, and d_0 is the initial pilot position. By defining $\mathbf{X'}_m = [S_0, \ldots, S_{m-1}, 0, \ldots, S_{M+m-1}, 0, \ldots, S_{N-1}]$ with $m \in [0, M)$ denoting the initial pilot position and its inverse DFT (IDFT) as $\mathbf{x'}_m = [x'_{m,0}, \ldots, x'_{m,N-1}]$, the distortion level due to the removal of the data tones at indexes $\psi_m = \{m, M+m, \ldots, (N_p-1)M+m\}$ is quantified in [5][8] as $\varepsilon_m = \sum_{n=0}^{N-1} |s_n - x'_{m,n}|^2$, and accordingly d_0 is selected based on its minimization as

$$d_0 = \arg\min_m \varepsilon_m = \arg\min_m \mathbf{\Phi}_m^H \mathbf{\Phi}_m \tag{3}$$

where $\mathbf{\Phi}_m = \left[S_m, S_{M+m}, \ldots, S_{(N_p-1)M+m}\right]$ and $(\cdot)^H$ denotes complex conjugate transpose. It is easy to find that the above strategy is in fact equivalent to minimize the loss of the power of $\mathbf{s}$ since we have $\sum_{n=0}^{N-1} |s_n|^2 = \sum_{k=0}^{N-1} |S_k|^2$ based on the Parseval theorem. Before transmission, $\mathbf{X}_{d_0}$ is first fed into an N-point IDFT and then appended by the length-G cyclic prefix (CP).

At the receiver, an N-point DFT is performed to convert the received signal without the CP to the frequency domain as $\mathbf{R}_{d_0} = [R_{d_0,0}, \ldots, R_{d_0,N-1}]$. By adopting the pilot position detection techniques proposed in [8]-[10], the pilot tones, i.e., $\{R_{d_0,k}, k \,(\mathrm{mod}\ M) = d_0\}$, can be easily and properly extracted from $\mathbf{R}_{d_0}$. Following that, the channel frequency responses (CFRs) on the pilot tones are estimated by dividing the extracted signals by the prior known pilots and those on the data tones are then obtained by performing interpolation. Many interpolation techniques can be used, such as linear interpolation, trigonometric interpolation, and etc., and their performance comparison has been investigated in [12]. For analytical simplicity, in this paper we assume perfect CSI.

Once CSI is known, the decision of the SC-FDE symbols should be made coupled with the iterative signal reconstruction (ISR) technique to improve the overall system performance [8] [13]. This is implemented as follows. Firstly, by equalizing the elements on data positions and nulling those on pilot positions, the outputs in the frequency domain can be readily obtained as

$$\tilde{S}_{d_0,k}^{(0)} = \begin{cases} 0, & k \,(\mathrm{mod}\ M) = d_0 \\ S_k + V_k/H_k, & otherwise \end{cases} \tag{4}$$

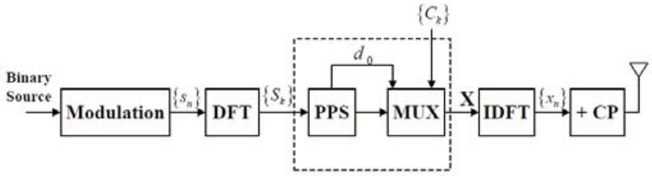

Fig. 1. The block diagram of the transmitter with the FDPMT.

with $k = 0, \ldots, N-1$, where H_k is the CFR on the kth tone, $\mathbb{E}\left[|H_k|^2\right] = 1$, and V_k is the zero-mean complex additive white Gaussian noise (AWGN) with variance σ_v^2. Without loss of generality, zero forcing (ZF) equalization is assumed in (4). Secondly, the initial estimates of SC-FDE symbols are derived by performing coherent demodulation to the N-point IDFT of those outputs in (4). Finally, the ith ($i \geq 1$) estimates are iteratively obtained by the IDFT of

$$\tilde{S}_{d_0,k}^{(i)} = \begin{cases} \hat{S}_{d_0,k}^{(i-1)}, & k \,(\mathrm{mod}\ M) = d_0 \\ \tilde{S}_{d_0,k}^{(0)}, & otherwise \end{cases} \tag{5}$$

with $k = 0, \ldots, N-1$, where $\{\hat{S}_{d_0,k}^{(i-1)}\}_{k=0}^{N-1}$ are the DFT of the $(i-1)$th estimates, i.e., $\{\hat{s}_{d_0,n}^{(i-1)}\}_{n=0}^{N-1}$. Notable is that by jointly considering the complexity and performance, the suggested iteration number should not exceed one.

III. Optimal Pilot Position Analysis

The non-linear operation of ISR would easily incur error propagation, consequently to guarantee the system performance, reliable initial estimates must be satisfied [13]. To get further insight, let us consider the initial input for a demodulation, which turns out to be the IDFT of (4), i.e.,

$$\begin{aligned} \hat{s}_{d_0,n}^{(0)} &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \alpha_{d_0,k} \left(S_k + \frac{V_k}{H_k}\right) e^{j\frac{2\pi kn}{N}} \\ &= \underbrace{s_n + s'_{d_0,n}}_{x'_{d_0,n}} + v'_{d_0,n}, \quad n = 0, \ldots, N-1 \end{aligned} \tag{6}$$

where $\alpha_{d_0,k} = 0$ for $k \,(\mathrm{mod} M) = d_0$, otherwise $\alpha_{d_0,k} = 1$, $s'_{d_0,n} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\alpha_{d_0,k} - 1) S_k e^{j\frac{2\pi kn}{N}}$ indicates the distortion caused by superimposing pilots, and $v'_{d_0,n} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\alpha_{d_0,k} V_k / H_k) e^{j\frac{2\pi kn}{N}}$ denotes the equalized noise.

Intuitively, the PPS criterion according to the minimization of the power loss in (3) should be significantly helpful in reconstructing the desired signals $\{s_n\}_{n=0}^{N-1}$ when $\{s'_{d_0,n}\}_{n=0}^{N-1}$ in (6) are considered as part of noise. However, for SC-FDE systems in which $\{s_n\}_{n=0}^{N-1}$ are drawn from a finite alphabet set, the distortion actually contributes as a deterministic interference from coherent demodulation perspective. This raises several questions, such as whether the minimization of the distortion of SC-FDE signals optimizes the symbol decision, and if not how to design the PPS technique to obtain the optimal symbol decision. In this section, we will address the above questions.

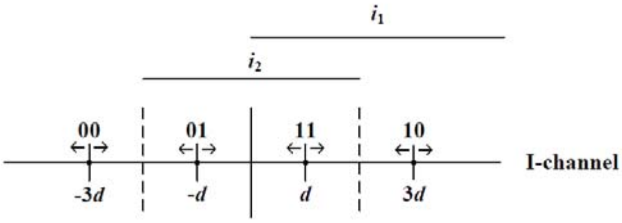

Fig. 2. Signal space diagram for 4-PAM.

A. Description of the suboptimal property of the current PPS technique

Without loss of generality, suppose that 16-QAM modulation is adopted. In this case, a modulated symbol consists of 4 bits, i.e., $i_1 i_2 q_1 q_2$, with the first two bits representing the in-phase (I-) component and the last two bits representing the quadrature (Q-) component. Since the analysis for I-channel and Q-channel scenarios is similar, the I-channel scenario is taken as an example for detailed analysis. Fig. 2 shows the I-channel signal constellation (4-PAM) using Gray mapping and its decision boundaries, where $\underline{i_1}$ and $\underline{i_2}$ denote the regions in which $i_1 = 1$ and $i_2 = 1$, respectively, and $d = \sqrt{2E_b/5}$ with E_b denoting the signal energy per information bit.

To verify that the PPS criterion in (3) is a suboptimal strategy, we resort to the following example.

Example: Suppose that two options are available for superimposing pilots: one is at tones ψ_{m_1} and the other is at tones ψ_{m_2}. To make this example easy to follow, we assume the accumulated distortion level on all SC-FDE symbols excluding the real part of the nth sample in one block for both options are equal, i.e., $\sum_{u\neq n} |{s'}_{m_1,u}|^2 + |\Im[{s'}_{m_1,n}]|^2 = \sum_{u\neq n} |{s'}_{m_2,u}|^2 + |\Im[{s'}_{m_2,n}]|^2$. In this case, the selection between these two options mainly depends on the distortion level on the real part of the nth sample, s_n^r. If $s_n^r = 3d$ and ${s'}^r_{m_1,n} < 0 < {s'}^r_{m_2,n}$ $\left({s'}^r_{\mathcal{X},n} = \Re[{s'}_{\mathcal{X},n}], \mathcal{X} \in \{m_1, m_2\}\right)$, i.e., the signal point labeled "10" of the nth sample is shifted to the left for the first option and to the right for the second option due to the distortion. It can be concluded that compared with the unshifted case, the error probabilities of both bits, i_1 and i_2, would become bigger for the first option while smaller for the second option. This is because the signal point gets closer to the decision boundaries or even moves to the wrong decision range for the left-shifted case while gets further from the decision boundaries for the right-shifted case. However, when the left-shifted amount is smaller than the right-shifted amount, i.e., $\left|{s'}^r_{m_1,n}\right|^2 < \left|{s'}^r_{m_2,n}\right|^2$, the first option will be chosen instead according to the current PPS criterion. Similarly, for $s_n^r = -3d$ and ${s'}^r_{m_1,n} < 0 < {s'}^r_{m_2,n}$, it will be seen that the current PPS criterion would erroneously pick up the second option provided that the right-shifted amount is smaller than the left-shifted amount. The analysis for the suboptimal property of the current PPS criterion in the case of $s_n^r = \pm d$ and ${s'}^r_{m_1,n} < 0 < {s'}^r_{m_2,n}$ seems a bit tedious since the comparison of the error probabilities for both options is highly relevant to the specific realization of channel, we omit it here due to the space limit. Note that there exist many other counterexamples for the validation of the suboptimal property of the current PPS technique.

B. An optimal PPS technique

Subsection III.A clearly demonstrates that the minimization of the distortion of SC-FDE signals cannot optimize the symbol decision. This subsection will give the answer about how to design the PPS technique to obtain the optimal symbol decision. From the above analysis, it is clear that an optimal PPS criterion should aim at minimizing the overall impact of the superimposition pilots on the SC-FDE systems, which can be characterized by using the average BER in one SC-FDE block. Motivated by this, in the following we first derive the closed-form expression of the average BER and then propose the realization of the optimal PPS criterion.

Let us begin with the simple case of an AWGN channel. Without loss of generality, consider the error probability of the nth sample, s_n, with the assumption that pilots are inserted at tones ψ_m. The initial input for demodulation can be obtained from (6) as $\hat{s}^{(0)}_{m,n} = {x'}_{m,n} + {v'}_{m,n}$, where ${v'}_{m,n}$ is AWGN with variance $\sigma^2_{v'} = \frac{N-N_p}{N}\sigma^2_v$ due to a linear combination of Gaussian variables. For the I-component of s_n, $s_n^r = \Re[s_n]$, there are two possible cases (Case I and Case II) for the probabilities, $P^{i_l}_{m,n}$, that the bit i_l $(l = 1, 2)$ is in error. To simplify the following analysis, we introduce parameters $\xi_{n,l,1}$, $\xi_{n,l,2}$ $(l = 1, 2)$ to further fractionize each class into four situations according to the possible positions of s_n^r and ${x'}^r_{m,n}$ $\left({x'}^r_{m,n} = \Re[{x'}_{m,n}]\right)$ in the constellation with each corresponding to $\xi_{n,l,1} = \xi_{n,l,2} = 1$, $\xi_{n,l,1} = -1$ and $\xi_{n,l,2} = 1$, $\xi_{n,l,1} = \xi_{n,l,2} = -1$, and $\xi_{n,l,1} = 1$ and $\xi_{n,l,2} = -1$, which are in sequence referred to as "s1", "s2", "s3", and "s4", respectively, in this paper. Here, $\xi_{n,l,1} = 1$ indicates that s_n^r belongs to any signal point in the region $\underline{i_l}$ otherwise $\xi_{n,l,1} = -1$ and $\xi_{n,l,2} = 1$ stands for ${x'}^r_{m,n}$ lying in the region $\underline{i_l}$ otherwise $\xi_{n,l,2} = -1$.

For Case I, only the bit i_1 is considered. The constellation, as shown in Fig. 2, can be separated into two regions based on the decision boundary represented by the solid line at the origin. As mentioned above, in total four situations for Cass I will be encountered. Specifically, for "s1", i.e., $\xi_{n,1,1} = \xi_{n,1,2} = 1$, a bit error will occur if the noise is smaller than $\left(-{x'}^r_{m,n}\right)$. In this case, the probability that the bit i_1 for "s1" is in error is [14]

$$P^{i_1}_{m,n}|_{s1} = Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}\right)^2}{\sigma^2_{v'}}}\right) \tag{7}$$

where $Q(x) = \left(1/\sqrt{2\pi}\right)\int_x^\infty e^{-t^2/2}dt$. For "s2", i.e., $\xi_{n,1,1} = -1$ and $\xi_{n,1,2} = 1$, a bit error will occur if the noise exceeds $(-{x'}^r_{m,n})$, yielding

$$P^{i_1}_{m,n}|_{s2} = 1 - Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}\right)^2}{\sigma^2_{v'}}}\right). \tag{8}$$

Due to symmetry, the bit error probability of i_1 for "s3", i.e., $\xi_{n,1,1} = \xi_{n,1,2} = -1$, and "s4", i.e., $\xi_{n,1,1} = 1$

and $\xi_{n,1,2} = -1$, will be equal to that for "s1" and "s2" respectively, i.e., $P^{i_1}_{m,n}|_{s3} = P^{i_1}_{m,n}|_{s1}$ and $P^{i_1}_{m,n}|_{s4} = P^{i_1}_{m,n}|_{s2}$. Therefore, putting these four situations together, we can obtain a generic form for the error probability of the bit i_1 as

$$P^{i_1}_{m,n} = \frac{1-\xi_{n,1,1}\xi_{n,1,2}}{2} + \xi_{n,1,1}\xi_{n,1,2} Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}\right)^2}{\sigma^2_{v'}}}\right). \tag{9}$$

Note that there are much more possible manners to generalize the form. For example, we could have used $\xi_{n,1,1}/\xi_{n,1,2}$ or $(|\xi_{n,1,1}+\xi_{n,1,2}|-1)$ in place of $\xi_{n,1,1}\xi_{n,1,2}$, where $|\cdot|$ denotes the absolute value.

For Case II, we consider only the bit i_2 while ignore the bit i_1. The dashed lines crossing $-2d$ and $2d$ are given as the decision boundaries of the decision region in Fig. 2. Firstly, let us focus on "s1", i.e., $\xi_{n,2,1} = \xi_{n,2,2} = 1$. A bit error will occur if the noise is smaller than $(-2d - {x'}^r_{m,n})$ or exceeds $(2d - {x'}^r_{m,n})$. Therefore, the error probability of the bit i_2 for "s1" can be expressed as

$$P^{i_2}_{m,n}|_{s1} = Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}-2d\right)^2}{\sigma^2_{v'}}}\right) + Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}+2d\right)^2}{\sigma^2_{v'}}}\right). \tag{10}$$

Secondly for "s2", i.e., $\xi_{n,2,1} = -1$ and $\xi_{n,2,2} = 1$, a bit error will occur if the noise is larger than $(-2d - {x'}^r_{m,n})$ and smaller than $(2d - {x'}^r_{m,n})$, resulting in

$$P^{i_2}_{m,n}|_{s2} = 1 - Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}-2d\right)^2}{\sigma^2_{v'}}}\right) - Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}+2d\right)^2}{\sigma^2_{v'}}}\right). \tag{11}$$

Thirdly in a similar manner, we can calculate the bit error probabilities of i_2 for "s3", i.e., $\xi_{n,2,1} = \xi_{n,2,2} = -1$, and "s4", i.e., $\xi_{n,2,1} = 1$ and $\xi_{n,2,2} = -1$, as

$$P^{i_2}_{m,n}|_{s3} = \left|Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}-2d\right)^2}{\sigma^2_{v'}}}\right) - Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}+2d\right)^2}{\sigma^2_{v'}}}\right)\right| \tag{12}$$

and

$$P^{i_2}_{m,n}|_{s4} = 1 - \left|Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}-2d\right)^2}{\sigma^2_{v'}}}\right) - Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}+2d\right)^2}{\sigma^2_{v'}}}\right)\right| \tag{13}$$

respectively. Finally, careful inspection of (10)-(13) reveals a generic expression for the error probability of the bit i_2, which is of a form

$$P^{i_2}_{m,n} = \frac{1-\xi_{n,2,1}\xi_{n,2,2}}{2} + \xi_{n,2,1}\xi_{n,2,2}\left|Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}-2d\right)^2}{\sigma^2_{v'}}}\right) + \xi_{n,2,2} Q\left(\sqrt{\frac{2\left({x'}^r_{m,n}+2d\right)^2}{\sigma^2_{v'}}}\right)\right|. \tag{14}$$

Since the demodulation of two quadrature channels are independent, from symmetry the error probabilities of the bit, q_1, and the bit, q_2, $P^{q_1}_{m,n}$ and $P^{q_2}_{m,n}$, can be calculated similar to (9) and (14) but with the terms ${x'}^r_{m,n}$, $\xi_{n,l,1}$, and $\xi_{n,l,2}$ be substituted by ${x'}^i_{m,n}$, $\zeta_{n,l,1}$, and $\zeta_{n,l,2}$, respectively, where $\zeta_{n,l,1}$ and $\zeta_{n,l,2}$ have similar definitions to $\xi_{n,l,1}$ and $\xi_{n,l,2}$, respectively ($l = 1, 2$), and ${x'}^i_{m,n} = \Im\left[{x'}_{m,n}\right]$. It can be seen from (9) and (14) that the expression of bit error probability consists of a sum of Gaussian *Q*-Function, with a general form $Q\left(\sqrt{2\rho}\right)$, where ρ depends on SNR, distortion, and decision boundary.

In the case of a flat fading channel, the CFRs for all tones are the same, i.e., $H_k = H$ for all $k = 0, \ldots, N-1$. The error probabilities of bits i_1 and i_2 carried on s^r_n can be obtained by first substituting the common term $\sigma^2_{v'}$ in (9) and (14) with $\sigma^2_{v'}/|H|^2$ and then taking the expectation with respect to $|H|^2$. For example, assume a Rayleigh fading channel in which $|H|^2$ is characterized by exponential distribution. Utilizing the result $\int_0^\infty Q\left(\sqrt{2u\rho}\right)e^{-u}du = \frac{1}{2}\left(1 - \sqrt{\frac{\rho}{1+\rho}}\right)$, the expression for the error probability of each bit for a Rayleigh flat fading channel can be derived by using the form $\frac{1}{2}\left(1 - \sqrt{\frac{\rho}{1+\rho}}\right)$ in place of the form $Q\left(\sqrt{2\rho}\right)$ in an AWGN case. The case for bits q_1 and q_2 is straightforward.

In the case of a frequency-selective fading channel, the conditional output SNR of $\hat{s}^{(0)}_{m,n}$ on the CFRs can be readily expressed from (6) by

$$\gamma_{m,n} = 4E_b\left(\sum_{k=0}^{N-1}\frac{\alpha_{m,k}\sigma^2_{v'}}{|H_k|^2}\right)^{-1} \tag{15}$$

which is the harmonic mean of the SNRs on data tones. Interestingly, this SNR expression has been also encountered in the multi-hop CSI-assisted amplify-and-forward (AF) relaying [15][16]. As demonstrated in [17], it is difficult to derive the analytical BER by adopting the commonly used moment-generating function (MGF) method in [14] since there is so far not closed-form expression for the MGF of the instantaneous SNR as in (15) except for two chi-squared random variables, i.e., $N = 2$ [18]. The authors in [17]-[20] have suggested an alternative but mathematically tractable method based on the characterization of the statistical properties of the equalized noise, i.e., ${v'}_{m,n}$. However, this method is only applicable under the assumption of independence among the CFRs and not straightforward for dependent case. To cope with this problem, we notice that it is inessential to obtain the explicit BER while feasible to use its tight lower bound instead for the comparison of the achievable performance for different initial pilot positions. By adopting the tight upper bound on the output SNR proposed in [21] that is the minimal value among the SNRs on the data tones (and thus irrelevant to the correlation of the CFRs), the lower bounded BER (or say approximated BER) for a frequency-selective fading channel can be readily obtained using the results given in flat fading channel case. The feasibility of this treatment will be verified by the simulation in Section IV.

Up to the present, the error probability of each bit carried

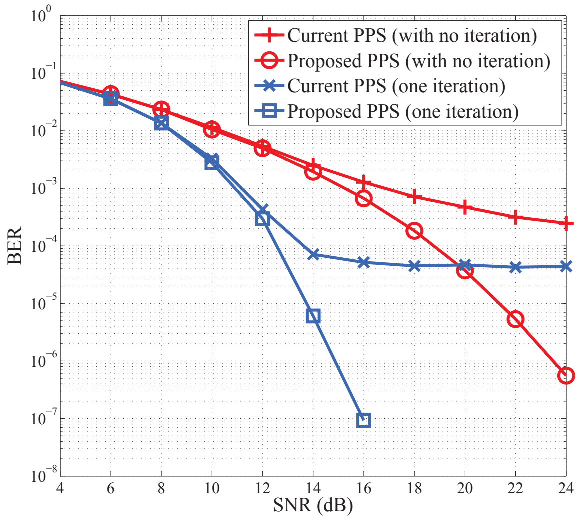

Fig. 3. BER performance comparison between the current PPS and proposed PPS techniques in an AWGN channel.

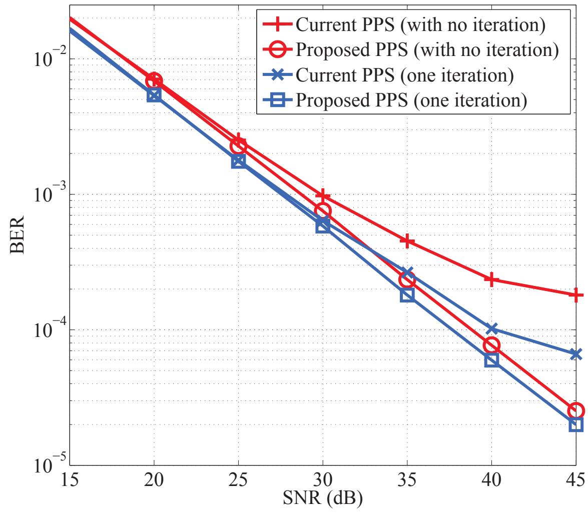

Fig. 4. BER performance comparison between the current PPS and proposed PPS techniques in a Rayleigh flat-fading channel.

on s_n has been derived. Based on this, the average BER in one block due to the removal of the data tones at index ψ_m can be written as

$$P_{m,e} = \frac{1}{4N}\sum_{n=0}^{N-1}\left(P_{m,n}^{i_1} + P_{m,n}^{i_2} + P_{m,n}^{q_1} + P_{m,n}^{q_2}\right). \quad (16)$$

By following steps similar to those given above, the average BER can be easily obtained for higher-order QAM, e.g., 64-QAM, 128-QAM, and so on.

According to current analysis, the optimal initial pilot position should be selected to minimize the average BER, such that we have

$$d_0 = \arg\min_m P_{m,e}. \quad (17)$$

IV. Simulation Results and Discussions

In this section, BER simulations are provided to verify the utility of the proposed PPS technique. An uncoded SC-FDE system with 16-QAM modulation and FDPMT is considered. The transmit block consists of 512 modulated symbols and a guard of length 26. The pilot overhead ratio is chosen as 3.125% ($N_p = 16$). Since we focus on the performance comparison between the current PPS technique [5][8] and the proposed one, the initial pilot position is assumed to be perfectly detected at the receiver. Note that this assumption is reasonable due to the guarantee of the present pilot detection techniques in [8]-[10]. To make a fair comparison, the same values of SNR, which is defined as E_b/σ_v^2, have been used.

Fig. 3 depicts the BER performance of two PPS techniques in an AWGN channel. At low SNRs, the BER is almost determined by noise and in consequence the superiority of the proposed PPS technique has not emerged. However, with the increase of SNR, the effect of the deviation of the constellation caused by the distortion becomes dominant; the performance of the proposed PPS technique is as expected overwhelming that of the current one. In addition, for a higher SNR where the noise effect can be ignored, the error probabilities of bits, i_2 and q_2, block the decrease of BER because of the current PPS criterion, and therefore result in an error floor. Fortunately, this error floor is completely removed by the use of the proposed PPS technique.

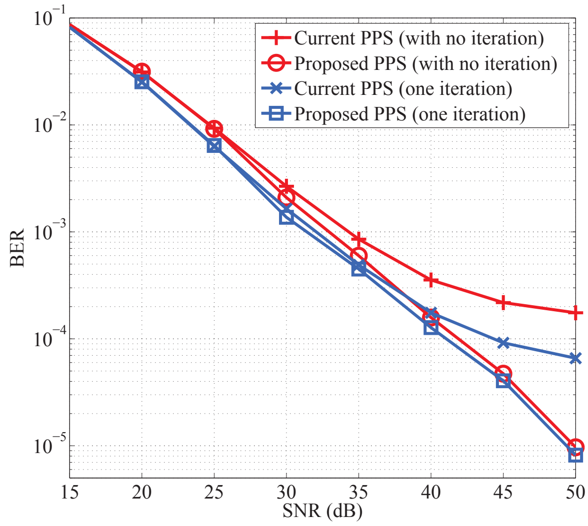

Fig. 5. BER performance comparison between the current PPS and proposed PPS techniques in a frequency-selective Rayleigh fading channel.

Fig. 4 shows the BER performance of two PPS techniques in a Rayleigh flat-fading channel, where perfect CSI is assumed to be available at the receiver. As can be seen, by jointly considering the effect of channel in selecting pilot position, the proposed PPS technique increases the reliability of the initial estimates compared with the current one, and thus improves the overall system performance.

Fig. 5 illustrates the BER performance of two PPS techniques in a frequency-selective fading channel which is modeled by a tapped-delay line filter. In the simulation, CFRs are estimated through the superimposed pilots (*Chu* sequences) and trigonometric interpolation. An exponential power delay profile is used with a root-mean-square (RMS) delay spread of 10 taps, where each tap follows a Rayleigh distribution. It can be observed from Fig. 5 that except for larger BER degradation compared with the flat fading channel case, the performance of the proposed PPS technique is still superior to that of the current one, which verifies the feasibility of the use of the lower bound on average BER in selecting the initial pilot position for frequency-selective fading channels. Actually, since the lower bound is tight enough that the approximated average BER can be roughly regarded to be equal to the exact average BER multiplied by a constant scale factor, the relative size among the approximated average BERs for different initial pilot positions would be the same as that among the exact average BERs. Therefore, based on the proposed PPS criterion which only cares about the minimal value of the average BERs, an accurate selection of the initial pilot position from the comparison of the approximated BERs is still guaranteed.

It can be observed from Figs. 3-5 that for different channel conditions, i.e., AWGN channels, flat fading channels, and frequency-selective fading channels, the proposed PPS technique with no iteration always outperforms the current one with one iteration at high SNRs. This implies that the complexity of receiver can be reduced by adopting the proposed PPS technique since one iteration additionally occupies $O(N \log N)$ computation time. On the other hand, by taking the flat fading channel case as an example, it can be readily found that the extra majority of computation in selecting pilot position at the transmitter for the proposed PPS technique with respect to the current one is the construction of $\{\mathbf{x}'_m\}_{m=0}^{M-1}$, which also requires about $O(N \log N)$ computation time. Therefore, for higher SNRs the proposed PPS technique would express similar overall system complexity to the current one while still preserves the performance gain.

V. Conclusions

In this paper, we have verified the suboptimal property of the conventional PPS technique and consequently proposed an optimal PPS technique for a SC-FDE system with the FDPMT when the ZF equalization is adopted. The proposed PPS technique is based on the minimization of the average BER whose closed-form expression has been derived. Simulation results have shown that the proposed PPS technique outperforms the conventional one in terms of the BER performance while having similar overall system complexity. By following the same principle, the proposed PPS technique can be readily applied to a scenario when the MMSE equalization is adopted.

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References

[1] D. Falconer, S. L. Ariyavisitakul, A. B. Seeyar, and B. Edison, "Frequency domain equalization for single-carrier broadband wireless systems," *IEEE Commun. Mag.*, vol. 40, no. 4, pp. 58–66, Apr. 2002.

[2] Y. Li, "Pilot-symbol-aided channel estimation for OFDM in wireless systems," *IEEE Trans. Veh. Technol.*, vol. 49, no. 4, pp. 1207–1215, July 2000.

[3] C. T. Lam, D. Falconer, F. Dan-lemoine, and R. Dinis, "Channel estimation for SC-FDE systems using frequency domain multiplexed pilots," in *Proc. IEEE VTC'2006*, pp. 1–5, Sept. 2006.

[4] R. Dinis, C. T. Lam, and D. Falconer, "Joint frequency-domain equalization and channel estimation using superimposed pilots," in *Proc. IEEE WCNC'2008*, pp. 447–452, Apr. 2008.

[5] U. K. Kwon, D. Kim, and G. H. Im, "Frequency domain pilot multiplexing technique for channel estimation of SC-FDE," *Electron. Lett.*, vol. 44, no. 5, pp. 364–365, Feb. 2008.

[6] D. Kim, U. K. Kwon, G. H. Im, and C. Shin, "A pilot design technique for single-carrier transmission over fast fading relay channels," in *Proc. IEEE GLOBECOM'2008*, pp. 1–5, Dec. 2008.

[7] D. Y. Seol, U. K. Kwon, and G. H. Im, "Performance of single carrier transmission with cooperative diversity over fast fading channels," *IEEE Trans. Commun.*, vol. 57, no. 9, pp. 2799–2807, Sept. 2009.

[8] D. Kim, U. K. Kwon, and G. H. Im, "Pilot position selection and detection for channel estimation of SC-FDE," *IEEE Commun. Lett.*, vol. 12, no. 5, pp. 350–352, May 2008.

[9] B. Sheng, X. Wang, X. You, and L. Chen, "A simple pilot position detection technique for channel estimation of SC-FDE," *IEEE Commun. Lett.*, vol. 14, no. 5, pp. 420–422, May 2010.

[10] L. Lu, H. C. Wu, and H. Jiang, "Novel robust pilot detection method for channel estimation in single-carrier frequency-domain equalization systems," in *Proc. IEEE CMC'2011*, pp. 437–440, Apr. 2011.

[11] W. C. Huang, C. P. Li, and H. J. Li, "On the power allocation and system capacity of OFDM systems using superimposed training schemes," *IEEE Trans. Veh. Technol.*, vol. 58, no. 4, pp. 1731–1740, May 2009.

[12] Y. Mostofi, "ICI mitigation and time synchronization for pilot-aided OFDM mobile systems," Ph.D. thesis, Stanford University, Nov. 2003.

[13] U. K. Kwon, D. Kim, and G. H. Im, "Amplitude clipping and iterative reconstruction of MIMO-OFDM signals with optimum equalization," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 268–277, Jan. 2009.

[14] M. K. Simon and M. -S. Alouini, *Digital communication over fading channels*, 2nd ed. New York: Wiley, 2004.

[15] N. C. Beaulieu, G. Farhadi, and Y. Chen, "A precise approximation for performance evaluation of amplify-and-forward multihop relaying systems," *IEEE Trans. Wireless Commun.* vol. 10, no. 12, pp. 3985–3989, Dec. 2011.

[16] M. Wen, X. Cheng, A. Huang, and B. Jiao, "Asymptotic performance analysis of multihop relaying with co-channel interference in Nakagami-m fading channels," *IEEE Commun. Lett.*, vol. 16, no. 9, pp. 1450–1453, Sept. 2012.

[17] J. J. Sánchez-Sánchez, M. C. Aguayo-Torres, and U. Fernández-Plazaola, "BER analysis for zero-forcing SC-FDMA over Nakagami-m fading channels," *IEEE Trans. Veh. Technol.*, vol. 60, no. 8, pp. 4077–4081, Oct. 2011.

[18] M. O. Hasna and M. -S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126–1131, Nov. 2003.

[19] M. Wen, X. Cheng, M. Wang, B. Ai, and B. Jiao, "Error probability analysis of interleaved SC-FDMA systems over Nakagami-m frequency-selective fading channels," *IEEE Trans. Veh. Technol.*, vol. 62, no. 2, pp. 748–761, Feb. 2013.

[20] M. Wen, X. Cheng, X. Yin, L. Yang, and B. Jiao, "A general framework for BER analysis of OFDMA and zero-forcing interleaved SC-FDMA over multipath Nakagami-m fading channels with arbitrary m," *IEEE Wireless Commun. Lett.*, vol. 2, no. 4, pp. 395–398, Aug. 2013.

[21] M. O. Hasna, "Average BER of multihop communication systems over fading channels," in *Proc. 10th IEEE International Conf. Electron., Circuits Syst.*, vol. 2, Dec. 2003, pp. 723–726.