Capacity Analysis of Finite Scatterer MIMO Wireless Channels

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***Abstract*—This paper analyzes the capacity of the finite scatterer (FS) multiple-input multiple-output (MIMO) channel model with finite angular resolution. The rank of the channel is bounded by not only the number of antennas but also the finite angular resolution of antenna arrays. The probability mass function (PMF) of rank is studied and derived based on the Stirling number of the second kind. As a result, the closed-form expression of the capacity of a FS MIMO channel model is obtained. Furthermore, asymptotic analyses such as the large-scale system analysis, infinite scatterer analysis, and the multiplexing gain analysis are investigated.**

***Keywords* – Finite scatterer model, Stirling number of the second kind, asymptotic analysis.**

I. Introduction

It has been demonstrated that MIMO systems are able to significantly increase spectral efficiency and improve link reliability [1]–[5]. Capacities of MIMO systems under various channel models were extensively analyzed in the previous literature. The capacity of classic MIMO channels with independent and identically distributed (i.i.d.) Rayleigh sub-channels was investigated in [1]–[5]. Besides, [2] and [5] investigated the impact of antenna correlations of the Kronecker MIMO channel model on the channel capacity. For the Kronecker MIMO channel model, correlations of the transmitter and receiver are assumed to be separated. On the other hand, the capacity of the Weichselberger MIMO channel model, which considers joint correlations between the transmitter and receiver, was studied in [6]. The channel capacity of another correlation-based MIMO channel model known as the virtual representation channel was given in [4] and [7]. It was shown that the channel capacity would be greatly reduced if correlated MIMO models were used.

Also, asymptotic capacity analysis of MIMO systems was given in the literature. The authors in [8] derived the asymptotic capacity of MIMO channels in low signal-to-noise ratio (SNR) regime when SNR tends to zero. Conversely, behaviors of MIMO channel capacity in high SNR regime were discussed in [4], [5], [9]. Furthermore, [10] and [11] evaluated the asymptotic capacity of Kronecker channel models for large-scale MIMO systems, i.e., the numbers of transmit and receive antennas tend to infinity. Asymptotic capacities of other MIMO channel models, such as the i.i.d Rayleigh model and the Weichselberger model, in large-scale MIMO systems can be found in [4] and [12], respectively.

In this paper, attentions are paid to the FS MIMO channel model [13], which assumes that there are only a limited number of multipath components in the radio environment. In this case, the capacity of the channel will be constrained by the limited number of scatterers. Limited efforts were devoted to calculating capacities of FS MIMO models. In [13], Burr was the first to provide the general description of the FS MIMO channel model and derived the capacity bound of the FS MIMO channel model with the large-scale system assumption. Then, [14] extended these results and investigated the performance of FS MIMO model in cellular networks. Also, the multiplexing gain of the FS MIMO channel model with multi-user MIMO (MU-MIMO) was studied in [15], while [16] analyzed the performance of MIMO systems with MU-MIMO and large MIMO in cellular networks using the FS MIMO channel model. However, the analysis methods used in [13]–[16] implied that the angular resolution of antenna arrays is infinite, which is not practical in realistic scenarios.

The **contributions** of this paper are listed as below:

1) This paper analyzes the capacity of the FS MIMO channel model with finite angular resolution. In this case, the rank of the FS channel is constrained by the number of scatterers and the angular resolution. The closed-form expression for the PMF of rank in terms of different numbers of scatterers and angular resolutions is derived based on the Stirling number of the second kind [17].
2) Asymptotic analyses for the large-scale system capacity, infinite scatterer capacity, and multiplexing gain of the finite scatterer MIMO channel model are provided with simulation results.

The remainder of this paper is organized as follows. Section II gives the general description of the FS MIMO channel model. Section III calculates the capacity and asymptotic results of the FS MIMO channel model based on the derivation of the PMF of rank. Simulation results are shown in Section IV and conclusions are finally drawn in Section V.

II. Finite Scatterer MIMO Channel Model

We consider a MIMO system with M_R and M_T uniform linear omnidirectional receive and transmit antennas. Let us assume that there are a finite number of scatterers, say n_s scatterers, with angles of arrival (AoAs) $\phi_{R,p}$ $(p = 1, \cdots, n_s)$

and angles of departure (AoDs) $\phi_{T,p}$. The FS MIMO channel matrix $\mathbf{H}$ can be expressed as

$$\mathbf{H} = \mathbf{\Psi}_R \mathbf{\Xi} \mathbf{\Psi}_T^{\mathrm{H}} \quad (1)$$

where $\mathbf{\Psi}_R$ and $\mathbf{\Psi}_T$ are the steering matrices of the receiver and transmitter, respectively. The dimensions of $\mathbf{\Psi}_R$ and $\mathbf{\Psi}_T$ are $M_R \times n_s$ and $M_T \times n_s$, respectively. The columns of $\mathbf{\Psi}_R$ and $\mathbf{\Psi}_T$ are $\psi_R(\phi_{R,p})$ $(p = 1, \cdots, n_s)$ and $\psi_T(\phi_{T,p})$, respectively, while $\mathbf{\Xi}$ is an $n_s \times n_s$ diagonal matrix whose diagonal elements are ξ_p denoting the amplitude gains of scatterers. Additionally, normalization has been performed so that the power of the channel equals the number of transmit antennas

$$\mathrm{E}\left[\mathrm{trace}(\mathbf{H}^{\mathrm{H}}\mathbf{H})\right] = M_T \quad (2)$$

where $\mathrm{E}[\cdot]$ denotes the expectation and the operator $\mathrm{trace}(\cdot)$ is defined as the sum of the elements on the main diagonal of a square matrix.

In [13], only the channel capacity with an infinite number of antennas was analyzed. An infinite number of antennas means infinite angular resolution. In this case, two scatterers, say scatterer i with ϕ_i and scatterer j with ϕ_j, are irresolvable if and only if $|\phi_i - \phi_j| = 0$. Let us assume that $\phi_i, \phi_j \in [0, 2\pi)$ and define $\Omega_{i(j)} = \cos\phi_{i(j)}$. The condition $|\phi_i - \phi_j| = 0$ can be equivalently expressed using the absolute difference $|\Delta\Omega|$, i.e.,

$$|\Delta\Omega| = |\Omega_i - \Omega_j| = |\cos\phi_i - \cos\phi_j| = 0. \quad (3)$$

However, a finite-element antenna array should have a finite angular resolution [4]. An example is illustrated in Fig. 1. Scatterers 1 to 4 surround the antenna array, the absolute directional difference between Scatterers 1 and 2 is $|\Delta\Omega_1|$, and the absolute directional difference between Scatterers 3 and 4 is $|\Delta\Omega_2|$. In this example, Scatters 1 and 2 are not resolvable for the antenna array because $|\Delta\Omega_1|$ is less than the minimum resolvable angular difference $\epsilon_{\min}$. As a result, Scatters 1 and 2 are regarded as coming from one effective direction. On the other hand, since the absolute directional difference $|\Delta\Omega_2|$ between Scatterers 3 and 4 is larger than $\epsilon_{\min}$, Scatterers 3 and 4 can be resolved as they are from two distinct directions.

Next, to obtain the angular resolution of the receive and transmit antenna arrays, let us assume a signal is arriving from direction Ω, where Ω is the cosine value of AoA for the receiver or the cosine value of AoD for the transmitter. Then, the array responses can be computed as [4]

$$\psi_R(\Omega) = \frac{1}{M_R}\left[1 \; e^{j2\pi\delta_R\Omega} \cdots e^{j2\pi(M_R-1)\delta_R\Omega}\right]^{\mathrm{T}} \quad (4)$$

$$\psi_T(\Omega) = \frac{1}{M_T}\left[1 \; e^{j2\pi\delta_T\Omega} \cdots e^{j2\pi(M_T-1)\delta_T\Omega}\right]^{\mathrm{T}} \quad (5)$$

where δ_R and δ_T are the normalized antenna spacings of the receive array and transmit array, respectively. When two signals are arriving from Ω and $\Omega + \Delta\Omega$, the angular correlation

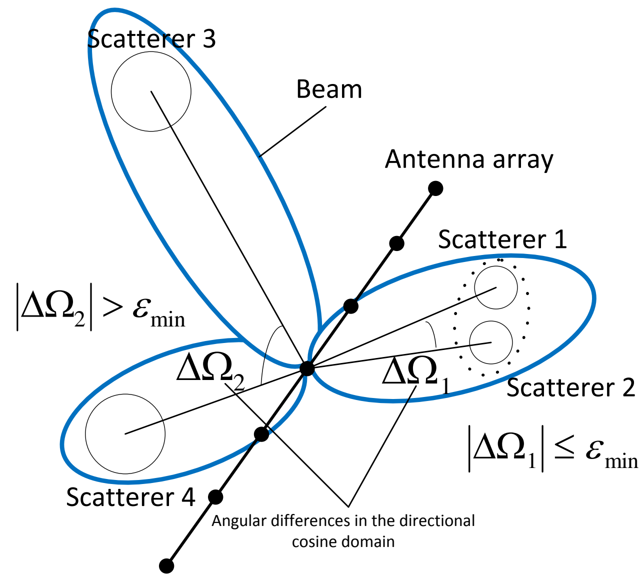

Fig. 1: An example of antenna array with finite angular resolution, resolvable scatterers, and irresolvable scatterers in the directional cosine domain.

functions of the receiver $f_R(\Delta\Omega)$ and transmitter $f_T(\Delta\Omega)$ are defined as [4]

$$f_R(\Delta\Omega) = \psi_R(\Omega)^{\mathrm{H}}\psi_R(\Omega + \Delta\Omega) = \frac{1}{M_R}\exp(j\pi\delta_R\Delta\Omega(M_R - 1))\frac{\sin(M_R\pi\delta_R\Delta\Omega)}{\sin(\pi\delta_R\Delta\Omega)} \quad (6)$$

$$f_T(\Delta\Omega) = \psi_T(\Omega)^{\mathrm{H}}\psi_T(\Omega + \Delta\Omega) = \frac{1}{M_T}\exp(j\pi\delta_T\Delta\Omega(M_T - 1))\frac{\sin(M_T\pi\delta_T\Delta\Omega)}{\sin(\pi\delta_T\Delta\Omega)} \quad (7)$$

respectively. It can be seen that $f_R(\frac{l}{M_R\delta_R}) = 0$ $(l = 1, 2, \cdots, M_R - 1)$ and $f_T(\frac{k}{M_T\delta_T}) = 0$ $(k = 1, 2, \cdots, M_T - 1)$. Therefore, the angular resolutions at the receiver and transmitter sides are $\frac{1}{M_R\delta_R}$ and $\frac{1}{M_T\delta_T}$, respectively. Let us define the minimum resolvable angular difference $\epsilon_{\min}^R$ at the receiver side and the minimum resolvable angular difference $\epsilon_{\min}^T$ at the transmitter side as one half of their angular resolutions, i.e.,

$$\epsilon_{\min}^R = \frac{1}{2M_R\delta_R} \quad (8)$$

$$\epsilon_{\min}^T = \frac{1}{2M_T\delta_T}. \quad (9)$$

Two scatterers are irresolvable at the receiver (transmitter) side if their absolute directional difference $|\Delta\Omega|$ is smaller than $\epsilon_{\min}^R$ ($\epsilon_{\min}^T$). Then, the entire two dimensional space is divided into N virtually separated parallel direction zones. As a result, N can be calculated as

$$N = \min\left\{M_R, M_T, \lfloor\frac{2}{\epsilon_{\min}^T}\rfloor, \lfloor\frac{2}{\epsilon_{\min}^R}\rfloor\right\} \tag{10}$$

where $\lfloor\cdot\rfloor$ is known as the floor function. Without loss of any generality, we use the same antenna configurations at both the transmitter and receiver, i.e., $M_T = M_R$ and $\delta_T = \delta_R$. Then (10) reduces to

$$N = \min\left\{M_R, \lfloor\frac{2}{\epsilon_{\min}^R}\rfloor\right\}. \tag{11}$$

Physical scatterers appeared within the same direction zone are not resolvable by the antenna array because of the limit of angular resolution. As a result, the number of parallel single-input single-output (SISO) channels , or equivalently the rank r of the channel, is bounded by

$$r \leq N. \tag{12}$$

III. Capacity Analysis

A. Ergodic Capacity

With the singular value decomposition (SVD), the channel matrix in (1) can be decomposed into

$$\mathbf{H} = \mathbf{V}\mathbf{\Lambda}\mathbf{U}^{\mathrm{H}} \tag{13}$$

where $\mathbf{V}$ and $\mathbf{U}$ are unitary matrices whose columns are the eigenvectors at the receiver and transmitter, respectively. Each eigenvector in $\mathbf{V}$ or $\mathbf{U}$ represents a beam direction of the antenna array. The entries of the diagonal matrix $\mathbf{\Lambda}$ are the square roots of the eigenvalues of $\mathbf{H}^{\mathrm{H}}\mathbf{H}$. Let us denote λ_i ($i = 1, \cdots, r$) as the i-th eigenvalue of $\mathbf{H}^{\mathrm{H}}\mathbf{H}$. Then, the normalized channel capacity without channel knowledge at the transmitter can be calculated as [5]

$$C = \sum_{i=1}^{r} \log_2(1 + \frac{\rho\lambda_i}{M_T}) \tag{14}$$

where ρ is the SNR. In this case, the channel capacity is equivalent to the total capacity of r parallel SISO channels [5].

Furthermore, let us assume that scatterers appear independently and uniformly in the directional cosine domain. The probability of their appearances in each of the N virtually separated parallel direction zones is equal. Let us denote the PMF of the rank r as $p_R(r)$ and the probability density function (PDF) of λ_i when rank is r as $p_{\Lambda_i}(\lambda_i|r)$. Then, from (14) the ergodic capacity of the channel can be obtained as

$$\mathrm{E}\left[C\right] = \sum_{r=1}^{N}\sum_{i=1}^{r}\left[\int_0^\infty \log_2(1 + \frac{\rho\lambda_i}{M_T}) p_{\Lambda_i}(\lambda_i|r) d\lambda_i\right] p_R(r). \tag{15}$$

It is further assumed that each virtual parallel SISO channel encounters the same power attenuation on average. Then, the mean power of a virtual parallel SISO channel X_r can be calculated as $X_r = \frac{M_T}{r}$. Assuming that the power gain of parallel channels obeys exponential distribution, the PDF of λ_i when rank is r can be acquired as

$$p_{\Lambda_i}(\lambda_i|r) = \frac{1}{X_r}\exp\left(-\frac{\lambda_i}{X_r}\right). \tag{16}$$

Then, (15) can be further expressed as

$$\mathrm{E}\left[C\right] = \sum_{r=1}^{N}\left[-\frac{r}{\ln 2}\exp\left(\frac{r}{\rho}\right)\mathrm{Ei}\left(-\frac{r}{\rho}\right)\right]\cdot p_R(r) \tag{17}$$

where $\mathrm{Ei}\left(\cdot\right)$ is the exponential integral function and is given by [18]

$$\mathrm{Ei}\left(-\frac{r}{\rho}\right) = -\int_{\frac{r}{\rho}}^{\infty}\frac{e^{-t}}{t}dt. \tag{18}$$

In order to derive $p_R(r)$, we need to calculate the ratio of the number of ways that n_s scatterers appear in N direction zones with r direction zones not empty to the total number of ways that n_s scatterers appear in N direction zones. With the assumption that scatterers appear in direction zones with equal probabilities, those falling into the same direction zone cannot be resolved by the antenna array due to its limited angular resolution. Consequently, the rank r of the channel is equivalent to the number of non-empty virtually separated parallel direction zones after randomly placing these n_s scatterers in the space. This problem is to count the number of ways of partitioning a set of n_s labeled objects into r nonempty unlabeled subsets, which can be described by the Stirling number of the second kind $\left\{\begin{matrix} n_s \\ r \end{matrix}\right\}$ and given by [17]

$$\left\{\begin{matrix} n_s \\ r \end{matrix}\right\} = \frac{1}{r!}\sum_{j=0}^{r}(-1)^j\binom{r}{j}(r-j)^{n_s} \tag{19}$$

where

$$\binom{r}{j} = \frac{r!}{j!(r-j)!} \tag{20}$$

and $x! = x(x-1)\cdots 1$. For the Stirling number of the second kind, $\left\{\begin{matrix} x \\ y \end{matrix}\right\} = 0$ if $x < y$ and $\left\{\begin{matrix} x \\ y \end{matrix}\right\} = 1$ if $x = y$, where x and y are integers.

Next, there are A_N^r ways to select r direction zones among N with orders, where A_N^r is the permutation of r objects among a set of N objects and can be given by

$$A_N^r = \frac{N!}{(N-r)!}. \tag{21}$$

Additionally, it can be obtained that the total number of ways of scatterers randomly falling into N direction zones is N^{n_s}. Hence, when the total number of scatterers is n_s, the probability that there are r of N virtually separated parallel zones with scatterers can be calculated as

$$p_R(r) = \frac{A_N^r}{N^{n_s}}\left\{\begin{matrix} n_s \\ r \end{matrix}\right\}. \tag{22}$$

It should be noticed that (22) is also the PMF of r.

B. Asymptotic Analysis

Asymptotic results are derived in this subsection, under the assumption that antenna spacings are normalized to a half of the wavelength.

1) Large-scale system analysis ($M_R \to \infty, M_T \to \infty$): When the numbers of receive and transmit antennas tend to infinity, the number of resolvable parallel zones N tends to infinity as well. In this case, the PMF $p_R(r)$ of rank can be calculated as

$$\lim_{N\to\infty} p_R(r) = \lim_{N\to\infty} \frac{A_N^r}{N^{n_s}} \left\{ \begin{matrix} n_s \\ r \end{matrix} \right\} = \left\{ \begin{matrix} n_s \\ r \end{matrix} \right\} \lim_{N\to\infty} \frac{\prod_{t=N-r+1}^{N} t}{N^{n_s}}. \tag{23}$$

When $r > n_s$, (23) equals 0 because $\left\{ \begin{matrix} n_s \\ r \end{matrix} \right\}$ is 0 [17]. When $r \leq n_s$, $\prod_{t=N-r+1}^{N} t$ has finite terms (r terms) and can be expanded as a polynomial of N with the highest order of r. Hence, $\frac{\prod_{t=N-r+1}^{N} t}{N^{n_s}}$ equals 1 if and only if $r = n_s$. Otherwise, $\frac{\prod_{t=N-r+1}^{N} t}{N^{n_s}}$ equals 0. Therefore, (23) reduces to a discrete Dirac delta function, i.e.,

$$\lim_{N\to\infty} p_R(r) = \delta[r - n_s] \tag{24}$$

where $\delta[\cdot]$ is the discrete Dirac delta function defined as

$$\delta[x] = \begin{cases} 1, & x = 0 \\ 0, & \text{otherwise.} \end{cases} \tag{25}$$

Then, the ergodic capacity in (17) becomes

$$\mathrm{E}[C] = -\frac{n_s}{\ln 2} \exp\left(\frac{n_s}{\rho}\right) \mathrm{Ei}\left(-\frac{n_s}{\rho}\right). \tag{26}$$

2) Infinite scatterer analysis ($n_s \to \infty$): When the number of scatterers n_s tends to infinity, the PMF of rank $p_R(r)$ can be calculated as

$$\begin{aligned} \lim_{n_s\to\infty} p_R(r) &= \lim_{n_s\to\infty} \frac{A_N^r}{N^{n_s}} \left\{ \begin{matrix} n_s \\ r \end{matrix} \right\} \\ &= \lim_{n_s\to\infty} \frac{A_N^r}{r!} \sum_{j=0}^{r} (-1)^j \binom{r}{j} (\frac{r-j}{N})^{n_s}. \end{aligned} \tag{27}$$

As $\frac{r-j}{N} \leq 1$, $(\frac{r-j}{N})^{n_s}$ equals 1 if and only if $r - j = N$; otherwise it equals 0. Additionally, the summation in (27) has finite terms ($r+1$ terms). Therefore, (27) reduces to a discrete Dirac delta function, i.e.,

$$\lim_{n_s\to\infty} p_R(r) = \delta[r - N]. \tag{28}$$

In this case, the ergodic capacity in (17) becomes

$$\mathrm{E}[C] = -\frac{N}{\ln 2} \exp\left(\frac{N}{\rho}\right) \mathrm{Ei}\left(-\frac{N}{\rho}\right). \tag{29}$$

3) Multiplexing gain analysis ($\rho \to \infty$): Following the definition in [4] and [9], the multiplexing gain g_{mul} can be presented as the ratio of MIMO capacity to SISO capacity, i.e.,

$$\begin{aligned} g_{\mathrm{mul}} &= \lim_{\rho\to\infty} \frac{\mathrm{E}[C]}{\log_2(\rho)} \\ &= \lim_{\rho\to\infty} \frac{\sum_{r=1}^{N} \sum_{i=1}^{r} \left[\int_0^\infty \log_2(\frac{\rho\lambda_i}{M_T}) \frac{1}{X_r} \exp\left(-\frac{\lambda_i}{X_r}\right) d\lambda_i \right] \cdot p_R(r)}{\ln 2 \log_2(\rho)} \\ &= \lim_{\rho\to\infty} \frac{\sum_{r=1}^{N} \sum_{i=1}^{r} [\log_2 \rho - \log_2 r + \mathrm{const}] \cdot p_R(r)}{\log_2(\rho)} \\ &= \lim_{\rho\to\infty} \sum_{r=1}^{N} \sum_{i=1}^{r} \left[1 + O\left(\frac{\log_2(r)}{\log_2(\rho)}\right) \right] \cdot p_R(r) \\ &= \sum_{r=1}^{N} r \cdot p_R(r). \end{aligned} \tag{30}$$

IV. Simulation Results and Analysis

In this section, simulations results are presented and analyzed. Fig. 2 depicts the angular correlation in terms of different antenna numbers in the directional cosine domain. It can be observed that as the antenna number increases, the main lobe of the angular correlation function becomes narrower and the correlations of side lobes reduce as well. These indicate that a larger number of antennas is equivalent to a better angular resolution.

Next, the PMF $p_R(r)$ of rank r for a finite number of scatters and parallel channels is illustrated in Fig. 3, where two scenarios are presented. Both curves start climbing at the beginning when the value of r is increasing. However, after they reach their own peaks, the PMFs drop gradually to zero. It should also be noticed that there is only a single peak in the PMF of rank. Also, the excellent match between analytical and simulation results demonstrates the correctness of both derivations and simulations.

Furthermore, the PMFs in asymptotic cases are depicted in Fig. 4. It can be seen that as N and n_s tend to infinity, the PMFs become discrete Dirac delta functions as derived in (23) and (27), respectively. These two theoretical results can only be approximated with large values of n_s and N.

The theoretical as well as simulated capacities of the finite scatter MIMO model are illustrated in Fig. 5. When n_s is fixed as 8, the theoretical upper bound in (26) (i.e., the number of direction zones N tends to infinity) is depicted. Then, as N increases from 6 to 170, their corresponding capacities gradually approach the upper bound. On the other hand, when N is fixed as 6, another theoretical upper bound in (29) (i.e., the number of direction zones n_s tends to infinity) is shown. As n_s increases from 8 to 40, the gaps between the corresponding capacities and the theoretical upper bound slowly reduce. Specifically, when $n_s = 40$, the gap is small enough to be omitted.

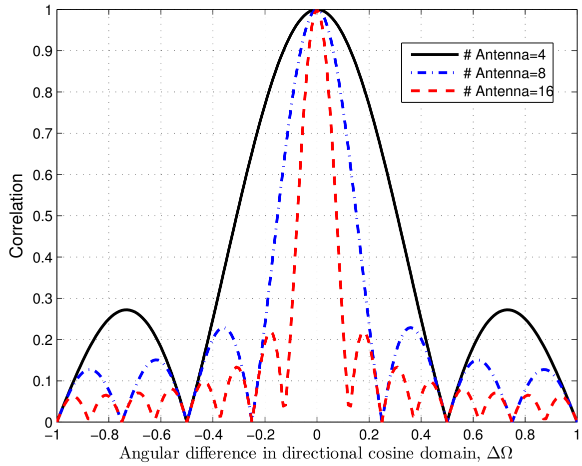

Fig. 2: Angular resolution in the directional cosine domain in terms of different antenna numbers.

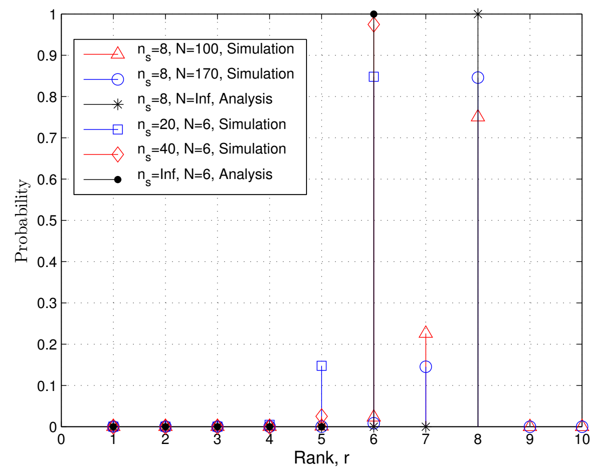

Fig. 4: The asymptotic PMF of r.

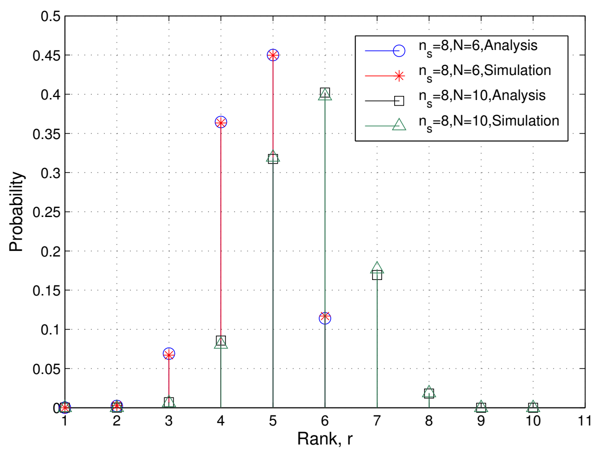

Fig. 3: The PMF of r.

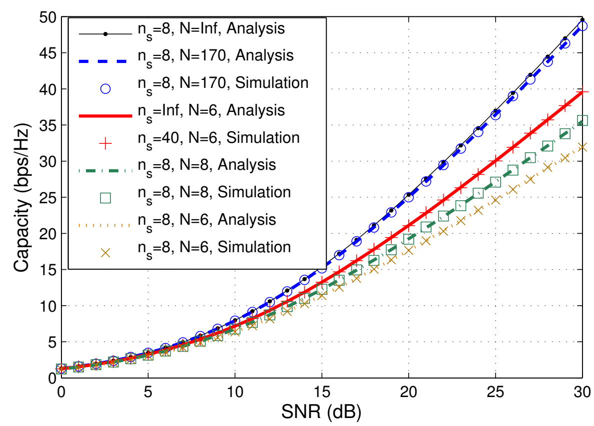

Fig. 5: The capacity of the FS MIMO channel model.

Last, the multiplexing gain of the FS MIMO model in terms of various numbers of scatterers and direction zones is presented in Fig. 6. It can be seen that the multiplexing gain is not only bounded by the number of scatterers, but also the angular resolution of antenna arrays, i.e., the number of direction zones. In low angular resolution cases, say $N \leq 5$, g_{mul} does not vary greatly for different numbers of scatterres. However, this will change as the angular resolution continues to grow. As a result, gaps can be observed and g_{mul} will be larger with more scatterers.

V. Conclusions

The capacity of the FS MIMO channel model with finite angular resolution has been studied in this paper. With the assumption of finite angular resolution, the rank of the channel is bounded by not only the number of antennas but also the number of resolvable direction zones. The closed-form PMF of the rank has been derived using the Stirling number

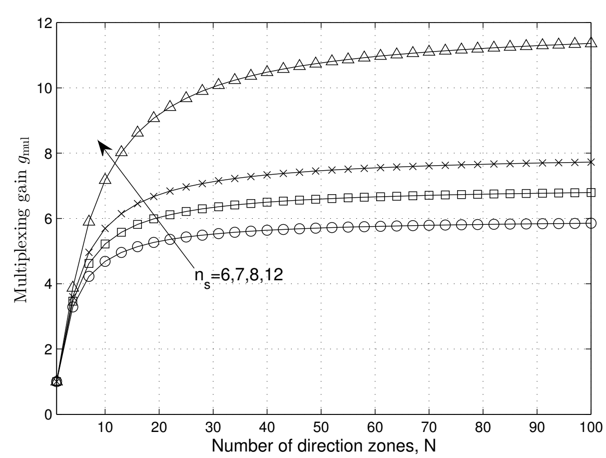

Fig. 6: The multiplexing gain of the FS MIMO channel model in terms of different scatterer numbers.

of the second kind. Thus, the capacity of the FS MIMO channel model with different numbers of scatterers and angular resolutions has been investigated based on the derived PMF of the rank. Then, it has been proved that PMFs of rank converge to discrete Dirac delta functions as the number of scatterers or direction zones tends to infinity. Based on these, asymptotic analyses such as large-scale system analysis, infinite scatter analysis, and multiplexing gain analysis have been studied. In addition, upper bounds of large-scale systems as well as infinite scatters have been provided. Finally, simulation results have shown good match with theoretical analysis, which demonstrates the correctness of both derivations and simulations.

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