

A Dual-Directional Path-loss Model in 5G Wireless Fractal Small Cell Networks

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Abstract—With the anticipated increase in the number of low power base stations (BSs) deployed in small cell networks, blockage effects becoming more sensitive on wireless transmissions over high spectrums, variable propagation fading scenarios make it hard to describe coverage of small cell networks. In this paper, we propose a dual-directional path loss model cooperating with Line-of-Sight (LoS) and Non-Line-of-Sight (NLoS) transmissions for the fifth generation (5G) fractal small cell networks. Based on the proposed path loss model, a LoS transmission probability is derived as a function of the coordinate azimuth of the BS and the distance between the mobile user (MU) and the BS. Moreover, the coverage probability and the average achievable rate are analyzed for 5G fractal small cell networks. Numerical results imply that the minimum intensity of blockages and the maximum intensity of BSs can not guarantee the maximum average achievable rate in 5G fractal small cell networks. Our results explore the relationship between the anisotropic path loss fading and the small cell coverage in 5G fractal small cell networks.

I. INTRODUCTION

It is now widely accepted that the fifth generation (5G) cellular networks will have significant changes that include low-power base stations (BSs) with dense deployments, higher spectrums, massive bandwidths and large numbers of antennas [1]–[4]. With the anticipated increase in the number of low-power BSs deployed in small cell networks, variable propagation losses make it hard to predict coverage of small cell networks. Moreover, blockage effects become more sensitive for wireless communication systems with higher spectrums [5] such that the user association and the system performance become more serious. In general, blockage effect is contained by a shadowing fading modeled as a log-normal distributed random variable with a uniform path loss exponent. However, different path loss exponents of special environments [6], [7] and distance-dependence of blockage effects [8] are ignored in the traditional performance analysis of cellular networks.

Due to the deployment of a variety of low-power BSs adopting millimeter wave (mmWave) technologies, the coverage of small cell networks is less regular. In 5G small cell networks, wireless transmissions are often divided into Line-of-Sight (LoS) and Non-Line-of-Sight (NLoS) wireless transmissions. Up to now, a simplistic path loss model with a fixed path loss exponent in the entire plane is widely considered in previous literatures [9] [10]. These studies focused on random

heterogeneous cellular networks, but the difference of LoS and NLoS wireless transmissions is ignored in cellular networks. The differences between LoS and NLoS wireless transmissions were investigated by path loss models in [8] [11]–[13]. A path loss model incorporated blockage effects was proposed in [8], which matched experimental trends that the distribution of the number of blockages in a link was proved to be a Poisson random variable with parameters depending on the length of links. Based on the results in [8], different LoS and NLoS path loss laws with distance dependent LOS probability functions were used to analyze the mmWave performance of coverages and transmission rates [11]. A multi-slope path loss model was proposed in [12], where different distance ranges were subject to different path loss exponents. The impact of a sophisticated piece-wise path loss model with probabilistic LoS and NLoS transmissions on the performance of dense small cell networks was investigated in [13], where the LoS probability was assumed as a linear function with the distance between a mobile user (MU) and a BS.

It was proved that the practical coverage of cellular networks has the fractal characteristic due to anisotropic path loss exponents [14]. Measurement results of path loss exponent in [15] indicated that the path loss exponent varies with the angles of the receiver relative to the BS. In fact, the probability of LoS transmission in a link is associated with the distance of the link and the direction of the link relative to the BS. In this paper, based on the fractal characteristics of 5G wireless small cell networks, a dual-directional path loss model is proposed in which the probability of LoS transmission is derived by the coordinate azimuth of a BS and the distance between a MU and the BS. Moreover, all studies in [8] [11]–[13] assumed that a user is associated with the nearest BS, or the nearest visible BS. Considering the random blockage effect and the large number of low-power BSs, the channel state of the invisible BS is better than the state of the visible BS if the distance of an invisible BS is shorter than the nearest visible BS. Based on the analysis result of channel states, a coverage probability and an average achievable rate are derived for 5G fractal small cell networks accounting for the proposed dual-directional path loss model. The main contributions of this paper are as follows:

- 1) A dual-directional path loss model incorporating both LoS and NLoS transmissions is proposed for 5G wireless fractal small cell networks.
- 2) Based on the proposed dual-directional path loss model, the coverage probability and the average achievable rate are derived for 5G wireless fractal small cell networks.
- 3) Numerical results indicate that the average achievable rate increases with the increase of the intensity of BSs and the decrease of the intensity of blockages, respectively. However, the minimum intensity of blockages and the maximum intensity of BSs can not guarantee to achieve the maximum average achievable rate in 5G fractal small cell networks.

The remainder of this paper is structured as follows. Section II describes the system model. The coverage probability and the average achievable rate are presented in Section III. The numerical results are discussed in Section IV. Finally, the conclusions are drawn in Section VII.

II. SYSTEM MODEL

In this paper, BSs and rectangle blockages are assumed to be independently and randomly deployed in a plane. Moreover, the main assumptions of 5G fractal small cell networks are summarized as follows.

Assumption 1: BSs are assumed to be deployed in the infinite plane \mathbb{R}^2 according to a homogenous Poisson point process (PPP) Φ_B of intensity λ_B , which is denoted as

$$\Phi_B = \{x_i, i = 1, 2, 3, \dots\}, \quad (1)$$

where x_i is a two dimensional Cartesian coordinate, denoting the location of BS_i . x_i is further extended as a Polar coordinate, i.e., (r_i, θ_i) , where r_i is the distance between BS_i and the coordinate origin, and satisfies $r_1 < r_2 < r_3 < \dots < r_{i-1} < r_i < \dots$, θ_i is the coordinate azimuth of BS_i .

Assumption 2: Blockages are assumed to form a Boolean scheme of rectangles [8]. The centers of the rectangles form a homogeneous PPP Φ_C of intensity λ_C , which is denoted as

$$\Phi_C = \{y_j, j = 1, 2, 3, \dots\}, \quad (2)$$

where y_j denotes the location of the center of a rectangle in a two dimensional Cartesian coordinate. The length l_j and width w_j of a rectangle are independent of each other. The coordinate azimuth of a rectangle is denoted as φ_j and is uniformly distributed in $[0, 2\pi)$. To simplify calculations, the length l_j , width w_j and coordinate azimuth φ_j are assumed to be constants l , w and φ , respectively. Hence, in this paper the rectangle blockages can be modeled by marked Poisson point process $\Phi_C \times \{l, w, \varphi\}$.

Assumption 3: To evaluate the received signal at a typical MU, in this paper the MU is assumed to be located at the coordinate origin. Channel models of BSs are assumed to be governed by independent identically distributed (i.i.d.) Rayleigh fading models. Furthermore, the received signal power P_i at the typical MU from BS_i can be expressed as

$$P_i = P_T L_i h_i, \quad (3)$$

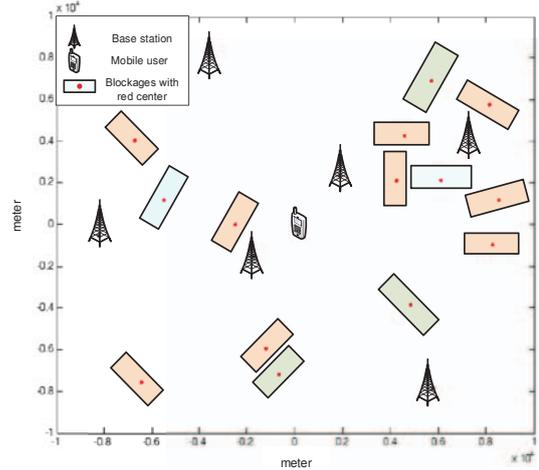


Fig. 1. The system model of 5G wireless fractal small cell networks.

where h_i denotes the Rayleigh fading between the typical MU and BS_i , which is distributed as an exponential distribution with the mean of 1 [16]. For simplicity but without loss of generality, all BSs are assumed to transmit with the same transmission power P_T . L_i denotes the path loss between the typical MU and BS_i . Moreover, the 5G wireless fractal small cell network is assumed to be interference limited and thermal noise is ignored. The system model of 5G wireless fractal small cell networks is illustrated as Fig. 1.

Assumption 4: A MU is assumed to associate with the BS from which the MU receives the maximum signal power [17]. Without loss of generality, the BS associated with the typical MU is denoted as BS_k , where

$$k = \arg \max_{i \in \Phi_B} P_i. \quad (4)$$

The path loss is associated with the distance and direction of the link relative to the BS. The wireless transmission between the typical MU and BS_i is NLoS when blockages are existed in the link, whereas the wireless transmission is LoS. Thus, the dual-directional path loss L_i considering differences between LoS and NLoS wireless transmissions is expressed as

$$L_i(x_i, \Phi_C \times \{l, w, \varphi\}) = \begin{cases} r_i^{-\alpha_{LoS}}, & \text{with } \Pr_i^{LoS}(x_i, \Phi_C \times \{l, w, \varphi\}) \\ r_i^{-\alpha_{NLoS}}, & \text{with } \Pr_i^{NLoS}(x_i, \Phi_C \times \{l, w, \varphi\}) \end{cases}, \quad (5)$$

where α_{LoS} and α_{NLoS} are LoS and NLoS wireless transmission path loss exponents, respectively. And two path loss exponents satisfy $\alpha_{LoS} < \alpha_{NLoS}$. $\Pr_i^{LoS}(x_i, \Phi_C \times \{l, w, \varphi\}) = \Pr_i^{LoS}(x_i)$ and $\Pr_i^{NLoS}(x_i, \Phi_C \times \{l, w, \varphi\}) = \Pr_i^{NLoS}(x_i)$ denote probabilities of LoS and NLoS wireless transmissions between the typical MU and BS_i , respectively. $\Pr\{\cdot\}$ is a probability operation.

III. COVERAGE PROBABILITY AND AVERAGE ACHIEVABLE RATE

In this section, the coverage probability and the average achievable rate of 5G wireless fractal small cell networks are

derived based on dual-directional path loss model.

A. Coverage Probability

The coverage probability of the 5G wireless fractal small cell network is defined as the probability that the received signal-to-interference-ratio (SIR) at the typical MU is larger than a threshold value T . The general expression of SIR at the typical MU associated with BS_k can be expressed as

$$SIR_k = \frac{P_k}{\sum_{\Phi_B, i \neq k} P_i} = \frac{L_k h_k}{\sum_{\Phi_B, i \neq k} L_i h_i} = \frac{L_k h_k}{I_k}, \quad (6)$$

where P_i and h_i ($x_i \in \Phi_B, i \neq k$) denote the received signal power and the channel fading from the interfering BS_i , respectively. L_k is the path loss of the associated BS_k and L_i ($i = 1, 2, \dots, k-1, k+1, \dots$) is the path loss of the interfering BS_i . I_k is the interference aggregated at the typical MU. Furthermore, when the typical MU associated with BS_k , the coverage probability of the 5G wireless fractal small cell network can be derived by

$$\begin{aligned} p_k(T) &= \Pr\{SIR_k > T\} = \mathbb{E}_{I_k, L_k} \left[e^{-\frac{TI_k}{L_k}} \right] \\ &= \int_0^\infty \frac{1}{2\pi} \left(\int_0^{2\pi} \mathcal{L}_{I_k} \left(\frac{T}{r_k^{-\alpha_N}} \right) \Pr_k^{NLoS}(x_k) d\theta_k \right. \\ &\quad \left. + \int_0^{2\pi} \mathcal{L}_{I_k} \left(\frac{T}{r_k^{-\alpha_L}} \right) \Pr_k^{LoS}(x_k) d\theta_k \right) f_r(r_k) dr, \end{aligned} \quad (7)$$

where $\mathcal{L}_{I_k}(s)$ denotes the Laplace transform of I_k evaluated at s , $f_r(r_k)$ is the probability density function (PDF) of the distance r_k between the typical MU and BS_k . $\mathbb{E}[\cdot]$ is an expectation operation.

Before calculating the coverage probability of the 5G wireless fractal small cell network, probabilities of the LoS transmission and the NLoS transmission from BS_i to the typical MU should be considered firstly. The transmission from BS_i is LoS when no blockage is existed in the link between the typical MU and BS_i . Based on the Lemma 2 in [8], the number of blockages N in the link is a Poisson random variable with the mean of

$$\mathbb{E}[N] = \lambda_c (r_i l |\sin(\varphi - \theta_i)| + r_i w |\cos(\varphi - \theta_i)| + wl). \quad (8)$$

Furthermore, the probability of the LoS transmission from BS_i to the typical MU is derived by

$$\begin{aligned} \Pr_i^{LoS}(x_i) &= \Pr\{N = 0\} \\ &= e^{-\lambda_c (r_i l |\sin(\varphi - \theta_i)| + r_i w |\cos(\varphi - \theta_i)| + wl)}. \end{aligned} \quad (9)$$

And the probability of the NLoS transmission is derived by

$$\begin{aligned} \Pr_i^{NLoS}(x_i) &= 1 - \Pr_i^{LoS}(x_i) \\ &= 1 - e^{-\lambda_c (r_i l |\sin(\varphi - \theta_i)| + r_i w |\cos(\varphi - \theta_i)| + wl)}. \end{aligned} \quad (10)$$

The interference I_k aggregated at the typical MU associated with BS_k is further expressed as

$$I_k = \sum_{\Phi_B/x_k} L_i h_i = \sum_{\Phi_B/x_k} r_i^{-\alpha_i} h_i. \quad (11)$$

Based on the dual-directional path loss model and probabilities of the LoS transmission and the NLoS transmission from BS_i to the typical MU, it is straightforward to derive the Laplace transform $\mathcal{L}_{I_k}(s)$ of the aggregated interference I_k as

$$\begin{aligned} \mathcal{L}_{I_k}(s) &= \mathbb{E}_{I_k} [e^{-sI_k}] \\ &= \mathbb{E}_{\Phi_B/x_k, \{\alpha_i\}, \{h_i\}} \left[\exp \left(-s \sum_{\Phi_B/x_k} r_i^{-\alpha_i} h_i \right) \right] \\ &= \mathbb{E}_{\Phi_B/x_k} \left[\prod_{\Phi_B/x_k} \left[\frac{e^{-\lambda_c (r_i l |\sin(\varphi - \theta_i)| + r_i w |\cos(\varphi - \theta_i)| + wl)}}{sr_i^{-\alpha_L + 1}} \right. \right. \\ &\quad \left. \left. + \frac{1 - e^{-\lambda_c (r_i l |\sin(\varphi - \theta_i)| + r_i w |\cos(\varphi - \theta_i)| + wl)}}{sr_i^{-\alpha_N + 1}} \right] \right] \\ &\stackrel{(a)}{=} \exp \left(-\lambda_B \int_0^{2\pi} \int_0^\infty \left(\frac{sr^{-\alpha_N}}{(sr^{-\alpha_N} + 1)} - \frac{(sr^{-\alpha_N} - sr^{-\alpha_L})}{(sr^{-\alpha_L} + 1)} \right) \right. \\ &\quad \left. \times \frac{e^{-\lambda_c (r_l |\sin(\varphi - \theta)| + r_l w |\cos(\varphi - \theta)| + wl)}}{(sr^{-\alpha_N} + 1)} \right) r dr d\theta \Big) \\ &\stackrel{(b)}{\leq} \exp \left(-2\pi \lambda_B \int_{r_k}^\infty \left(\frac{sr^{-\alpha_N}}{(sr^{-\alpha_N} + 1)} - \frac{(sr^{-\alpha_N} - sr^{-\alpha_L})}{(sr^{-\alpha_L} + 1)} \right) \right. \\ &\quad \left. \times \frac{e^{-\lambda_c w l} I_0(\lambda_c r \sqrt{l^2 + w^2})}{(sr^{-\alpha_N} + 1)} \right) r dr \Big), \end{aligned} \quad (12)$$

where the step (a) of (12) is obtained by the probability generating functional (PGFL) of PPP [18] and like terms merger. The step (b) is the upper bound of the Laplace transform $\mathcal{L}_{I_k}(s)$ due to $r_i l |\sin(\varphi - \theta_i)| + r_i w |\cos(\varphi - \theta_i)| + wl \geq r_i l \sin(\varphi - \theta_i) + r_i w \cos(\varphi - \theta_i) + wl$. For simplicity but without loss of generality, φ is assumed to be 0. It can be obtained that $\int_0^{2\pi} e^{-\lambda_c (r_i l \sin(\theta_i) + r_i w \cos(\theta_i) + wl)} d\theta = 2\pi e^{-\lambda_c w l} I_0(\lambda_c r \sqrt{l^2 + w^2})$, where $I_0(\cdot)$ is the modified Bessel function of the first kind. Submitting $s = \frac{T}{r_k^{-\alpha_k}}$ into (12), the coverage probability of the 5G wireless fractal small cell network is derived by

$$\begin{aligned} p_k(T) &= \int_0^\infty \frac{1}{2\pi} \left(\mathcal{L}_{I_k} \left(\frac{T}{r_k^{-\alpha_N}} \right) (2\pi - Q(r_k)) \right. \\ &\quad \left. + \mathcal{L}_{I_k} \left(\frac{T}{r_k^{-\alpha_L}} \right) Q(r_k) \right) f_r(r_k) dr_k, \end{aligned} \quad (13)$$

where $Q(r_k) = \int_0^{2\pi} e^{-\lambda_c (r_k l |\sin(\theta_k)| + r_k w |\cos(\theta_k)| + wl)} d\theta_k$, the PDF of $f_r(r_k)$ [19] is

$$f_r(r_k) = \frac{2(\pi \lambda_B)^k}{(k-1)!} r_k^{2k-1} e^{-\pi \lambda_B r_k^2}. \quad (14)$$

B. Average Achievable Rate

The average achievable rate is another important performance metric in a wireless communication system. It gives a maximum data rate that a cellular network can support on the fixed bandwidth. The average achievable rate normalized by the bandwidth is expressed as

$$R_{avg} = \mathbb{E}[\log_2(1 + SIR)]. \quad (15)$$

According to the Assumption 4, the typical MU is assumed to associate with the BS from which the typical MU can receive the maximum signal power. The selected BS could be

TABLE I
THE SIMPLIFIED USER ASSOCIATION SCHEME.

Input: Received signal powers of the typical MU, ignoring the channel fading.

Output: The selected BS.

For $i = 1$ to ∞ do

If $r_1^{-\alpha_1} > r_2^{-\alpha_2}$, the selected BS is the nearest BS, BS_1 ;

Else $r_2^{-\alpha_2} > r_1^{-\alpha_1} \& r_2^{-\alpha_2} > r_3^{-\alpha_3}$, the selected BS is BS_2 ;

⋮

Else $r_k^{-\alpha_k} = \arg \max \{r_{k+1}^{-\alpha_{k+1}}, r_k^{-\alpha_k}, \dots, r_2^{-\alpha_2}, r_1^{-\alpha_1}\}$
(local maximum value), the selected BS is BS_k ;

End

End

As a result, the distance r_i and the corresponding path loss exponent α_i are used to determine the selected BS.

either the nearest BS_1 , or other BS_i ($i = 2, 3, \dots$) due to the fractal characteristics of 5G wireless small cell networks [14]. Thus, the average achievable rate of the 5G wireless fractal small cell network is expressed as

$$R_{avg} = \mathbb{E}_k [\mathbb{E}_{SIR_k} [\log_2 (1 + SIR_k)]]$$

$$= \sum_{k=1}^{\infty} p_{ass}^k \times \mathbb{E}_{SIR_k} [\log_2 (1 + SIR_k)], \quad (16)$$

where p_{ass}^k is the probability of the typical MU associated with BS_k . Due to the complexity of the user association scheme in the Assumption 4, the user association scheme is simplified by Table I. Based on the simplified user association scheme, the probability of the typical MU associated with BS_k is derived by

$$p_{ass}^k = \int \dots \int \left(\frac{1}{(2\pi)^{k+1}} \left(\prod_{i=1}^{k-1} \Pr_i^{NLoS}(x_i) \right) \right)$$

$$\times \Pr_k^{LoS}(x_k) \Pr \left\{ r_1 > r_k^{\alpha_L/\alpha_N} \right\}$$

$$+ \left(\prod_{i=1}^k \Pr_i^{NLoS}(x_i) \right) \Pr_{k+1}^{NLoS}(x_{k+1})$$

$$+ \left(\prod_{i=1}^k \Pr_i^{NLoS}(x_i) \right) \Pr_{k+1}^{LoS}(x_{k+1}) \Pr \left\{ r_k < r_{k+1}^{\alpha_L/\alpha_N} \right\}$$

$$\times f(r_1, r_2, r_3, \dots, r_k, r_{k+1}) dr_1 \dots dr_{k+1} d\theta_1 \dots d\theta_{k+1} \quad (17)$$

Proof: See Appendix.

Moreover, based on the definition of the coverage probability $p_k(T)$ in (7) which is the complementary cumulative distribution function (CCDF) of SIR_k , the PDF of SIR_k is calculated by

$$f_{SIR_k}(t) = \frac{\partial(1-p_k(t))}{\partial t}$$

$$= - \int_0^{\infty} \frac{1}{2\pi} \left(\frac{\partial \mathcal{L}_{I_k} \left(\frac{t}{r_k^{-\alpha_N}} \right)}{\partial T} (2\pi - Q(r_k)) \right)$$

$$+ \frac{\partial \mathcal{L}_{I_k} \left(\frac{t}{r_k^{-\alpha_L}} \right)}{\partial t} Q(r_k) \right) f_r(r_k) dr_k, \quad (18a)$$

with

$$\frac{\partial \mathcal{L}_{I_k} \left(\frac{t}{r_k^{-\alpha_N}} \right)}{\partial t} \stackrel{s = \frac{t}{r_k^{-\alpha_N}}}{=} \frac{1}{r_k^{-\alpha_N}} \frac{\partial \mathcal{L}_{I_k}(s)}{\partial s}, \quad (18b)$$

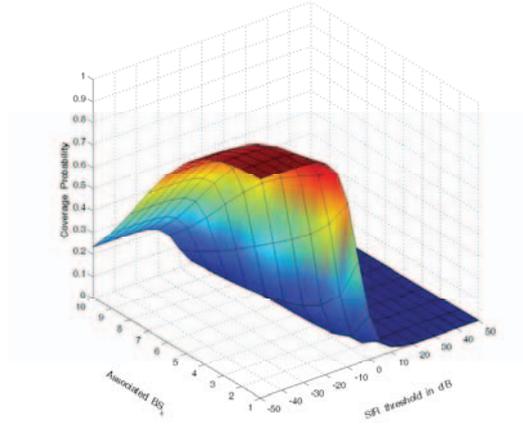


Fig. 2. The coverage probability of the 5G wireless fractal small cell network with respect to the associated BS_k and the SIR threshold T .

$$\frac{\partial \mathcal{L}_{I_k} \left(\frac{t}{r_k^{-\alpha_L}} \right)}{\partial t} \stackrel{s = \frac{t}{r_k^{-\alpha_L}}}{=} \frac{1}{r_k^{-\alpha_L}} \frac{\partial \mathcal{L}_{I_k}(s)}{\partial s}, \quad (18c)$$

$$\frac{\partial \mathcal{L}_{I_k}(s)}{\partial s} = \exp \left(-2\pi\lambda_B \int_{r_k}^{\infty} \left(\frac{sr^{-\alpha_N}}{(sr^{-\alpha_N}+1)} \right) \right.$$

$$\left. - \frac{(sr^{-\alpha_N} - sr^{-\alpha_L}) e^{-\lambda_c w l} I_0(\lambda_c r \sqrt{l^2 + w^2})}{(sr^{-\alpha_L}+1)(sr^{-\alpha_N}+1)} \right) r dr$$

$$\times \left(-2\pi\lambda_B \int_{r_k}^{\infty} \frac{1}{(sr^{-\alpha_N}+1)^2} - \frac{(s^2 r^{-\alpha_N - \alpha_L} - 1)}{(sr^{-\alpha_L}+1)^2} \right)$$

$$\times \left(\frac{r^{-\alpha_L} - r^{-\alpha_N}}{(sr^{-\alpha_N}+1)^2} e^{-\lambda_c w l} I_0(\lambda_c r \sqrt{l^2 + w^2}) \right) r dr. \quad (18d)$$

With the condition that the typical MU associates with BS_k , the conditional average achievable rate is expressed as

$$\mathbb{E}_{SIR_k} [\log_2 (1 + SIR_k)] = \int \log_2 (1 + T) f_{SIR_k}(T) dT$$

$$= - \int_0^{\infty} \frac{1}{2\pi} \int \log_2 (1 + T) \left(\frac{\partial \mathcal{L}_{I_k} \left(\frac{T}{r_k^{-\alpha_N}} \right)}{\partial T} (2\pi - Q(r_k)) \right)$$

$$+ \frac{\partial \mathcal{L}_{I_k} \left(\frac{T}{r_k^{-\alpha_L}} \right)}{\partial T} Q(r_k) \right) dT f_r(r_k) dr_k. \quad (19)$$

As a result, the average achievable rate of the 5G wireless fractal small cell network is obtained by submitting formulas (16), (18), and (19) into (15).

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, numerical results of the coverage probability and the average achievable rate of the 5G wireless fractal small cell network are analyzed. The default parameters are configured as: $\lambda_B = \frac{1}{800^2 \pi}$, $\lambda_C = 0.002$; $l = 15m$, $w = 10m$; $\alpha_L = 2$, $\alpha_N = 5$; $T = -5dB$.

Fig. 2 illustrates the coverage probability of the 5G wireless fractal small cell network with respect to the associated BS_k and the SIR threshold T . When the SIR threshold T is fixed, the coverage probability decreases with the increase of the value of k marking the associated BS. The coverage

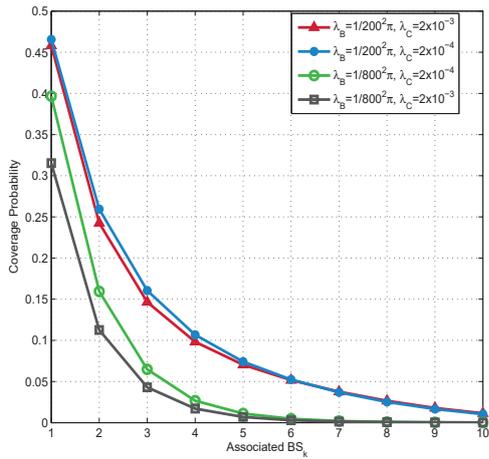


Fig. 3. The coverage probability of the 5G wireless fractal small cell network with respect to the associated BS_k considering different intensities of BSs and blockages.

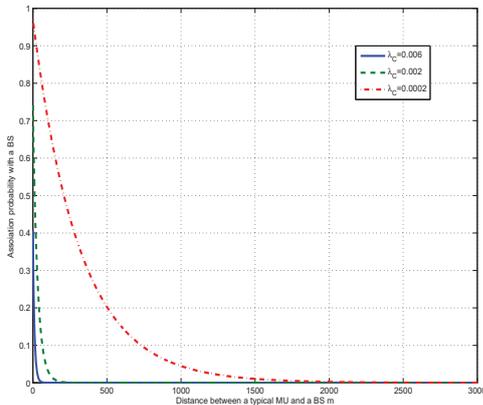


Fig. 4. The probability of LoS transmission with respect to the distance between a BS and a MU considering different intensities of blockages.

probability decreases with the increase of the SIR threshold T when the associated BS is fixed. Moreover, the coverage probability decreases more significantly when the value of k is larger.

Fig. 3 shows the coverage probability of the 5G wireless fractal small cell network with respect to the associated BS_k considering different intensities of BSs and blockages. When the intensity of BSs is fixed, the coverage probability decreases with the increase of the intensity of blockages from 0.0002 to 0.002. When the intensity of blockages is fixed, the coverage probability increases with the increase of the intensity of BSs from $\frac{1}{800^2\pi}$ to $\frac{1}{200^2\pi}$.

Fig. 4 illustrates the probability of LoS transmission with respect to the distance between a BS and a MU considering different intensities of blockages. The probability of the LoS transmission decreases with the increase of the distance between a BS and a MU. The curve of the probability of the

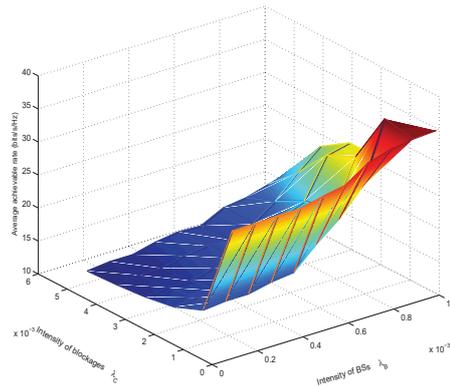


Fig. 5. The average achievable rate of the 5G wireless fractal small cell network with respect to intensities of BSs and blockages.

LoS transmission declines more rapidly when the intensity of blockages is larger.

Fig. 5 illustrates the average achievable rate of the 5G wireless fractal small cell network with respect to intensities of BSs and blockages. The average achievable rate increases with the increase of the intensity of BSs when the intensity of blockages is fixed and larger than 0.001. The average achievable rate decreases with the increase of the intensity of blockages when the intensity of BSs is fixed and smaller than 0.0008. The peak value comes out when the intensity of BSs and blockages are in the intervals $[0.0008, 0.001]$ and $[0.0001, 0.001]$, respectively.

V. CONCLUSIONS

In this paper a dual-directional path loss model cooperating with LoS and NLoS transmissions is proposed for fractal small cell networks where the probability of LoS transmission is determined by the coordinate azimuth of the BS and the distance between a typical random MU and the BS. The coverage probability and the average achievable rate are analyzed based on the proposed path loss model. Numerical results indicate that the average achievable rate increases with increasing the intensity of BSs and blockages, respectively. Moreover, our results imply that the maximum average achievable rate is not achieved with the minimum intensity of blockages and the maximum intensity of BSs. Our results provide some guidelines for deployment of 5G fractal small cell networks.

APPENDIX

Based on the simplified user association scheme shown in Table I, the received signal power of the selected BS has the maximum value. When the selected BS is the nearest BS_1 ,

the conditional probability of the typical MU associated with BS_1 which includes four items is expressed as

$$\begin{aligned}
 & p_{ass}^1(r_1, r_2) \\
 &= \Pr \{r_1^{-\alpha_N} > r_2^{-\alpha_N} | BS_1 \text{ is NLOS}, BS_2 \text{ is NLOS}\} \\
 &\times \Pr \{BS_1 \text{ is NLOS}, BS_2 \text{ is NLOS}\} \\
 &+ \Pr \{r_1^{-\alpha_N} > r_2^{-\alpha_L} | BS_1 \text{ is NLOS}, BS_2 \text{ is LOS}\} \\
 &\times \Pr \{BS_1 \text{ is NLOS}, BS_2 \text{ is LOS}\} \\
 &+ \Pr \{r_1^{-\alpha_L} > r_2^{-\alpha_N} | BS_1 \text{ is LOS}, BS_2 \text{ is NLOS}\} \\
 &\times \Pr \{BS_1 \text{ is LOS}, BS_2 \text{ is NLOS}\} \\
 &+ \Pr \{r_1^{-\alpha_L} > r_2^{-\alpha_L} | BS_1 \text{ is LOS}, BS_2 \text{ is LOS}\} \\
 &\times \Pr \{BS_1 \text{ is LOS}, BS_2 \text{ is LOS}\}, \tag{20}
 \end{aligned}$$

with $r_2 > r_1$, and $\alpha_N > \alpha_L$. Furthermore, the probability p_{ass}^1 of the typical MU associated with BS_1 can be derived by

$$\begin{aligned}
 p_{ass}^1 &= \mathbb{E} [p_{ass}^1(r_1, r_2)] \\
 &= \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \int_0^\infty \int_0^\infty p_{ass}^1(r_1, r_2) f(r_1, r_2) dr_1 dr_2 d\theta_1 d\theta_2, \tag{21a}
 \end{aligned}$$

with

$$\begin{aligned}
 & p_{ass}^1(r_1, r_2) \\
 &= \Pr_1^{NLoS}(x_1) \times \Pr_2^{NLoS}(x_2) \\
 &+ \Pr \left\{ r_1 < r_2^{\alpha_L/\alpha_N} \right\} \Pr_1^{NLoS}(x_1) \times \Pr_2^{LoS}(x_2), \tag{21b} \\
 &+ \Pr_1^{LoS}(x_1)
 \end{aligned}$$

where $f(r_1, r_2)$ is the joint probability density function of r_1 and r_2 . The entire expression of p_{ass}^1 is written as

$$\begin{aligned}
 p_{ass}^1 &= \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \int_0^\infty \int_0^\infty \Pr_1^{NLoS}(x_1) \\
 &\times \Pr_2^{NLoS}(x_2) f(r_1, r_2) dr_1 dr_2 d\theta_1 d\theta_2 \\
 &+ \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \int_0^\infty \int_0^{r_2^{\alpha_L/\alpha_N}} \Pr_1^{NLoS}(x_1) \\
 &\times \Pr_2^{LoS}(x_2) f(r_1, r_2) dr_1 dr_2 d\theta_1 d\theta_2 \\
 &+ \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty \int_0^\infty \Pr_1^{LoS}(x_1) f(r_1, r_2) dr_1 dr_2 d\theta_1 \tag{22}
 \end{aligned}$$

When the selected base station is BS_k , received signal powers of BSs satisfy $r_1^{-\alpha_1} < r_2^{-\alpha_2} < \dots < r_k^{-\alpha_k} > r_{k+1}^{-\alpha_{k+1}}$. The probability of the typical MU associated with BS_k is derived by

$$\begin{aligned}
 p_{ass}^k &= \frac{1}{(2\pi)^{k+1}} \int \dots \int p_{ass}^k(r_1, \dots, r_k, r_{k+1}) \\
 &\times f(r_1, \dots, r_k, r_{k+1}) dr_1 \dots dr_{k+1} d\theta_1 \dots d\theta_{k+1}, \tag{23a}
 \end{aligned}$$

with

$$\begin{aligned}
 & p_{ass}^k(r_1, r_2, r_3, \dots, r_k, r_{k+1}) \\
 &= \left(\prod_{i=1}^{k-1} \Pr_i^{NLoS}(x_i) \right) \Pr_k^{LoS}(x_k) \Pr \left\{ r_1 > r_k^{\alpha_L/\alpha_N} \right\} \\
 &+ \left(\prod_{i=1}^k \Pr_i^{NLoS}(x_i) \right) \Pr_{k+1}^{NLoS}(x_{k+1}) \\
 &+ \left(\prod_{i=1}^k \Pr_i^{NLoS}(x_i) \right) \Pr_{k+1}^{LoS}(x_{k+1}) \Pr \left\{ r_k < r_{k+1}^{\alpha_L/\alpha_N} \right\}. \tag{23b}
 \end{aligned}$$

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