# A Statistical Weighting Average Approach for Cognitive Radio Networks

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***Abstract*—In this paper, a statistical weighting average (SWA) approach is proposed for cooperative spectrum sensing (CSS) system in Cognitive Radio Networks (CRNs). Two SWA algorithms, including ordered weighted average (OWA) and weighted ordered weighted average (WOWA), are initially designed in CRNs. In a weighting process, the final weighting factor (WF) for a distributed sensor is jointly determined by its previous sensing contributions and its currently gathered primary user (PU) samples. The WF is initially calculated by the sensing contributions, which are numerically calibrated by probability of detection. The gathered PU samples are used to further adjust the corresponding WF. Simulation results show that the proposed SWA approach can outperform the conventional weighting schemes.**

***Index Terms* – CRNs, spectrum sensing, statistical weighting average, weighting factor, ordered weighted average, weighted ordered weighted average.**

## I. Introduction

CRNs as emerging and promising wireless communication solutions have been proposed in [1]–[4]. In CRNs, Secondary User (SU) systems dynamically access licensed frequency bands, which belong to PU systems but are not occupied. As one of CRNs' key functions, SS is exploited to indicate the presence or absence of PU signal and the usability of target frequency bands. For SU systems, the sensing performance is mainly determined by two following factors. First, the prior information of PU signal for SU system is limited. Second, the transmission circumstances including multi-path fading, shadowing and noise uncertainty are non-ideal. In order to conquer the above limitations and achieve reliable sensing performance, cooperative SS (CSS) schemes have been heavily discussed in the literatures [5]–[7]. There are two main structures of CSS, with fusion center (FC) (centralized) [8] and without FC (distributed) [9]. In the centralized CSS systems, the distributed sensors periodically send PU samples or local results to the FC under a given coordination mechanism. In this paper, it is assumed that the sensors do not make a local decision and only the gathered PU samples are sent to the FC.

However, the distributed sensors cannot equally contribute to the whole sensing performance due to some limitations including varied positions, non-ideal sensing circumstance and communication links between the sensors and the FC. In order to achieve better sensing performance and conquer the limitations, the weighting approach for CSS has been discussed in [10]–[13]. A weighting rule based on the likelihood ratio test has been proposed in [10]. A weighted average consensus (WAC) scheme has been used to construct weighted CSS in [11]. [12] combined the above two weighting schemes, in which mean signal-to-noise ratio (SNR) method and likelihood ratio test have been discussed. A weighting fusion scheme for CSS has been discussed in [13], in which the location decision of the distributed sensors are determined by their previous decisions. In [11], [12], the final WF has been determined by PU signal SNR, which is assumed to be known for sensors. Such straightforward assumption is apparently convenient to be used. However, from a practical point of view, the information of PU signal including SNR and features cannot be directly accessed by SU systems through synchronization or information sharing.

In order to determine reliable WFs and improve sensing performance, we propose a statistical weighting scheme which utilizes past sensing contributions and currently gathered PU samples. In the proposed scheme, OWA scheme [14] and WOWA scheme [15] are utilized to integrate past sensing contributions and currently gathered PU samples. The OWA scheme is used to evaluate the previous sensing contributions and the WOWA is used to combine the past contributions and the current sensing results. The final WFs for distributed sensors are statistically calculated by the WOWA scheme.

The remainder of this paper is given as follows. In section II we describe cooperative spectrum sensing model using weighting scheme. The main weighting scheme is proposed in section III. In section IV, we present simulations results for the proposed scheme. Conclusions are offered in section V.

## II. System Models

A weighted CSS system model is shown in Fig. 1, where there are PU systems and SU systems. The weighted CSS system is consisted of the FC and distributed SUs working as sensors to detect PU link signal (down-link or up-link) between PU base station and distributed PUs. The distributed sensors periodically sample PU signal and send the samples to the FC through a common channel according to a coordination system. At the FC, the received sample vector $\mathbf{x}^t$ can be

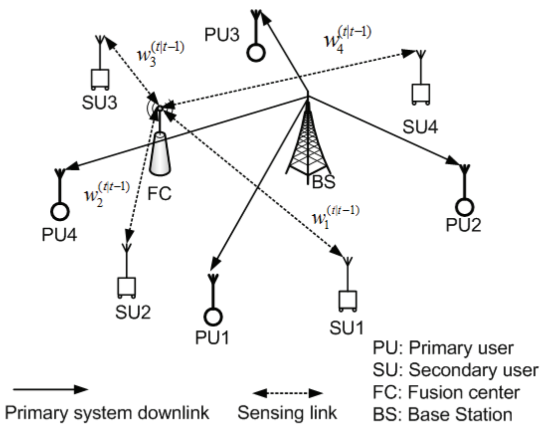


Fig. 1. Cooperative spectrum sensing model with WFs.

expressed as

$$\mathbf{x}^t = \sum_{m=1}^{M} w_m^{(t|t-1)} \cdot \mathbf{s}_m^t \tag{1}$$

where $M$ denotes the number of sensors, $m$ and $t$ are sensor index and time index, respectively, $\mathbf{s}_m^t$ denotes an $N$-length sample vector sampled by the $m$-th sensor at time $t$. The WF $w_m^{(t|t-1)}$ is jointly determined at time $t$ and time $t-1$. The signal sesing problem can be formulated by the binary hypothesis

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{x}^t = \sum_{m=1}^{M} w_m^{(t|t-1)} \cdot \mathbf{n}^t \\ \mathcal{H}_1 &: \mathbf{x}^t = \sum_{m=1}^{M} w_m^{(t|t-1)} \cdot (\sqrt{\beta_m^t}\mathbf{s}_m^t + \mathbf{n}^t) \end{aligned} \tag{2}$$

where $\mathbf{x}^t$ is the received sample vector including $N$ samples at the FC, $\mathbf{n}^t$ follows standard normal distribution $\mathcal{N}(\mathbf{0}, \mathbf{I})$, $\mathbf{I}$ is the identity vector of size $N$. The PU signal $\mathbf{s}$ including channel effect follows the distribution $\mathbf{s}^t \sim \mathcal{N}(\mu_s, \sigma_s)$, where $\mu_s$ and $\sigma_s$ denote the mean and variance, respectively. The PU signal SNR is denoted by $\beta_m^t$. Note that the hypothesis test is provided based on the $t$-th sensing process.

The mean and variance of $\mathbf{x}^t$ are denoted by $\mu_x$ and $\sigma_x$, respectively. For $\mathcal{H}_0$,

$$\begin{aligned} \mu_x &= \mathbf{0} \\ \sigma_x &= \sigma_w^2 \mathbf{I} \end{aligned} \tag{3}$$

where $\sigma_w$ is the variance of WF for $M$ sensors. For $\mathcal{H}_1$,

$$\begin{aligned} \mu_x &= \mu_w \overline{\mu}_s \\ \sigma_x &= \sigma_w^2 \overline{\sigma}_s \end{aligned} \tag{4}$$

where $\mu_w$ is the mean of WF and $\overline{\mu}_s$, $\overline{\sigma}_s$ denote the mean variance of received PU signal. The mean of WF can be generated as $\mu_w = 1/M$ and the variance $\sigma_w$ is affected by the different detection contributions of different sensors. The energy detection (ED) at the FC with $N$ samples at time $t$ can be defined as [16]

$$D_E = \sum_{n=1}^{N} (|x^{tn}|)^2 \tag{5}$$

where $x^{tn}$ is the $n$-th entry of sample vector and $D_E$ follows a $\chi^2$-distribution with $N$ degree of freedom.

## III. WOWA based Cooperative spectrum sensing scheme

The WOWA approach is introduced in this section, followed by the proposed weighted CSS scheme. The WFs for distributed sensors are generated using a statistical way based on WOWA approach.

### A. Weighted ordered weighted average approach

Before the introduction of WOWA approach, weighted mean approach and OWA approach are described. In weighted mean approach, the WF is determined by the reliability of the information source, i.e., distributed sensing nodes. A larger WF is distributed to a reliable sensor due to its past better sensing contributions. In OWA approach, the WF is generated based on the current samples gathered from different sensors. The reliability of the sensors are not considered in OWA approach. The gathered samples at the FC are weighted according to the properties of samples.

Both above weighting approaches have drawbacks when considering the reliability of distributed sensors and the current gathered samples. More practical sensing system should consider both factors when the sensing circumstance is variable. Both the reliability of distributed sensors and the instant samples should be considered to determine the WF. We combine the above weighting approaches and propose a new weighting scheme using revised WOWA approach.

In the WOWA, there are two weighting vectors, $\mathbf{u}$ corresponding to the relevance of $M$ sensing nodes and $\mathbf{v}$ corresponding to the relevance of the sensing data value. Two weighting vectors $\mathbf{u} = [u_1, ..., u_M]$ and $\mathbf{v} = [v_1, ..., v_M]$ are defined as

$$\begin{aligned} u_i &\in [0,1], \sum_i u_i = 1 \\ v_i &\in [0,1], \sum_i v_i = 1. \end{aligned} \tag{6}$$

Based on two weighting vectors $\mathbf{u}$ and $\mathbf{v}$, a WOWA operator is defined as

$$f_W(a_1, ..., a_M) = \sum_i w_i \cdot a_{\sigma(i)} \tag{7}$$

where the $\{\sigma(1), ..., \sigma(M)\}$ is the permutation of $[1, ..., M]$ and $a_{\sigma(i-1)} \geqslant a_{\sigma(i)}$ for all $i = 1, ..., M$.

The WF $w_i$ in WOWA can be calculated by

$$w_i = v^*(\sum_{j \leqslant i} u_j) - v^*(\sum_{j<i} u_j) \tag{8}$$

where the $v^*$ is a monotone increasing function which interpolates the points $(i/M, \sum_{j\leqslant i} v_j)$ together with the point $(0,0)$. The interpolation function $v^*()$ can be defined as

$$\begin{aligned} y &= \sqrt{0.5 * x}, \qquad if \quad x < 0.5 \\ y &= 1 - \sqrt{0.5 * (1-x)}, \quad if \quad x \geq 0.5. \end{aligned} \tag{9}$$

Based on the interpolation function and the vectors $\mathbf{v}$ and $\mathbf{u}$, the WF $\mathbf{w}$ can be determined.

*B. Weighted cooperative spectrum sensing based on WOWA*

Two weighting vectors $\mathbf{u}$ and $\mathbf{v}$ can denote the reliability of the distributed sensors and the importance of the gathered samples, respectively. The WF $\mathbf{u}$ can be determined by the previous sensing contributions for a weighted CSS system. For a specific sensor, such sensing contribution can be evaluated by the probability of detection $P_D$. Therefore, the WF $u_m^{t+1}$ is expressed as [10], [17]

$$u_m^{t+1} = \frac{u_m^t \cdot P_{D_m}^t}{(\sum_{m=1}^{M} u_m^t \cdot P_{D_m}^t)/M} \tag{10}$$

where $u_m^t$ and $P_{D_m}^t$ are the WF and the probability of detection of $m$-th sensor in the $t$-th sensing process, respectively. Initially, we can assume the same WF $u_m^0 = 1$ for each sensor.

The WF vectors for $M$ sensors system can be generated as

$$\mathbf{u}^i = \{u_1^i, \cdots, u_M^i\} \tag{11}$$

where $I$ states of PU systems can be used to indicate different sensing models.

The weighting vector $\mathbf{v} = [v_1, ..., v_M]$ is used to indicate the importance of current gathered sample. The weighting vector is used to adjust the total WF $\mathbf{w}$ for all $M$ sensors based on their current gathered samples. The WF $v_m$ for the $m$-th sensor can be generated as

$$v_m^t = \kappa \cdot \sum_{n=1}^{N} |x_m^t(n)|^2 \tag{12}$$

where $x_m^t(n)$ denotes the $n$-th sample of the $m$-th sensor and the parameter $\kappa$ is used to adjust the WF based on the current signal energy. Note that $\kappa$ can be evaluated in a practical weighting scheme based on the energy results of all $M$ sensors and it can be defined as a piecewise function. Based on the above two weighting vectors $\mathbf{u}$ and $\mathbf{v}$, the WF $\mathbf{w}$ for $M$ sensors can be generated using (8). In this paper, a fitting interpolation function is used to calculate $\mathbf{w}$ when the inputs $\mathbf{u}$ follows straight line or convex function.

For hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, $D_E$ follows central $\chi^2$-distribution and noncentral $\chi^2$-distribution, respectively. For noncentral $\chi^2$-distribution, the non-centrality parameter $\lambda$ can be defined as

$$\lambda = \frac{1}{N}\sum_{n=1}^{N}(\frac{\mu_{y_{tn}}}{\sigma_{y_{tn}}})^2. \tag{13}$$

Considering (4), the above formula can be rewritten as

$$\lambda = (\frac{\mu_w \overline{\mu}_s}{\sigma_w \overline{\sigma}_s})^2 = (\frac{\overline{\mu}_s}{K\sigma_w \overline{\sigma}_s})^2. \tag{14}$$

For the noncentral $\chi^2$-distribution, the probability density function (PDF) of $D_E$ can be written as [18]

$$f_K^1(d;\lambda) = \sum_{i=0}^{\infty} \frac{e^{-\lambda/2}(\lambda/2)^i}{i!} f_{K+2i}^0(d) \tag{15}$$

where $K$ is the degrees of freedom and $\lambda$ is the non-centrality parameter, the function $f_K^0(x)$ is the PDF of central $\chi^2$-distribution with $K$ degrees of freedom. The superscripts of 1 and 0 are used to denote the noncentral and central $\chi^2$-distribution, respectively. The PDF $f_K^0(x)$ can be written as

$$f_K^0(x) = \frac{x^{K/2-1}e^{-x/2}}{2^{K/2}\Gamma(K/2)} \tag{16}$$

where $\Gamma()$ is the Gamma function. The above PDF can be expressed in a closed-form expression with the hypergeometric functions [19]

$$f_K^1(d;\lambda) = e^{-\lambda/2}{}_0F_1(K; \tfrac{K}{2}; \tfrac{\lambda x}{4})\frac{1}{2^{K/2}\Gamma(\frac{K}{2})}e^{-\frac{d}{2}}d^{\frac{K}{2}-1} \tag{17}$$

where ${}_0F_1(;;)$ is the hypergeometric function.

The cumulative distribution function (CDF) of noncentral $\chi^2$-distribution for $D_E$ can be integrated from the corresponding PDF (15)

$$F_K^1(d;\lambda) = e^{-\lambda/2}\sum_{i=0}^{\infty}\frac{(\lambda/2)^i}{i!}F_{K+2i}^0(d) \tag{18}$$

where $F_K^0(x)$ is the CDF of central $\chi^2$-distribution with $K$ degrees of freedom. $F_K^0(x)$ can be expressed as

$$F_K^0(x) = \frac{\gamma(K/2, x/2)}{\Gamma(K/2)} \tag{19}$$

$\gamma(,)$ is the lower incomplete Gamma function. The CDF expressions in closed-form can also be found in [18]. The exact results are not easy to calculate due to high complexity. Note that the CDF can be generated using a numerical way based on the corresponding empirical PDF.

The CDFs of central and noncentral $\chi^2$-distribution are critical in the generation of theoretical thresholds. The corresponding hypothesis test results can be generated using central and non-central $\chi^2$ distributions with the same threshold $\gamma$. The total four results are the probability of false alarm ($P_F$), the probability of acquisition ($P_A$), the probability of miss detection ($P_M$) and $P_D$, respectively. $P_F$ and $P_A$ are generated based on the central $\chi^2$ distribution and $P_D$ and $P_M$ can be calculated based on the non-central $\chi^2$ distribution. It should note that such four results can completely describe the mechanism of CRNs. It is clear to see the relations between such four results,

$$\begin{aligned} P_F + P_A &= 1 \\ P_D + P_M &= 1. \end{aligned} \tag{20}$$

Conventional discussions on CRNs usually focus on the relation between $P_D$ and $P_F$, that is, keeping $P_F$ below given limitations and maximizing $P_D$. From the view of

SU systems which aim to utilize non occupied frequency bands and avoid harmful interference to PU systems, it is more important to discuss $P_A$ under the constraint that $P_M$ should not larger than the interference limitations. Some initial relative discussions using finite random matrix theory can be found in [20] and references therein. We should emphasize that $P_A$ is proposed here to indicate that the target frequency bands are not occupied by PU systems and can be accessed by SU systems. The other three results have been heavily discussed in relative CR literatures. For a specific CSS system, $P_F$ should below than a given value $\delta$

$$P_F \leq \delta. \tag{21}$$

Based on the CDF of $D_E$, $P_F$ can be generated as

$$P_F = 1 - F_K^0(\gamma) \tag{22}$$

where $\gamma$ is the required threshold. Therefore, the theoretical threshold $\gamma$ can be calculated under the condition that $P_F$ should be below than a given value,

$$\gamma = F_K^{0(-1)}(1-\delta) \tag{23}$$

where the superscript $(-1)$ denotes the inverse function. Based on the above discussions, the theoretical probability of detection $P_D$ can be calculated under the assumption that the expressions of $F_K^1(x)$ and $F_K^0(x)$ are known

$$P_D = 1 - F_K^1(\gamma;\lambda) = 1 - F_K^1(F_K^{0(-1)}(1-\delta);\lambda) \tag{24}$$

where $\delta$ is the required $P_F$ and $\lambda$ is the noncentral parameter which is determined by PU signal and $M$ WF for all sensors.

In the practical situations, it is not easy to obtain the exact theoretical expressions for $F_K^1(x)$ and $F_K^0(x)$. The accurate approximation for $P_D$ is necessary if the PDFs for $D_E$ can be generated using an empirical method. Including the expression of $f_K^1(x;\lambda)$ shown in Eq. 15, the theoretical $P_D$ can be approximately written as

$$P_D(\gamma, M, \lambda) = \int_{\gamma}^{\infty} \sum_{i=0}^{\infty} \frac{e^{-\frac{\lambda+x}{2}} (\frac{\lambda}{2})^i x^{\frac{M+2i-2}{2}}}{i! 2^{\frac{M+2i}{2}} \Gamma(\frac{M+2i}{2})} dx$$

where $M$ are the number of sensors and $\lambda$ is the non-centrality parameter. It should note that the theoretical result of $P_D$ is hard to evaluate due to the computational complexity the expression. Therefore the theoretical $P_D$ can be approximately evaluated using a numerical way.

## IV. Simulation Results

In this section, we firstly summarize the proposed WOWA-based scheme, in which the final WF for all $M$ sensors are determined by their past detection performance calibrated by weighting vector $\mathbf{u}$ and their current PU signal samples denoted by weighting vector $\mathbf{v}$. The final WF $\mathbf{w}$ are generated by (8) using $\mathbf{u}$ and $\mathbf{v}$. In the following simulations, we evaluate the proposed scheme WOWA, comparing to the conventional weighting schemes such as OWA and WAC [11]. The number of sensor $M$ and the number of samples per sensor $N$ are set to be $M = 3$ and $N = 10$, respectively.

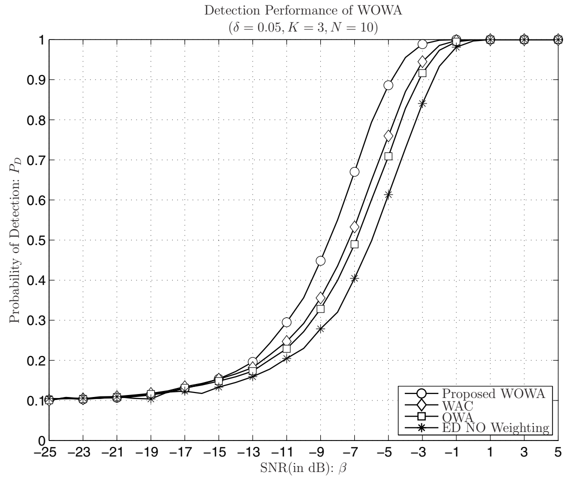


Fig. 2. Detection performances for varying PU SNR.

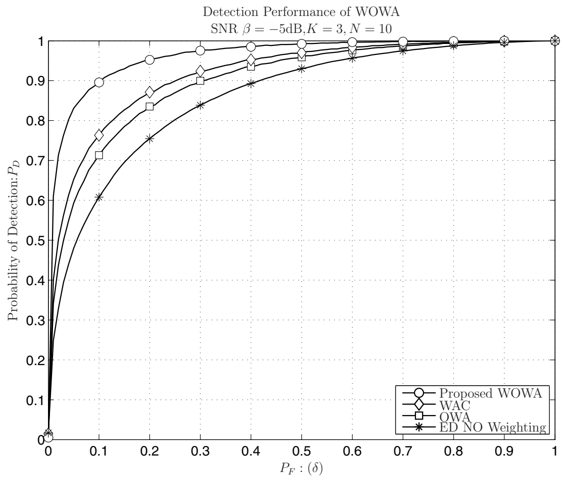


Fig. 3. ROC performance.

In Fig. 2, the given $P_F$ is set to $\delta = 0.05$. It can be seen that the proposed scheme outperforms the conventional weighting schemes, such as WAC and OWA. There is about 2 dB gain comparing to the conventional weighting schemes. The proposed scheme can achieve about 15% detection performance gain when SNR is -7dB. The ED without weighting operation is also included as a benchmark to indicate the total effect of weighting. It is obvious that the weighting scheme can achieve better detection performance comparing to the ED scheme. The main reason for the performance gain of the proposed weighting scheme is that both the past sensing contributions and the current sensing operation are considered to generate the final WFs.

In Fig. 3, the average SNR is set to be $-5$ dB and $M = 3, N = 10$. $P_F$ is set to be $0 : 0.1 : 1$. The sensing performance is evaluated at different $P_F$. There are about 14% and 19% gain when comparing the proposed scheme with the WAC

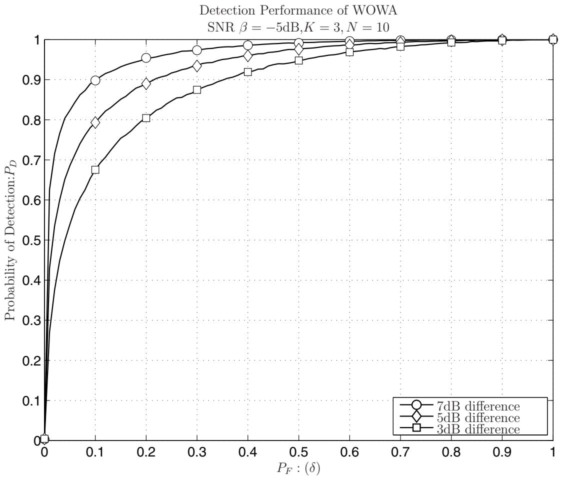


Fig. 4. The detection performance with SNR difference.

scheme and OWA scheme when $\delta = 0.01$. We can also see that the proposed scheme outperforms other weighting schemes for all $P_F$ in the same simulation circumstance.

In Fig. 4, we use different average SNR to indicate different PU signal. Based on their detection performances for different PU signal, the WFs $\mathbf{u}$ can be generated under the constraint that $u_1 + u_2 + u_3 = 1$. On the other hand, another WFs $\mathbf{v}$ can also be generated based on their current PU samples with different SNR. According to such method, the detection performance of the proposed scheme for different average SNR of PU signal is evaluated. The SNR difference is set to be 7dB, 5dB and 3dB, respectively. We can see that with the higher SNR, the detection performance is better with about $10\%$ gain. The simulation results indicate that the proposed scheme can achieve better detection performance when the PU signal has larger diversity.

## V. Conclusion

A new weighting scheme for the cooperative spectrum sensing scheme has been discussed in this paper. The WOWA is originally included to construct the proposed scheme, in which the two main weighting vectors are used to indicate the reliability of the sensors and the importance of the current gathered samples. The reliability of a specific sensor is indicated by its past detection contributions and the importance of the current samples is used to adjust the WF. Four hypothesis test results have also been discussed under the CRNs diagram and the logic relations are indicated. We point out that the relation between of $P_A$ and $P_M$ is more practical for SU system. Simulation results show that the new scheme outperforms other weighting schemes, in consequence of the fact that both past and current detection performance have been considered to generate the final WFs.

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