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A Novel Iterative Method for Turbo Equalization

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***Abstract*—A number of turbo equalization (TE) methods with reduced complexity have recently been introduced, in which the maximum a posteriori probability (MAP) equalizer is replaced by suboptimal and low complexity ones. These methods can save the computational complexity significantly, but with sacrificed system performance. In this paper, we first use the extrinsic information transfer (EXIT) chart tool to visually explain why these suboptimal methods have the worse performance. Then, we propose a novel iterative method for TE which takes advantage of both parallel and serial concatenation turbo-like schemes. In the novel iterative method, the EXIT chart switches between the two schemes as a balance. The transmitter side is also changed correspondingly by utilizing the concept of repetition code to acquire a structure similar to turbo codes. It is shown from both analytical and simulation results that the proposed iterative method for TE results in the excellent system performance while its realization complexity is kept relatively low.**

I. INTRODUCTION

Inter-symbol interference (ISI) channels are often encountered in high speed wireless communication systems. To combat the effects of ISI, coding and equalization are the two widely employed schemes. In this paper, we consider the problem of coded data transmission over a channel that is subject to ISI. For simplicity, the transmitter can be considered as a serial concatenation of a channel encoder and an ISI channel. A typical implementation of the receiver for such a system is to iteratively equalize and decode the received symbols, resulting in the so-called turbo equalizer pioneered by Douillard et al. [1]. The turbo equalizer in [1] consists of a soft-in-soft-out (SISO) channel equalizer and a MAP decoder. It has been shown in [3] that a turbo equalizer based on the MAP criterion (TE-MAP) outperforms all other turbo equalizers both in convergence rate and bit error rate (BER). Unfortunately, for channels with long impulse responses or signal constellations with higher orders, the TE-MAP quickly becomes unfeasible. A low-cost alternative based on the linear equalizer and decision feedback equalizer (DFE) were proposed in [2], [3]. The parameters of these equalizer filters can be designed according to a variety of optimization criteria, such as the zero forcing (ZF) or linear minimum mean-squared error (LMMSE) criterion [2], [3]. Compared to a TE-MAP, a turbo equalizer based on the LMMSE criterion (TE-LMMSE) [4] has less computational complexity but worse performance, especially at low signal-to-noise ratios (SNRs) and in early iterations.

In this paper, our aim is to find a novel TE method which has a good compromise between the realization complexity and the system performance. First, we use the EXIT chart to visually explain the reason of the performance gap between the TE-MAP and the TE-LMMSE. Then, inspired by the EXIT chart of turbo codes, at the transmitter side we utilize the concept of repetition code [9] to transmit the same coded bit twice in parallel. Note that an interleaver is used for one of the two transmitted code bits. At the receiver side, a novel iterative method for TE is proposed, which combines a parallel concatenation turbo-like equalizer (P-TE) and a serial concatenation turbo-like equalizer (S-TE). In the proposed novel iterative method, we use the EXIT chart to investigate the possibility of matching different schemes (P-TE and S-TE) to different ranges of the iteration process. Utilizing LMMSE equalizer and selecting suitable iterative schemes for every iteration and different SNR according to their EXIT charts in the novel iterative method can not only save computational complexity tremendously but also yield the system performance very close to the S-TE-MAP.

II. TRUBO EQUALIZATION (TE)

The idea behind turbo equalization comes from turbo decoding, in which the received signal contains two-dimensional redundancy in the form of two recursive systematic convolutional (RSC) codes separated by an interleaver. Decoding is accomplished via an iterative process in which extrinsic information is fed back and forth between the two RSC channel decoders.

Fig. 1 illustrates a coded data transmission system with a traditional S-TE. The transmitter can be considered as a serial concatenation of a single non-recursive and non-systematic convolutional code and an ISI channel. At the receiver, the S-TE consists of an equalizer, a decoder, a deinterleaver, and an interleaver. If the equalizer is based on the MAP criterion [3], the output of the equalizer $\mathbf{L}_E(x_n)$ is the a-posteriori log-likelihood ratio (LLR) value, which equals to the difference between the LLR value of the a-posteriori probabilities (APPs) $P(x_n = x \mid \mathbf{y})$ and the a-priori LLR value, i.e.,

$$L_E(x_n) \overset{\Delta}{=} \ln\frac{P(x_n=+1\mid \mathbf{y})}{P(x_n=-1\mid \mathbf{y})} - \ln\frac{P(x_n=+1)}{P(x_n=-1)}$$

$$= \ln\frac{\sum_{S+}\tilde{\beta}_{n+1}(s)\gamma_{ext,n}(s',s)\tilde{\alpha}_n(s')}{\sum_{S-}\tilde{\beta}_{n+1}(s)\gamma_{ext,n}(s',s)\tilde{\alpha}_n(s')} = \ln\frac{\mathbf{b}_{n+1}^{\mathbf{T}}(\mathbf{U}(+1)\boldsymbol{\Gamma}_{\mathbf{ext,n}})\mathbf{f_n}}{\mathbf{b}_{n+1}^{\mathbf{T}}(\mathbf{U}(-1)\boldsymbol{\Gamma}_{\mathbf{ext,n}})\mathbf{f_n}} \quad (1)$$

where the vectors $\mathbf{f_n}$ (forward recursion) and $\mathbf{b_{n+1}}$ (backward recursion) keep track of the quantities $\tilde{\alpha}_n(s)$ and $\tilde{\beta}_n(s)$, respectively, and the $\boldsymbol{\Gamma}_{\mathbf{ext,n}}$ are extrinsic transition matrices. In

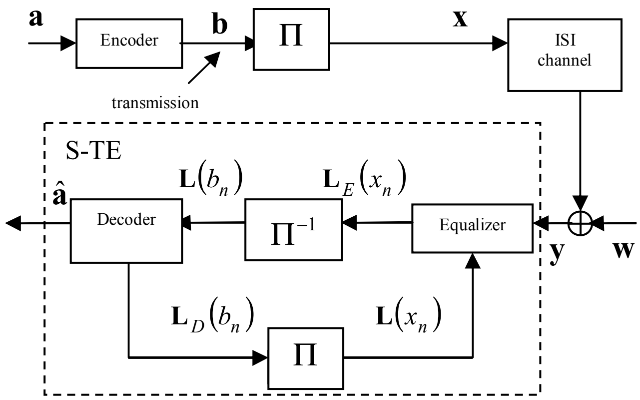

Fig. 1. A coded data transmission system with a S-TE.

(1), the a-priori LLR value $L(x_n)$=ln($P(x_n$=+1)/$P(x_n$=-1)) stands for the prior information on the occurrence probability of x_n and is provided by the decoder output after the interleaver. The MAP decoder computes the APPs $P(b_n=b|\mathbf{L}(b_n))$ and outputs the difference

$$L_D(b_n) \stackrel{\Delta}{=} \ln\frac{P(b_n=0\,|\,\mathbf{L}(b))}{P(b_n=1\,|\,\mathbf{L}(b))} - \ln\frac{P(b_n=0)}{P(b_n=1)} \tag{2}$$

where the equalizer outputs $\mathbf{L}_E(x_n)$ are considered to be the a-priori LLR values $\mathbf{L}(b_n)$ for the decoder. The interleaver $\Pi(\cdot)$ and the de-interleaver $\Pi^{-1}(\cdot)$ provide the correct ordering of the LLRs $\mathbf{L}(x)=\Pi(\mathbf{L}_D(b))$ and $\mathbf{L}(b)=\Pi^{-1}(\mathbf{L}_E(x))$, which are the inputs to the equalizer and decoder, respectively. The MAP decoder also computes the data bit estimates

$$\hat{a}_n \stackrel{\Delta}{=} \arg\max P(a_n = a\,|\,\mathbf{L}(b)) \,. \tag{3}$$

Applying the S-TE, after an initial detection of a block of received symbols, block-wise decoding and equalization operations are performed on the same set of received data. A suitably chosen termination criterion stops the iterative process.

It was shown in [3] that the MAP equalizer is suitable for S-TE in terms of both convergence rate and BER. However, for channels with a long impulse response or signal constellations with a higher order, the MAP equalizer quickly becomes unfeasible. A possible solution is to replace the MAP equalizer with a linear equalizer based on the LMMSE algorithm [2]. The LMMSE equalizer computes estimates $\hat{x}_n$ of the transmitted symbols x_n from the received symbols y_n by minimizing the cost function $E\{|\hat{x}_n - x_n|^2\}$ and outputs $L_E(x_n)$ as follows

$$L_E(x_n) \stackrel{\Delta}{=} \ln\frac{P(x_n=+1\,|\,\hat{x}_n)}{P(x_n=-1\,|\,\hat{x}_n)} - \ln\frac{P(x_n=+1)}{P(x_n=-1)}$$
$$= \frac{2\hat{x}}{1-s_n/(1+(1-\upsilon_n)s_n)} = \frac{2(\mathbf{f_n^T}(\mathbf{y_n}-\mathbf{H_n}\boldsymbol{\mu}_\mathbf{n})+\mu_n s_n)}{1-\upsilon_n s_n} \tag{4}$$

where we assume the probability density function (PDF) $p(e_n)$ of $e_n=\hat{x}_n-x_n$ is Gaussian distributed and $\mathbf{H_n}$ is the $W\times(W+L)$ channel convolution matrix with $W=W_1+W_2+1$ denoting the length of the filter (or sliding window) in the LMMSE equalizer. The vector $\boldsymbol{\mu}_\mathbf{n}$ includes W mean values of the sliding window and the vector $\mathbf{f_n}$ includes coefficients of a length-W FIR filter. The length of the vector $\mathbf{y_n}$ is also W. Note that in (4), the estimate $\hat{x}_n$ is used instead of $\mathbf{y}$.

III. A NOVEL ITERATIVE METHOD FOR TE

From the discussion in the above section, we know that the computational complexity of the S-TE-LMMSE is smaller than that of the S-TE-MAP. The complexity of the S-TE-MAP and S-TE-LMMSE are increased exponentially and linearly, respectively, with the increase of the ISI channel impulse response length and the order of modulation size. However, because of the Gaussian assumption made on the PDF of the estimation error $e_n=\hat{x}_n-x_n$ in the S-TE-LMMSE, the performance of S-TE-LMMSE is worse than that of the S-TE-MAP. On one hand, with the same iteration number, the performance of S-TE-LMMSE is worse than that of the S-TE-MAP. On the other hand, with a large number of iterations, the S-TE-LMMSE can achieve the similar performance to the S-TE-MAP. In this paper, the performance refers to both the BER and convergence behavior. The EXIT chart is a convenient tool to visualize the performance by means of mutual information between transmitted bits and LLR values used within the turbo-like scheme. A comprehensive overview of this method can be found in [10] [11]. In the following, we will briefly discuss the basic principle of the EXIT chart tool.

A. The Basic Principle of the EXIT Chart

Let us start with the equalizer. We assume perfect interleaving. Therefore, the input LLR values $\mathbf{L}(x_n)$ of the equalizer are modeled by independent and identically distributed (i.i.d.) random variables A_{Equ}. The PDF $p(l|x)$ of A_{Equ} conditioned on the transmitted bits x is assumed to be Gaussian distributed with mean $\sigma_A^2/2(1-2k)$ (k=0, 1) and variance σ_A^2. The information content $I_A(Equ)=I(x;A_{Equ})$ between x and A_{Equ} has a one-to-one relationship with σ_A. We can calculate $I_A(Equ)$ numerically [4]:

$$I_A(Equ) = \frac{1}{2}\sum_{k=0,1}\int_{-\infty}^{\infty} p(l\,|\,x=k)\log_2\Big(\frac{2p(l\,|\,x=k)}{p(l\,|\,x=0)+p(l\,|\,x=1)}\Big)dl \,. \tag{5}$$

Similarly, we denote the extrinsic equalizer output $\mathbf{L}_E(x_n)$ as a random variable E_{Equ}. Then, the mutual information $I_E(Equ) \stackrel{\Delta}{=} I(x;E_{Equ})$ between x and E_{Equ} can be determined by observing the PDF of E_{Equ} conditioned on the transmitted bits x. The mutual output information $I_E(Equ)$ as a function of the mutual input information $I_A(Equ)$ is called the EXIT characteristics of the equalizer. Since (5) is a monotonically increasing function, $I_A(Equ)$ as a function of σ_A^2 is invertible. Therefore, artificial LLR values with variance σ_A^2 and mean $\sigma_A^2/2(1-2k)$ may be generated for the given input information $I_A(Equ)$. These LLR values and the received symbol sequence $\mathbf{y}$ are then fed to the equalizer and the output PDF of E_{Equ} conditioned on the transmitted bits can be measured either by means of Monte Carlo simulations (histogram measurements) or the following equation to determine $I_E(Equ)$.

$$I_E(Equ) \approx 1 - \frac{1}{N}\sum_{n=0}^{N-1} \log_2(1+\exp(-x_n L(x_n))) \quad (6)$$

where $x_n = \pm 1$ and the PDF $p(l|x)$ satisfies both the symmetry and the consistency constraint. The above procedure can also be applied to characterize the decoder. We can denote the corresponding mutual input information of the decoder as $I_A(Dec)$ and the output information as $I_E(Dec)$, respectively.

B. Parallel Turbo Equalization (P-TE)

From the EXIT chart of the S-TE, we can find when the interval between the curve of equalization transfer function $I_E(Equ)$ and the decoding transfer function $I_E(Dec)$ is large, the S-TE will have a good convergence behavior. When the intersection between $I_E(Equ)$ and $I_E(Dec)$ is near $I_E(Dec) = 1$, the S-TE will have a good error floor. In general, in the EXIT chart of a turbo-like scheme if the interval between the two transfer functions is large and the intersection between the two transfer functions is near 1 (the transmitted bits are known) the performance of this turbo-like scheme will be very good. We know that the S-TE-LMMSE has comparatively bad BER performance and convergence behavior, which can be explained by the EXIT chart as the small interval between $I_E(Equ)$ and $I_E(Dec)$. Let us think further about the EXIT chart for turbo codes, in which the transfer function of decoder 2 $I_E(Dec2)$ is just the reverse between abscissa and vertical axis of $I_E(Dec1)$. If we can get the $I_E(Equ2)$, which is just the reverse between abscissa and vertical axis of the $I_E(Equ1)$ ($I_E(Equ1) = I_E(Equ)$), in the EXIT chart, we can find the interval between the two transfer functions ($I_E(Equ1)$ and $I_E(Equ2)$) is very large. Then we must construct a turbo-like scheme, which is like the structure of turbo codes, in equalizer. Then we utilize the concept of repetition code to get some useful change at the transmitter, so we can acquire the P-TE which is the same as the structure of turbo codes.

In case of the P-TE, during a retransmission of the same code bits, an interleaver is placed before the input to the ISI channel, as shown in Fig. 2. The transmission through the ISI channel can be considered as a concatenation of convolutional codes. When an interleaver is used in between the two transmissions, it represents a parallel concatenated turbo-like scheme where the ISI channels are the component codes. The two transmissions are combined using turbo decoder that iteratively forms estimates for the transmitted code bits $\mathbf{b}$ based on the received $\mathbf{y}_1$ and $\mathbf{y}_2$. The receiver consists of two component SISO equalizers.

For an easy understanding, we use one EXIT chart to express the convergence behavior of both the S-TE and P-TE, as shown in Fig. 4 and Fig. 5. In order to put them easily in one EXIT chart, we set the equalizer as the start of iteration. Here, we set Equalizer1 as the start of iteration in the S-TE and Equalizer2 in the P-TE. Using this sort of EXIT chart we can find some interesting results. First, the P-TE has a very good convergence behavior but also a very bad error floor. The S-TE, especially the S-TE-LMMSE, has a bad convergence behavior but a very low error floor compared to the P-TE. It follows that we should try to utilize the advantages and avoid disadvantages of both the S-TE and P-TE schemes. At the first iteration, we can use the P-TE to get the very good mutual information $I_E(Dec)$ and then based on the good mutual information offered by the P-TE, we can use the S-TE to get the final mutual information $I_E(Dec)$, which is very close to 1 only after a few iterations.

C. A Novel Iterative Method

Based on the above discussion, we incorporate the P-TE into the S-TE to construct a novel iterative method, as illustrated in Fig. 3. The first transmission goes through a traditional S-TE scheme, which can be considered as a serial concatenated turbo-like scheme. At the transmitted side, there is an interleaver between the channel encoder and the ISI channel. The retransmission from the channel encoder directly transmits to the other ISI channel. At the received side, we use the P-TE, which is used as a parallel concatenated turbo-like scheme, to combine the received sequence from the first transmission and retransmission. Note that in the proposed novel iterative method, the interleaver between the channel encoder and ISI channel in the first transmission is used again in both the S-TE and P-TE, to construct the serial and parallel concatenation schemes, respectively.

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, we consider a system employing a rate 1/2 systematic convolutional code with generator polynomials $[7\ 5]_8$. The information block length is 512 bits. A S-random (S=16) interleaver is used before mapping the coded bits into the BPSK and 8PSK signal constellations. We employ tap coefficients [0.407 0.815 0.407] [7] ($M=3$) and W=11 for the BPSK modulation. For the 8PSK modulation, we use [0.227 0.46 0.688 0.46 0.227] ($M=5$) [7] as tap coefficients and W=15. It is also assumed that the receiver has the exact knowledge of the channel fading coefficients.

Fig. 4 and Fig. 5 depict the EXIT charts of the novel iterative method for the BPSK modulation (M=3) and 8PSK modulation

(M=5), respectively. We also include the trajectory (measurement of the mutual information after each equalization and decoding (or equalization) step) of an actual system using data bit blocks of length 10^5. We can observe that the S-TE-MAP results in better BER performance than the S-TE-LMMSE with the same number of iterations. On the other hand, with more numbers of iterations, the S-TE-LMMSE can reach the same BER performance as the S-TE-MAP. We can also observe that the P-TE yields a significant error floor, but the S-TE-LMMSE can get the excellent BER performance after convergence. Unfortunately, the S-TE-LMMSE exhibits a poor starting behavior, which makes it unsuitable to be used in the first iteration. However, by combining the P-TE and the S-TE-LMMSE, after convergence we can achieve the performance as well as S-TE-MAP. In the first iteration we use the P-TE which needs the shorter trajectory to reach the intersection between the two equalization transfer functions (Intersection_1). Then, by increasing the number of iterations we change to utilize the S-TE-LMMSE to hop over the restriction of Intersection_1 and reach the intersection between the equalization and decoding transfer function (Intersection_2), which is nearer $I_E(Dec)$ =1 than Intersection_1.

Figs. 6 and 7 show the comparison of the BER performance of the novel iterative strategy based on the LMMSE criterion (NI-LMMSE), the S-TE-LMMSE, and the S-TE-MAP. The performance in an AWGN channel without any ISI and the performance in an ISI-free channel after 8 demapping-decoding iterations (BICM) are also included as reference schemes. The "it" is the abbreviation for iteration. It is clear that the performance of the NI-LMMSE is very close to the S-TE-MAP and much better than the S-TE-LMMSE.

Table 1 compares the number of multiplications and additions per symbol per iteration of the S-TE-MAP and the NI-LMMSE, assuming complex channel coefficients [3]. Because the MAP decoder is used in both schemes, the scheme complexity is solely determined by the equalizer. For channels with long impulse responses or signal constellations with higher orders, the NI-LMMSE requires less additions and multiplications than the S-TE-MAP. From Fig. 6, Fig. 7, and Table 1, we can see that the proposed novel iterative method not only provides better BER performance but also requires less complexity.

V. CONCLUSIONS

Based on the EXIT chart tool and inspired by the structure of turbo codes, we utilize the concept of repetition code at the transmitter in order to incorporate the P-TE into the S-TE at the receiver. This means that the proposed novel iterative method is based on the combination of the parallel and serial concatenation turbo-like schemes. Simulation results demonstrate that the proposed iterative method results in the excellent performance in terms of both the convergence behavior and the BER, while requing considerably reduced complexity.

The proposed iterative method is more appropriate for a Multiple Input Single Output (MISO) system. Also, we can use any other suboptimal approaches, such as DFE or Matched Filtering (MF), to replace the linear equalizer in the proposed iterative method. Different approaches used in the novel method will result in different performance. The performance comparison of different TE approaches in the novel iterative method will be our future work.

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TABLE I

A COST COMPARISON OF TWO SCHEMES. (UNIT: OPERATION NUMBERS PER SYMBOL PER ITERATION.)

Scheme	Multiplications	Additions
S-TE-MAP	$3\cdot 2^{mM}+2m2^{m(M-1)}+P$ (122880+P)	$3\cdot 2^{mM}+2(m-1)2^{m(M-1)}+Q$ (114688+Q)
NI-LMMSE	$32W^2+8M^2+20M-8W-8+P$ (7380+P)	$16W^2+4M^2-20W+4M+8+Q$ (3428+Q)

M: channel impulse response length; W: equalizer filter length; 2^m: order of the signal constellation; P/Q: the numbers of multiplications/additions for the MAP decoder (The numbers within the brackets are for the special case with W=15, m=3, M=5, and 8PSK modulation.)

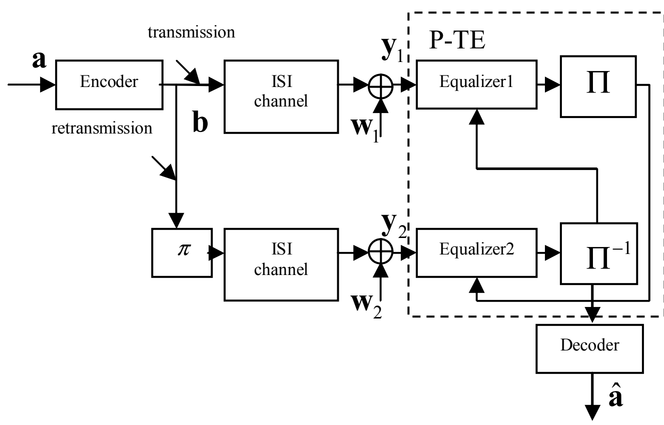

Fig. 2. A coded data transmission system with a P-TE.

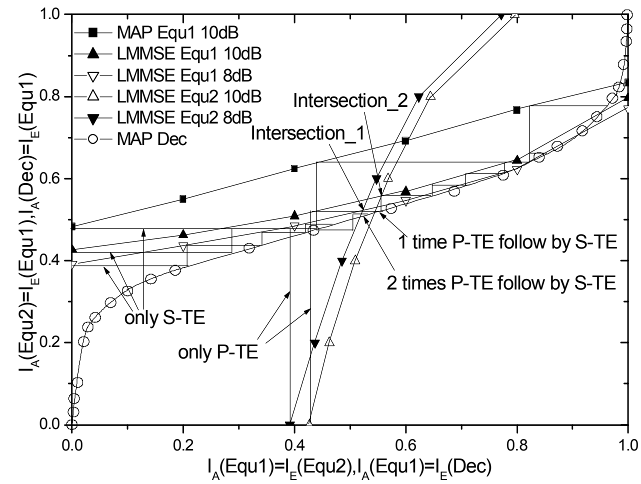

Fig. 5. The EXIT chart of the novel iterative method for the 8PSK modulation (M=5).

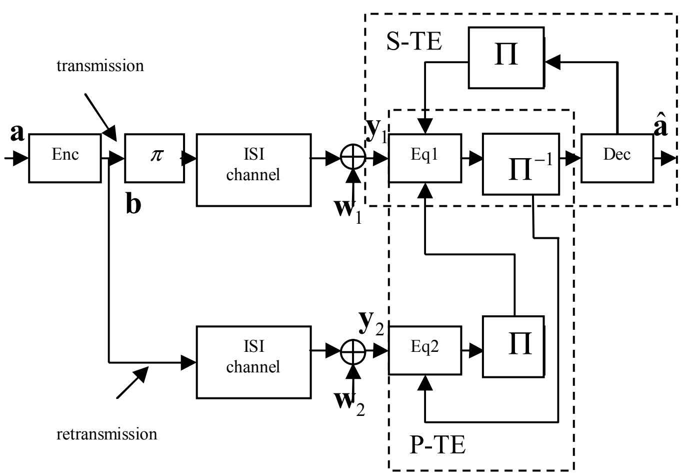

Fig. 3. A coded data transmission system with a novel iterative method for TE.

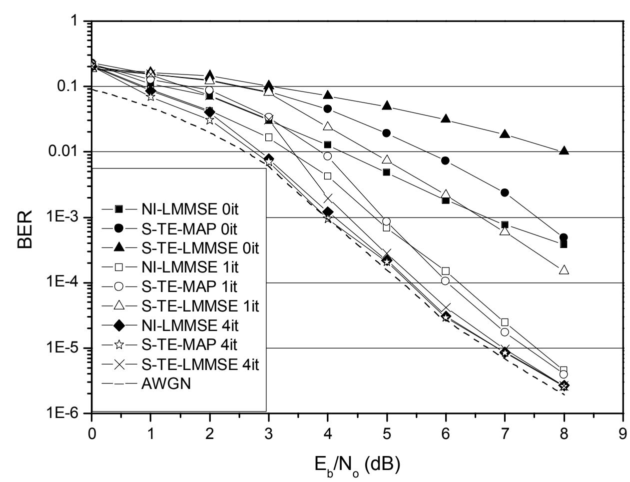

Fig. 6. Comparison of the BER performance between NI-LMMSE, S-TE-MAP and S-TE-LMMSE for the BPSK modulation and the ISI channel $M=3$.

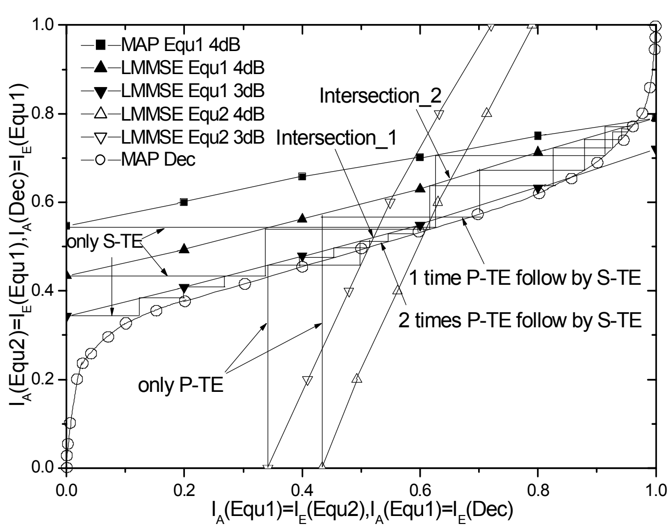

Fig. 4. The EXIT chart of the novel iterative method for the BPSK modulation M=3).

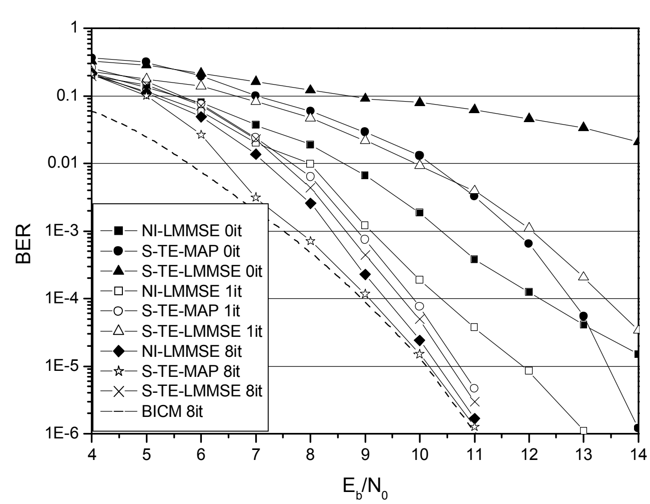

Fig. 7. Comparisons of the BER performance of NI-LMMSE, S-TE-MAP, and S-TE-LMMSE for the 8PSK modulation and the ISI channel $M=5$.