A Novel Kronecker-Based Stochastic Model for Massive MIMO Channels

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***Abstract*—This paper proposes a novel Kronecker-based stochastic model (KBSM) for massive multiple-input multiple-output (MIMO) channels. The proposed KBSM can not only capture antenna correlations but also the evolution of scatterer sets on the *array axis*. With the consideration of the evolution of scatterer sets, the overall correlation matrices of the transmitter and receiver are presented. In addition, upper and lower bounds of MIMO channel capacities in both the high and low signal-to-noise ratio (SNR) regimes are derived when the numbers of transmit and receive antennas are increasing unboundedly with a constant ratio. Furthermore, the evolution of scatterer sets on the array axis is shown to decrease spatial correlations of MIMO channels.**

***Keywords* – Massive MIMO, KBSM, birth-death process, channel capacity bounds, low and high SNR regimes.**

I. Introduction

Recently, massive multiple-input multiple-output (MIMO) systems [1] [2], which are equipped with tens or even hundreds of antennas, have been proposed to satisfy the demand of the fifth generation (5G) wireless communication networks because they are capable of providing enhanced reliability and increased capacity [1]–[3].

It is of crucial importance to develop efficient small-scale fading channel models which are able to capture massive MIMO channel characteristics in order to design and evaluate massive MIMO systems. The classic independently and identically distributed (i.i.d) Rayleigh fading channels were used in [4] to analyze massive MIMO system performance. However, the i.i.d assumption may over-simplify the channel model. Conventional correlation-based stochastic models (CBSMs) are usually utilized to study the performance of MIMO systems with spatial correlation due to their low complexity. The Kronecker-based stochastic model (KBSM) assumes that the correlation matrices of the transmitter and receiver are unrelated [5], [6]. Channel capacities of massive MIMO systems with KBSM were presented in [5] and [7]. Also, a KBSM with line-of-sight (LOS) components was assumed in [8] for capacity analysis of massive MIMO. On the other hand, joint correlation at both the transmitter and receiver was included in the Weichselberger MIMO model [9]. Sum rates of massive MIMO systems with the Weichselberger MIMO model were investigated in [10].

However, it was pointed out in [11] and [12] that all the antennas elements may not observe the same set of scatterers in massive MIMO systems. This property was not included in the abovementioned conventional MIMO channel models. A geometry-based stochastic model based on birth-death (BD) process on the array axis was proposed in [13]. Although this model is able to capture the massive MIMO characteristic of cluster evolution on the array axis mentioned in [11] and [12], it may not be feasible for capacity analysis because of its complexity. Therefore, a novel simple channel model that considers different sets of scatterers for different antennas is necessary for capacity analysis of massive MIMO channels.

The **contributions** of this paper are listed as below:

1) A novel KBSM with BD process on the array axis (KBSM–BD–AA) is proposed for massive MIMO channels. The BD process is abstracted by a survival probability matrix whose rows are values from exponential functions. The survival probability matrix reduces channel correlations.
2) Upper and lower bounds of channel capacities of the proposed KBSM–BD–AA in both the high and low signal-to-noise ratio (SNR) regimes are investigated.

The remainder of this paper is organized as follows. Section II describes the system model as well as the proposed KBSM–BD–AA. Channel capacity analysis of the proposed KBSM–BD–AA is derived in Section III. Results and discussions are presented in Section IV. Conclusions are drawn in Section V.

II. System Model

Let us consider an $M_R \times M_T$ MIMO system. The receiver and transmitter are equipped with M_R and M_T uniform linear antennas (ULAs), respectively. To study the performance of massive MIMO systems, the numbers of receive and transmit antennas are assumed to be unboundedly increasing with a constant ratio, i.e., $M_R, M_T \rightarrow \infty$ and $\gamma = \frac{M_R}{M_T}$. This assumption was widely used in [7], [10], [14], [15]. The MIMO channel can be characterized by an $M_R \times M_T$ complex matrix $\mathbf{H}$. Although only a narrowband channel is discussed in this paper, the same analysis procedure can easily be generalized to wideband channels by applying an orthogonal frequency-division multiplexing (OFDM) system. Then, let $\mathbf{v}$ be the transmitted symbol vector, the received vector $\mathbf{y}$ can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{v} + \mathbf{n} \tag{1}$$

where $\mathbf{n}$ is the zero-mean unit-variance additive white Gaussian noise (AWGN) vector. In addition, we assume that the mean power of the received signal via such an MIMO channel is normalized as $\mathrm{E}\left[\mathrm{trace}\left(\mathbf{H}\mathbf{H}^{\mathrm{H}}\right)\right] = M_R M_T$, where $\mathrm{E}\left[\cdot\right]$ calculates the expected value and $\mathrm{trace}\left(\cdot\right)$ calculates the trace of a matrix.

A. Conventional KBSM

The conventional KBSM assumes that spatial correlation matrices of the receive arrays and transmit arrays are unrelated. Hence, the channel matrix can be expressed as

$$\mathbf{H} = \mathbf{R}_R^{\frac{1}{2}} \mathbf{H}_{\mathrm{w}} \mathbf{R}_T^{\frac{\mathrm{T}}{2}} \tag{2}$$

where $\mathbf{H}_{\mathrm{w}}$ is an $M_R \times M_T$ matrix with zero-mean unit-variance complex i.i.d Gaussian entries, $\mathbf{R}_R$ and $\mathbf{R}_T$ are overall spatial correlation matrices at the receiver and transmitter, respectively. Additionally, if ULAs are deployed at the receiver and transmitter sides, $\mathbf{R}_R$ and $\mathbf{R}_T$ are Toeplitz matrices [7]. To avoid repeated analysis, we only analyze the receiver side in this paper as the analysis of the transmitter side follows the same procedure. Furthermore, let us denote the complex gain between the kth ($k = 1, 2, \cdots$) scatterer and the mth ($m = 1, 2, \cdots, M_R$) antenna as s_{mk}^R, and the complex gain between the kth scatterer and the nth ($n = 1, 2, \cdots, M_R$) antenna as s_{nk}^R. Let $T_{R,mn}$ be the spatial correlation coefficient between the mth and the nth antennas and the entry of matrix $\mathbf{T}_R$ in the mth row and nth column. Then, $T_{R,mn}$ can be computed as

$$T_{R,mn} = \frac{\sum\limits_k s_{mk}^R \left(s_{nk}^R\right)^*}{\sqrt{\sum\limits_k |s_{mk}^R|^2} \sqrt{\sum\limits_k |s_{nk}^R|^2}}. \tag{3}$$

In the conventional KBSM, the above discussion implies that all the antennas share the same set of scatterers. In this case, $\mathbf{R}_R$ is equivalent to $\mathbf{T}_R$, i.e., $\mathbf{T}_R = \mathbf{R}_R$. However, the equivalence between $\mathbf{T}_R$ and $\mathbf{R}_R$ may not hold if antennas do not share the same set of scatterers. This will be studied in later paragraphs.

B. Proposed KBSM–BD–AA

It was reported in [11] and [12] that each antenna on a large antenna array may not observe the same set of scatterers. Scatterers may appear or disappear on the array axis. Examples of scatter set evolution on the array axis are illustrated in Fig. 1. The same set of scatterers are observed by all the antenna elements for conventional MIMO as discussed in Section II-A. On the other hand, for massive MIMO, different antennas may observe different scatters. In Fig. 1, Antenna 1 in the massive MIMO observes Scatterers $1, 2$ and 3, while Antenna k observes Scatterers $1, 4$ and 5. Antennas 1 and k only have Scatterer 1 in common. We refer this effect as the evolution of scatterer sets on the *array axis*. Therefore, the proposed KBSM–BD–AA is developed to characterize this effect for massive MIMO channels.

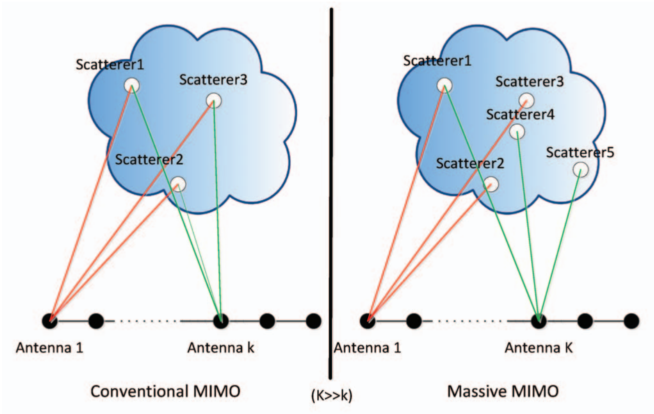

Fig. 1: Diagram of scatterer set evolution on the array axis for massive MIMO.

The BD process was applied to cluster evolution on the time axis in [16] and [17]. In this paper, the BD process is adapted to modeling the evolution of scatterer sets on the array axis. BD process on the array axis consists of two parts. First, scatterers survive during the evolution of the scatterer set on the array axis. Second, new scatterers are generated when the scatterer set evolves. As a result, the scatterers in the scatterer set of the mth antenna may not be the same as those of the nth antenna. The number of scatterers shared by both the mth and nth antennas is determined by the scatterer survival probability. According to the BD process, the survival probability $E_{R,mn}$ of scatterers when they evolve from the mth antenna to the nth antenna can be modeled as an exponential function [18]

$$E_{R,mn} = e^{-\beta|m-n|} \tag{4}$$

where $\beta \geqslant 0$ is a parameter describing how fast a scatterer disappears on the array axis. The value of $E_{R,mn}$ is decreasing if m and n differ more. This implies that less scatterers are shared if two antennas are more separated. Let $T'_{R,mn}$ be the antenna correlation between the mth and nth antennas considering the evolution of scatterer sets on the array axis. Then, $T'_{R,mn}$ can be modeled as

$$T'_{R,mn} = E_{R,mn} \frac{\sum\limits_k s_{mk}^R \left(s_{nk}^R\right)^*}{\sqrt{\sum\limits_k |s_{mk}^R|^2} \sqrt{\sum\limits_k |s_{nk}^R|^2}} = E_{R,mn} T_{R,mn}. \tag{5}$$

It can be observed that the antenna correlation of KBSM–BD–AA for massive MIMO is equal to the antenna correlation of KBSM for conventional MIMO multiplied by a factor of $E_{R,mn}$. This is because only $E_{R,mn}$ of the scatterers for the mth antenna are able to survive to be observed by the nth antenna. At the same time, although there may be new scatterers generated according to the BD process, these newly generated scatterers are uncorrelated to the survived scatterers. Therefore, the newly generated scatterers do not contribute to the antenna correlation.

C. Discussions

Let us now compare the conventional KBSM and the proposed KBSM–BD–AA. All antennas observe the same set of scatterers for conventional KBSM. The decrease of the spatial correlation function (3) of the conventional KBSM is purely determined by the spatial difference between antennas. On the other hand, the spatial correlation function (5) of the proposed KBSM–BD–AA is calculated by not only the spatial difference between antennas but also the scatterer set difference. The latter difference is more significant in massive MIMO when the antenna array is large.

Furthermore, the scatterer evolution on the array axis is memoryless. Namely, the survival probability of scatterers evolving from the nth antenna to the mth antenna (denoted as $\Pr\{n \to m\}$) is equal to that of scatterers evolving from the the nth antenna to the mth antenna via an intermediate vth antenna (denoted as $\Pr\{n \to v \to m\}$, $n \leqslant v \leqslant m$). This can be expressed as

$$\begin{aligned}\Pr\{n \to v \to m\} &= \Pr\{n \to v\}\Pr\{v \to m\} \\ &= e^{-\beta(m-v)}e^{-\beta(m-n)} = e^{-\beta(m-n)} \\ &= \Pr\{n \to m\}. \end{aligned} \quad (6)$$

Let $\mathbf{E}_R = [E_{R,mn}]_{M_R \times M_R}$ $(m, n = 1, 2, \cdots, M_R)$ and $\mathbf{E}_T = [E_{T,pq}]_{M_T \times M_T}$ $(p, q = 1, 2, \cdots, M_T)$ denote the survival probability matrices at the receiver side and transmitter side. The overall antenna correlation matrices $\mathbf{R}_R$ and $\mathbf{R}_T$ can be represented as $\mathbf{R}_R = \mathbf{T}_R \circ \mathbf{E}_R$ and $\mathbf{R}_T = \mathbf{T}_T \circ \mathbf{E}_T$ where $\circ$ denotes the Hadamard product.

III. Channel Capacity Analysis

In this section, channel capacity of the proposed KBSM–BD–AA in both the low and high SNR regimes will be investigated. The normalized channel capacity without channel knowledge at the transmitter side can be computed as [5]

$$C(\rho) = \frac{1}{M_R}\log_2 \det\left(\mathbf{I} + \frac{\rho}{M_T}\mathbf{R}_R^{\frac{1}{2}}\mathbf{H}_{\mathrm{w}}\mathbf{R}_T\mathbf{H}_{\mathrm{w}}^{\mathrm{H}}\mathbf{R}_R^{\frac{\mathrm{H}}{2}}\right) \quad (7)$$

where ρ is designated as SNR and $\mathbf{I}$ is the identity matrix.

For the sake of brevity, the term 'capacity' means the normalized capacity (normalized with respect to bandwidth and the number of receive antennas) unless further clarification is provided.

A. Low SNR Approximation

In the low SNR regime, the channel capacity can be analyzed with the classic Shannon's capacity function. However, it was stated in [14] and [15] that the channel capacity can be sufficiently well approximated in the first order with respect to energy per bit to noise ratio $\frac{E_b}{N_0}$ (decibels) in the low SNR regime. Therefore, the low SNR approximation is formulated as [14], [15]

$$\begin{aligned}\mathcal{C}\left(\frac{E_b}{N_0}\right) &\approx \frac{S_0}{M_R}\log_2\left(\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}_{\min}}\right) \\ &= \frac{S_0}{10M_R\log_{10}2}\left[\frac{E_b}{N_0}(\mathrm{dB}) - \frac{E_b}{N_0}_{\min}(\mathrm{dB})\right] \end{aligned} \quad (8)$$

where $\frac{E_b}{N_0}_{\min} = \frac{\ln 2}{M_R}$ is the minimum energy per bit to noise ratio for reliable communications which decreases as the number of receive antennas increases. The capacity slope S_0 is expressed as [15]

$$S_0 = \frac{2M_TM_R}{M_T\zeta(\mathbf{R}_R) + M_R\zeta(\mathbf{R}_T)} \quad (9)$$

where $\zeta(\cdot)$ is the dispersion of a matrix. For a $n \times n$ positive-definite matrix $\mathbf{\Theta}$, its dispersion can be calculated as $\zeta(\mathbf{\Theta}) = \frac{\mathrm{trace}(\mathbf{\Theta}^2)}{n}$ [15]. It can be easily seen in (8) that the channel capacity $\mathcal{C}\left(\frac{E_b}{N_0}\right)$ is a linear function of energy per bit to noise ratio $\frac{E_b}{N_0}$ (decibels) in the low SNR regime.

1) Capacity Upper Bound ($\rho \to 0$): Since the dispersion of the correlation matrix $\zeta(\mathbf{R}_R)$ is equal to or larger than 1, the slope S_0 is upper bounded by [15]

$$S_0 \leqslant \frac{2M_TM_R}{M_T + M_R}. \quad (10)$$

As a result, the capacity upper bound $\mathcal{C}^{\mathrm{U}}\left(\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}_{\min}}\right)$ in the low SNR regime can be shown as

$$\mathcal{C}\left(\frac{E_b}{N_0}\right) \leqslant \frac{2}{\gamma+1}\log_2\left(\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}_{\min}}\right) = \mathcal{C}^{\mathrm{U}}\left(\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}_{\min}}\right). \quad (11)$$

Equality holds true when the antennas are uncorrelated.

2) Capacity Lower Bound ($\rho \to 0$): Let $E_{R,z} = E_{R,mn}$ and $T_{R,z} = T_{R,mn}$ such that z is the absolute index difference defined by $z = |m-n|$. Then, the dispersion of the correlation matrix $\mathbf{R}_R$ can be calculated as [15]

$$\zeta(\mathbf{R}_R) = 1 + \frac{2}{M_R}\sum_{z=1}^{M_R-1}(M_R - z)|E_{R,z}|^2|T_{R,z}|^2. \quad (12)$$

It should be noticed that the absolute value of spatial correlation between two antennas is less than or equal to 1, i.e., $|T_{R,z}| \leqslant 1$. Consequently, the dispersion $\zeta(\mathbf{R}_R)$ is upper bounded by

$$\begin{aligned}\zeta(\mathbf{R}_R) &= 1 + \frac{2}{M_R}\sum_{z=1}^{M_R-1}(M_R - z)e^{-2\beta z}|T_{R,z}|^2 \\ &\leqslant 1 + \frac{2}{M_R}\sum_{z=1}^{M_R-1}(M_R - z)e^{-2\beta z} \to \frac{1+e^{-2\beta}}{1-e^{-2\beta}} \end{aligned} \quad (13)$$

as $M_R \to \infty$. In this case, the slope is bounded by

$$S_0 \geqslant \frac{2M_TM_R}{M_T + M_R}\left(\frac{1-e^{-2\beta}}{1+e^{-2\beta}}\right). \quad (14)$$

Consequently, the capacity lower bound $\mathcal{C}^{\mathrm{L}}\left(\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}_{\min}}\right)$ in the low SNR regime can be computed as

$$\begin{aligned}\mathcal{C}\left(\frac{E_b}{N_0}\right) &\geqslant \frac{2\left(1-e^{-2\beta}\right)}{(\gamma+1)\left(1+e^{-2\beta}\right)}\log_2\left(\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}_{\min}}\right) \\ &= \mathcal{C}^{\mathrm{L}}\left(\frac{\frac{E_b}{N_0}}{\frac{E_b}{N_0}_{\min}}\right). \end{aligned} \quad (15)$$

Equality holds true when $|T_{R,z}| = 1$, which implies that the antennas are fully correlated and the uncorrelated properties between antennas are solely dominated by the evolution of scatterer sets on the array axis.

B. High SNR Approximation

In the high SNR regime, the channel capacity can be approximated in terms of SNR ρ as [5]

$$C(\rho) \approx C_{\text{iid}}(\rho) + \frac{1}{M_R}\left[\log_2 \det\left(\mathbf{R}_R\right) + \log_2 \det\left(\mathbf{R}_T\right)\right]. \tag{16}$$

The channel capacity $C_{\text{iid}}(\rho)$ of an i.i.d channel with $M_R, M_T \to \infty$ can be expressed as [19]

$$\begin{aligned} C_{\text{iid}}(\rho) &= \frac{1}{M_R}\log_2 \det\left(\frac{\rho}{M_T}\mathbf{H}_{\text{w}}\mathbf{H}_{\text{w}}^{\text{H}}\right) \\ &= \begin{cases} F\left(1/\gamma, \gamma\rho\right) & \gamma \le 1 \\ \frac{1}{\gamma}F\left(\gamma, \rho\right) & \gamma \ge 1 \end{cases} \end{aligned} \tag{17}$$

where

$$\begin{aligned} &F\left(\gamma, \rho\right) \\ &= \log_2\left(1 + \rho(\sqrt{\gamma}+1)^2\right) + (\gamma+1)\log_2\left(\frac{1+\sqrt{1-a}}{2}\right) \\ &- (\log_2 e)\sqrt{\gamma}\frac{1-\sqrt{1-a}}{1+\sqrt{1-a}} + (\gamma-1)\log_2\left(\frac{1+\alpha}{\alpha+\sqrt{1-a}}\right) \end{aligned} \tag{18}$$

with $a = \frac{4\rho\sqrt{\gamma}}{1+\rho\left(\sqrt{\gamma}+1\right)^2}$ and $\alpha = \frac{\sqrt{\gamma}-1}{\sqrt{\gamma}+1}$.

1) Capacity Upper Bound ($\rho \to \infty$): It was shown in [5] that antenna correlation degrades the channel capacity because $\log_2 \det\left(\mathbf{R}_R\right), \log_2 \det\left(\mathbf{R}_T\right) \leqslant 0$. Then, the capacity upper bound $C^{\text{U}}(\rho)$ in the high SNR regime is equivalent to the capacity of i.i.d channels,

$$C(\rho) \leqslant C_{\text{iid}}(\rho) = C^{\text{U}}(\rho). \tag{19}$$

2) Capacity Lower Bound ($\rho \to \infty$): On the other hand, the capacity lower bound $C^{\text{L}}(\rho)$ in the high SNR regime can be derived from (16) as

$$\begin{aligned} &C(\rho) \\ &= C_{\text{iid}}(\rho) + \frac{1}{M_R}\left[\log_2 \det\left(\mathbf{T}_R \circ \mathbf{E}_R\right) + \log_2 \det\left(\mathbf{T}_T \circ \mathbf{E}_T\right)\right] \\ &\ge C_{\text{iid}}(\rho) + \frac{1}{M_R}\left[Q_R + Q_T\right] = C^{\text{L}}(\rho) \end{aligned} \tag{20}$$

where $Q_R = \max\left\{\log_2 \det\left(\mathbf{T}_R\right), \log_2 \det\left(\mathbf{E}_R\right)\right\}$ and $Q_T = \max\left\{\log_2 \det\left(\mathbf{T}_T\right), \log_2 \det\left(\mathbf{E}_T\right)\right\}$ are derived based on the Schur-Oppenheim inequality [20], [21].

Matrices $\mathbf{E}_R$ and $\mathbf{E}_T$ caused by scatterer set evolution decorrelate antennas in addition to antenna spacings. Since $\mathbf{E}_R$ and $\mathbf{E}_T$ are exponential correlation matrices with a Toeplitz structure and $M_R, M_T >> 1$, their determinants can be approximated as [22]

$$\det\left(\mathbf{E}_R\right) \approx \left(1 - e^{-2\beta}\right)^{M_R - 1} \tag{21}$$

$$\log_2 \det\left(\mathbf{E}_R\right) \approx \left(M_R - 1\right)\log_2\left(1 - e^{-2\beta}\right). \tag{22}$$

Moreover, as $\mathbf{T}_R$ and $\mathbf{T}_T$ are large-dimension Toeplitz matrices, the logarithms of their determinants are able to be solved by the Szego's theorem [23] and tend to constants as $M_R, M_T \to \infty$. Hence, it can be observed in (20) that, provided a scattering environment, the gap between the capacity lower bound and the capacity upper bound does not vary with SNR.

As a special case, we assume that the normalized antenna spacing is 0.5 and the scattering environment is isotropic. Then, the spatial correlation between antennas can be characterized by the zero-th order Bessel function of the first kind $J_0(\cdot)$, i.e., $T_{R,z} = J_0(\pi z)$ [24]. According to the Szego's theorem [23], if the entries of the first column of a large Toeplitz matrix (size tends to infinity) are able to be written as the Fourier transform of a function $f(\lambda)$ as

$$T_{R,z} = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\lambda)e^{-iz\lambda}d\lambda, \tag{23}$$

the logarithm of the determinant of the correlation matrix $\mathbf{T}_R$ can then be calculated as [23]

$$\log_2 \det\left(\mathbf{T}_R\right) = \frac{M_R}{2\pi}\int_{-\pi}^{\pi} \ln f(\lambda)e^{-iz\lambda}d\lambda \tag{24}$$

as $M_R \to \infty$. The function $f(\lambda)$ can be computed by taking the inverse Fourier transform of $T_{R,z}$ [25]. As a result, $f\left(\lambda\right)$ is computed as

$$\begin{aligned} f(\lambda) &= \int_{-\infty}^{\infty} T_{R,z}e^{iz\lambda}dz = \int_{-\infty}^{\infty} J_0(\pi z)e^{iz\lambda}dz \\ &= \frac{2}{\sqrt{\pi^2 - \lambda^2}} \quad \left(|\lambda| < \pi\right). \end{aligned} \tag{25}$$

With (25), (24) can be obtained as

$$\log_2 \det\left(\mathbf{T}_R\right) = \frac{M_R}{2\pi}\int_{-\pi}^{\pi} \ln\frac{2}{\sqrt{\pi^2 - \lambda^2}}d\lambda = M_R \log_2\left(\frac{e}{\pi}\right). \tag{26}$$

Therefore, combining the receiver and transmitter sides and substituting (22) and (26) into (20), the capacity lower bound of a massive MIMO system with half-wavelength arrays under isotropic scattering environment in the high SNR regime can be derived as

$$\begin{aligned} &C^{\text{L}}(\rho) \\ &\approx C_{\text{iid}}(\rho) + \left(1 + \frac{1}{\gamma}\right)\max\left\{\log_2\left(\frac{e}{\pi}\right), \log_2\left(1 - e^{-2\beta}\right)\right\} \end{aligned} \tag{27}$$

as $M_R, M_T \to \infty$. In this case, the gap between the capacity lower bound and the capacity upper bound is a constant $\left(1 + \frac{1}{\gamma}\right)\max\left\{\log_2\left(\frac{e}{\pi}\right), \log_2\left(1 - e^{-2\beta}\right)\right\}$.

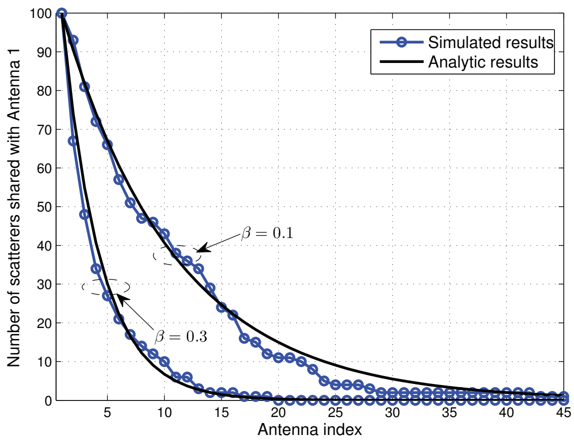

Fig. 2: Number of scatterers shared with Antenna 1 in terms of antenna indices (100 initial scatterers in Antenna 1).

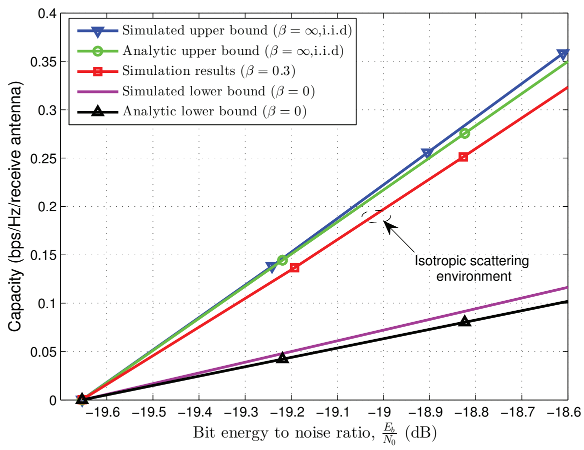

Fig. 4: Capacity analysis in the low SNR regime for the KBSM–BD–AA ($M_R = M_T = 64$).

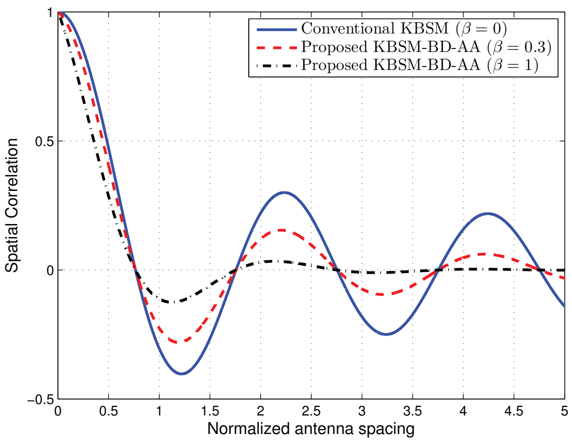

Fig. 3: Antenna spatial correlation for the KBSM–BD–AA (isotropic scattering environment).

IV. Results and Discussions

Fig. 2 shows the number of scatterers of different antennas shared with Antenna 1. Fewer scatterers are in common as the indices increase. Also, the parameter β controls the survival probability of scatterers. The value of β can be tuned to adapt to different scattering environments.

Antenna correlation with BD process of scatterers on the array axis for massive MIMO is compared with antenna correlation of conventional MIMO in Fig. 3. Lower antenna correlation with BD process can be observed because of the different scatterer sets between different antennas. The gap enlarges when the normalized antenna spacing increases. Moreover, larger values of β result in more significant drop of antenna correlations.

The capacity analysis of the proposed KBSM–BD–AA in the low SNR regime is depicted in Fig. 4. The analytic capacity upper bound is the linear approximation under the i.i.d channel assumption as derived in (11). The linearly approximated capacity upper bound aligns well with the simulated (exact) i.i.d channel capacity in the low SNR regime. Next, the analytic capacity lower bound of the KBSM–BD–AA is the linear approximation based on the fully correlated channel assumption as derived in (15). In this case, the scatterer evolution on the array axis is solely responsible for the uncorrelated properties of the channel. It can be observed that the linear approximation of the capacity lower bound of the KBSM–BD–AA has a significantly smaller slope than that of an i.i.d channel. The decrease of the slope is caused by antenna correlations. It is demonstrated by simulation that the linear approximation of the capacity lower bound matches the exact channel capacity properly. The simulated (exact) capacity of the KBSM–BD–AA in an isotropic scattering environment lies between the upper and lower bounds.

Fig. 5 illustrates the upper and lower capacity bounds of the KBSM–BD–AA in an isotropic scattering environment as well as the simulation results in the high SNR regime. It can be noticed that simulation results match analytic derivations well. In addition, the gap between the upper and lower bounds is constant when the SNR is high. Again, the simulated capacity of the KBSM–BD–AA lies within the upper and lower bounds.

Cumulative distribution functions (CDFs) of capacities of the KBSM–BD–AA in an isotropic scattering environment are shown in Fig. 6. SNR is set to be 25 dB. These CDFs of capacities are shifted by different values of β. The gap of capacity medians between $\beta = 0$ and $\beta = \infty$ is approximately 0.55 bps/Hz/receive antenna.

V. Conclusions

A novel KBSM–BD–AA for massive MIMO has been proposed in this paper. The evolution of scatterer sets on the array axis has been modeled by the BD process. With this consideration, the overall antenna correlation matrix is equivalent to the Hadamard product of the spatial correlation matrix and survival probability matrix. Upper and lower bounds of channel capacities have been analyzed in both the

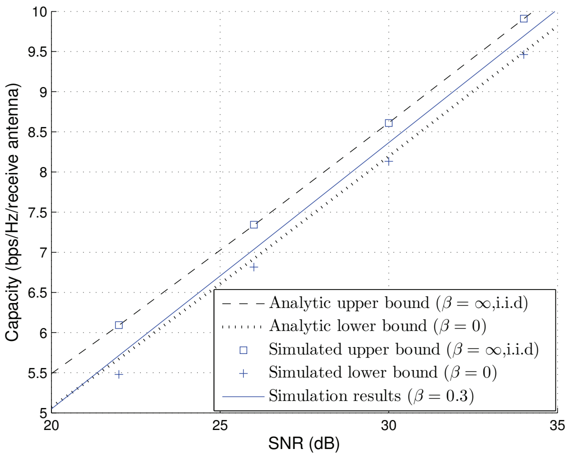

Fig. 5: Capacity analysis in the high SNR regime for the KBSM–BD–AA ($M_R = M_T = 64$, half-wavelength ULAs, isotropic scattering environment).

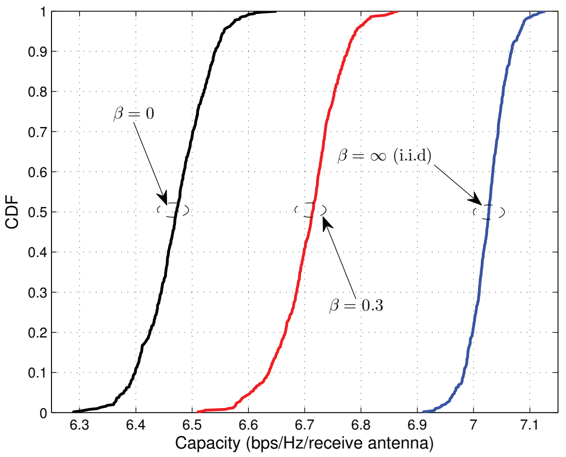

Fig. 6: CDFs of capacities of the KBSM–BD–AA with different values of β (SNR=25dB, $M_R = M_T = 64$, half-wavelength ULAs, isotropic scattering environment).

high and low SNR regimes when the numbers of transmit and receive antennas are increasing unboundedly with a constant ratio. The BD process on the array axis additionally decreases correlations between antennas since antennas in a large array may observe different sets of scatterers.

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